Half-quantum vortices in superfluid ^{3}He-*B*

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The structure of an axially asymmetric vortex and of a disclination vortex in ^{3}He-*B* in a strong magnetic field is studied. Near the phase transition to the *A* phase and also in the superheated *B* phase, beyond the line of the phase transition to the *A* phase, these entities are bound states of two half-quantum vortices (i.e., vortices with half a quantum of circulation) which are well isolated from each other and connected by a domain wall. The distance between the half-quantum vortices in a vortex pair diverges as the line of the absolute instability of the *B* phase with respect to the formation of a planar phase is approached.

Vortices of two types are observed in superfluid ^{3}He-*B*, separated by a line of first-order phase transition (see the review article in Ref. 1). The structure of vortices with $N=1$, i.e., with a single quantum of the circulation $h/(2m_3)$, has been studied theoretically in ^{3}He-*B* only near T_c, in the region in which the Ginzburg–Landau functional is applicable.[1] Numerical analysis of this region has revealed the existence of vortices of two types: an axisymmetric vortex with *A* phase in its core[2] and an axisymmetric vortex,[3,4] which has been interpreted as a bound pair of vortices (a vortex molecule), each of which has $N=1/2$ (Ref. 5). Recent NMR experiments have clearly shown that the line of the vortex phase transition does indeed separate symmetric and asymmetric vortices.[6] On the other hand, the interpretation of the axisymmetric vortex as a pair of half-integer vortices is not a settled matter, since in the numerical solutions these half-quantum vortices are poorly separated from each other. The reason is that the distance between them in a vortex molecule has a coherence length ξ which is comparable in magnitude to the size of the core of a half-vortex.

In the present letter we show that there is a certain region of values of the external parameters (the pressure *P*, the magnetic field *H*, and the temperature *T*) in which half-integer vortices in ^{3}He-*B* are well separated within a pair. The distance between them may increase to the size of the vortex cell.

This is the region of low *P*, where strong-coupling effects—i.e., deviations from the BCS theory—are small, and the energy of the *A* phase differs only slightly from the energy of the planar (*P*) phase. Specifically, we have $\eta=(F_P-F_A)/(F_N-F_A)\ll 1$, where F_N is the energy of the normal phase. Figure 1 is a sketch of the *H–T* phase diagram. The first letter in a group specifies the stable phase; the following letters, in parentheses, specify the metastable (or locally stable) phases in order of increasing energy. The heavy line is the line of the first-order phase transition between the *B* and *A* phases, on which the equality $F_B=F_A$ holds. Above this line, the *B* phase can exist as a metastable phase. On the dashed line, its energy becomes equal to the energy of the planar phase, $F_B=F_P$. The distance between the heavy line and the dashed line is

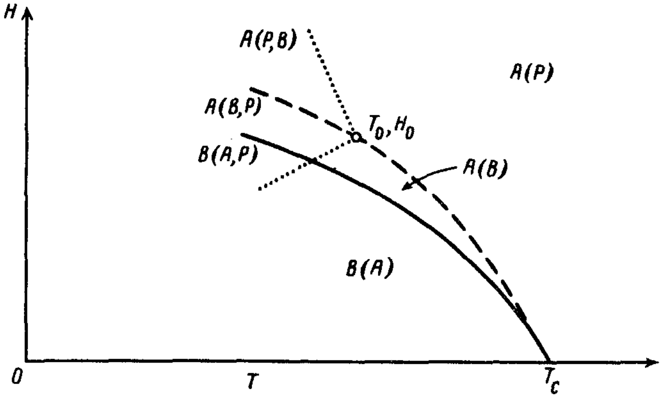

FIG. 1. Schematic H–T phase diagram of the superfluid phases of ^{3}He at low pressures. The first letter in a group of letters represents the stable phase; the following letters, in parentheses, specify the metastable (or locally stable) phases in order of increasing energy. Above the heavy line, of a first-order phase transition between the B and A phases, the B phase can exist as a metastable phase. On the dashed line, the energy of the superheated B phase becomes equal to the energy of the planar (P) phase. The distance between the heavy and dashed lines is small at low pressures to the extent that strong-coupling effects are weak. The point (T_0,H_0) is a tricritical point: At $T > T_0 \sim 0.8T_c$, the transition between the metastable B and P phases is a second-order transition, while at $T < T_0$ it is a first-order phase transition.

small to the extent that strong-coupling effects are small. According to Ref. 7, at $T > T_0$ the line of the transition between the metastable B phase and the metastable P phase is a line of a second-order transition, while at $T < T_0$ the transition is of first order. The point (T_0,H_0) is a tricritical point for these phases. The dotted lines emerging from this point are catastrophe lines, on which either the B phase or the planar phase becomes unstable. Theory[7] and experiment[8] yield $T_0 \sim 0.8T_c$ and $H_0 \sim 2$ kG.

The order parameter of the B phase in a magnetic field directed along the z axis is

$$A_{\alpha,i} = e^{i\Phi}[\Delta_1(\hat{x}_\alpha \hat{m}_i + \hat{y}_\alpha \hat{n}_i) + \Delta_2 \hat{z}_\alpha (\hat{m} \times \hat{n})_i] \ , \tag{1}$$

where Φ is the phase of the Bose condensate, and $\hat{m}$ and $\hat{n}$ are mutually perpendicular unit vectors. We are interested in the case $\Delta_2 \ll \Delta_1$, i.e., the case in which the B phase lies near the line of the second-order transition to the planar phase. According to Landau's theory for second-order transitions, the energy of the B phase is[7]

$$F_B = F_P - a(T,H)\Delta_2^2 + \frac{1}{2}b(T,H)\Delta_2^4 \ , \tag{2}$$

where $a(T,H)$ is negative in the planar phase and vanishes on the line of the transition from the planar phase to the B phase, while $b(T,H)$ is positive on the line of the second-order transition and vanishes at the tricritical point, at which we have $a(T_0,H_0) = b(T_0,H_0) = 0$. The equilibrium value $\Delta_{20}^2 = a/b$ is small not only in region $A(B)$, of a superheated B phase, beyond the line of the transition to the A phase, but also in the adjacent region of stable B phase, near the transition to the A phase at $T > T_0$. The reason is that on the line of the transition, with $F_A = F_B = F_P - (1/2)b\Delta_{20}^4$ we have $\Delta_{20}^2/\Delta_{10}^2 \sim \eta^{1/2} \ll 1$.

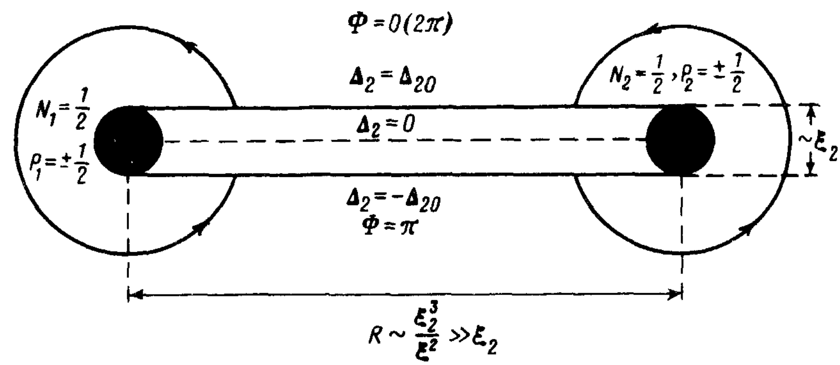

FIG. 2. Two vortices with a half-integer number ($N = 1/2$) of circulation quanta in the B phase. As each of these vortices is circumvented, the phase of the Bose condensate, Φ [and also the angle through which the order parameter rotates, not shown in this figure; see Eq. (5)], changes by π. The cut on which the phase and the angle jump by an amount π is a domain wall of thickness ξ_2, in which Δ_2 changes sign. Near the line of the transition to the planar phase, with $\xi_2 \gg \xi$, the size of the wall, R becomes large.

Half-quantum vortices in the B phase are not isolated entities, since such an isolation is forbidden by the topology. They may, on the other hand, be lines on which domain walls terminate.[9] A domain wall pulls two neighboring half-vortices together, leading to their confinement (Fig. 2). An important point is that when Δ_2 is small, the energy of the domain wall, at which Δ_2 changes sign, is small. It in fact vanishes at the transition to the P phase. Accordingly, the half-vortices connected by such a wall may lie a fair distance apart. Since this distance is greater than ξ, we can take a phenomenological (hydrodynamic) approach to find the structure of these entities.

The energy of the domain wall can be found by adding a gradient energy $(1/2)K(\partial_x \Delta_2)^2$, to Eq. (2), where x is the coordinate normal to the wall. Introducing a corresponding coherence length $\xi_2^2 = K/a(\xi_2 \to \infty$ upon the transition to the planar phase), we then find the expression

$$\Delta_2(x) = \Delta_{20} \tanh \frac{x}{\xi_2} \tag{3}$$

for the domain wall, and we find the energy per unit area to be

$$F_{wall} = \frac{8}{3}(F_P - F_B)\xi_2 \sim (F_N - F_B)\xi(\frac{\xi}{\xi_2})^3 \quad , \tag{4}$$

It follows that the energy of such a wall is smaller by $(\xi_2/\xi)^3$ than in the usual case, in which its size is on the order of the ordinary coherence length ξ.

We now consider the energy of a complex containing two half-vortices, separated by a distance R and linked by a domain wall. Each of the half-quantum vortices has the following structure. The condensate phase Φ and the unit vectors $\hat{m}$ and $\hat{n}$ vary in accordance with

$$\Phi = N\phi \quad , \quad \hat{m} = \hat{m}_0 \cos p\phi + \hat{n}_0 \sin p\phi \quad , \quad \hat{n} = \hat{n}_0 \cos p\phi - \hat{m}_0 \sin p\phi \ , \tag{5}$$

upon a circumvention of a half-vortex; here ϕ is the azimuthal angle around the axis of a half-vortex, and N (the number of circulation quanta) and the disclination index p are half-integers ($\pm 1/2$). As a half-vortex is circumvented, only the sign of the term with Δ_2 changes in the order parameter in Eq. (1), but this change is exactly canceled by a change in the sign Δ_2 during passage through the domain wall glued to the vortex.

The R dependence of the energy per unit length of this complex is the sum of the energy of the wall pulling the vortices together and the logarithmic "Coulomb" energy of the interaction of the half-vortices with charges N_1, N_2 and p_1, p_2:

$$F(R) = \frac{8}{3}(F_P - F_B)\xi_2 R - \pi\left(\frac{\hbar}{2m_3}\right)^2(\rho_s N_1 N_2 + \rho_{sp} p_1 p_2) \ln R \ . \tag{6}$$

Here ρ_s and ρ_{sp} are respectively the superfluid density and the spin rigidity. Far from T_c the relation $\rho_s > \rho_{sp}$ holds. The complex has a local minimum as a function of R if the charges N of the half-vortices have the same sign: $N_1 = N_2 = \pm 1/2$. In other words, the overall entity has a quantum of circulation, $N = N_1 + N_2 = \pm 1$. The overall "Coulomb" interaction is then repulsive; this situation, combined with the pulling effect of the wall, results in a local equilibrium. The equilibrium size of the complex, R, given by

$$R = \frac{3\pi}{32}\left(\frac{\hbar}{2m_3}\right)^2 \frac{\rho_s \pm \rho_{sp}}{(F_P - F_B)\xi_2} \sim \frac{\xi_2^3}{\xi^2} \gg \xi_2 \gg \xi \tag{7}$$

may be substantially greater than both the coherence length and the thickness of the domain wall, ξ_2, especially near the transition to the planar phase, where we have $\xi_2 \to \infty$. The minus sign in (7) refers to the case of opposite charges, $p_1 = -p_2$, and the plus sign to the case of like charges.

The first case describes a pure vortex with $N = 1$. This is a greatly stretched-out modification of the axially asymmetric vortex with two cores which was found through a numerical analysis in Ref. 3 and 4. The spatial separation of the half-quantum vortices constituting the asymmetric vortex is much more apparent in this case. The second case, with a nonzero disclination index, $p = p_1 + p_2 = \pm 1$, describes a combination of a vortex with a disclination. A similar entity with two cores was found through a numerical analysis in Ref. 10 near T_c in a zero field. We can identify each of the cores in this numerical solution with a half-quantum vortex. We then see why the distance T between the cores in the vortex-disclination in the numerical solution[20] ($\sim 20\xi$) is much greater than the distance R between the cores in a pure vortex ($\sim 5\xi$). The explanation follows from (7) when we note that we have $\rho_s + \rho_{sp} \gg \rho_s - \rho_{sp}$. In our case, of an extremely large R, we also see why a combination of a vortex with a disclination is locally stable with respect to decay into a vortex and a disclination separated from each other. For such a dissociation to occur, there must be a tearing of the domain wall, accompanied by the formation of a pair of new half-vortices (Fig. 3). This process requires the surmounting of an energy barrier.

The region of low pressures and strong fields should also be very interesting in the

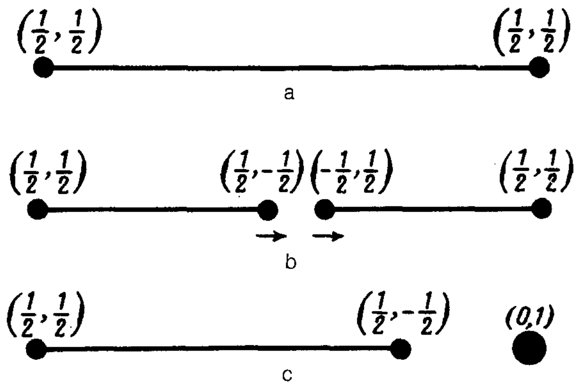

FIG. 3. Dissociation of a composite entity (Fig. 3a) consisting of a vortex and a disclination with $(N,p) = (1,1)$ (N is the vortex charge, and p the disclination charge). This entity consists of a pair of identical half-vortices, each of which has a half-integer charge: $(N/p) = (1/2,1/2)$. In the intermediate state (Fig. 3b), the domain wall is ruptured, and a pair of new half-vortices forms, with mutually opposite charges, $(N,p) = (1/2), -1/2)$ and $(N,p) = (1/2, -1/2)$. Their subsequent motion is indicated by the arrows. As a result (Fig. 3c), a disclination with $(N,p) = (0,1)$ and a vortex with $(N,p) = (1,0)$ form. They are separated from each other. The vortex consists of a pair of half-vortices with charges $(N,p) = (1/2,1/2)$ and $(N,p) = (1/2, -1/2)$.

experimental arena. In particular, the B phase would have to be superheated only slightly to reach the metastable planar phase, which has yet to be produced in a volume. In addition, according to the experiments of Ref. 11, one cannot rule out the possibility that a stable new phase exists. It was shown in Ref. 11 that a governing combination of coefficients of the fourth-order terms in the Ginzburg–Landau functional near T_c, $\beta_2 + \beta_4 + 2\beta_5$, crosses zero at $P = P_0 \sim 1$ atm. According to the one-parameter paramagnon model of strong-coupling effects,[12] this combination vanishes at the same time as $F_A - F_P$. If, on the other hand, we have $F_P < F_A$, then a so-called axi-planar phase,[13] which is an intermediate phase between the A and P phases, has an even lower energy according to the same model. Consequently, if that model is correct, then in strong fields at $P < P_0$ we should expect the existence of a stable axi-planar phase.

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