Structure of surface layer in superfluid ^{3}He-B

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Four types of surface states are described at a boundary between ^{3}He-B and a vessel near the transition to ^{3}He-A. In this region, these states are interfaces [(^{3}He-B) — (^{3}He-A) or (^{3}He-B) — (planar phase) interfaces] which are pinned by the boundary of the vessel.

The structure of the order parameter in ^{3}He-B near the boundary with the vessel was studied in Refs. 1–7. In addition to the ordinary symmetric surface state,[1–3] which is a layer with a thickness on the order of the coherence length ξ, in which the B phase in the volume undergoes a continuous transition into the planar phase (P) at the boundary, other possible surface states,[4–7] with a lowered symmetry, have been considered in efforts to explain several unusual experimental results.[8–11] In these states, the B phase undergoes a transition not to the planar phase but to another phase, in particular, the A phase, at the boundary. The structure of the surface layer is usually found through an elaborate numerical analysis of either the Ginzburg–Landau equations for a multicomponent order parameter or of equations for semiclassical Green's functions. Consequently, a physical interpretation is only rarely possible. An exceptional case is the structure which was found analytically in Ref. 7 near the transition to the A phase. That structure consists of a well-defined interface between A and B phases. It separates the B phase in the volume from a fairly wide surface layer of A phase.

Below we show that there are regions of the temperature, the pressure, and the magnetic field in which all four of the most important surface structures—P^1, P^2, A^1, and A^2 (Figs. 1a–1d)—can be determined analytically. Each of these structures is an interface near the boundary of the vessel. In the P^1 and P^2 structures, there is planar phase between the boundary of the vessel and the interface, while in structures A^1 and A^2 there is A phase in this layer. Structures with a superscript 2 differ from those with a 1 in that there is a nonuniform texture of the orbital vector l in the layer of the P or A phase. This vector changes from an orientation normal to the layer at the boundary of

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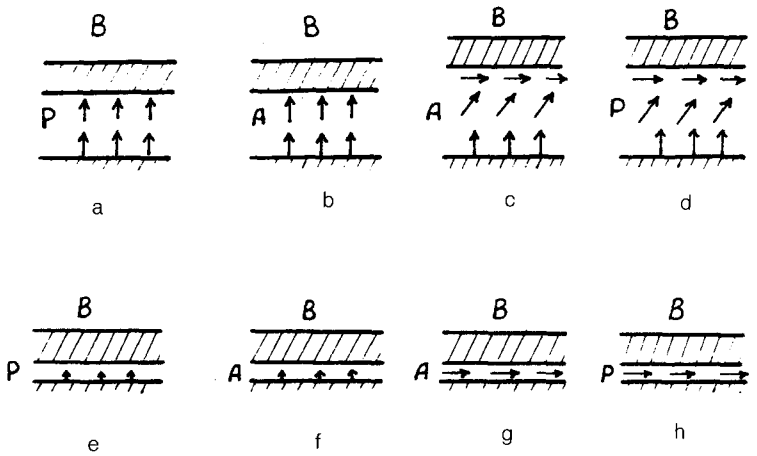

FIG. 1. The surface states of superfluid ^{3}He-B can, in certain situations, be represented as a wide surface layer of a planar (P) or A phase which is separated from the bulk B phase by a well-defined interface. a—Structure P^1; b—A^1; c—A^2; d—P^2. In states P^2 and A^2 in parts c and d, there is a nonuniform texture of the orbital vector l in the surface layer (as shown by the arrows). The state $\tilde{P}^1$ in part e has the same symmetry as P^1 in part a, but the interface is glued to the surface. f—Structure $\tilde{A}^1$; g—$\tilde{A}^2$; h—$\tilde{P}^2$. The symmetry of this state is higher than that of structure P^2, since there is no texture of the vector $\vec{l}$.

the vessel to a tangential orientation at the surface of the interface. Structures P^1, P^2, A^1, and A^2 differ in symmetry (in the classification scheme of Ref. 5, their symmetry classes are 17, 8, 11, and 3, respectively), so in principle they might also exist outside the regions in which they were determined analytically. Structure A^2 was found in Ref. 7. The correspondence between the states found through the numerical analysis and the structures which we are discussing here is discussed below. The structure P^2 is seen here for the first time.

The reason for the existence of these structures is that the b phase does not wet the boundary of the vessel, so a layer of a different phase should arise near the boundary. This other phase will be either the planar phase or the A phase, since other wettable phases have a higher energy. The width of the layer of this other phase depends on the proximity of the first-order transition between this phase and ^{3}He-B. We first consider surface layers of the A^1 and A^2 types near the line of a first-order phase transition between the B and A phases, with a small difference between the condensation energies of the A and B phases: $F_A - F_B \ll |F_B|$. There are two competing structures because there are two types of interfaces between the A and B phases.[12,13] These two types of interfaces differ in the orientation of the orbital vector $\vec{l}$. The AB boundary with a vector $\vec{l}$ lying in the plane of the boundary has an energy lower than that of an AB boundary with $\vec{l}$ perpendicular to the boundary:[12,13] $\sigma_{\parallel}^{AB} < \sigma_{\perp}^{AB}$ (near T_c we have $\sigma_{\parallel}^{AB} \sim 1.2\xi |F_B|$ and $\sigma_{\perp}^{AB} \sim 1.5\xi |F_B|$; Ref. 13). As a result, structure A^2 is preferred. This situation may be offset, however, by the circumstance that the A^2 structure should have a nonuniform texture of the vector $\vec{l}$ (Fig. 1c).

and thus a related additional energy, because of the normal boundary conditions for the vector $\vec{l}$ at the surface of the vessel.

Let us find the energy of structures A^1 and A^2 as a function of the proximity to the AB transition. The energy of a surface layer with a distance L between the AB boundary and the surface of the vessel is the sum of the energy of the corresponding AB boundary, the energy of the interaction of the AB boundary with the surface, and the difference between the energies of the A and B phases in the surface layer of A phase. In structure A^1, the repulsion of the AB boundary from the surface is of an exponential nature, because of the exponential decay far from the AB of those components of the order parameter boundary which correspond to deviations from the A phase. In structure A^2, this interaction is of a power-law nature, because of the gradient of the vector l (the gradient energy in a layer of thickness L is inversely proportional to L). As a result, we find the following expressions for the energies of the structures:

$$\sigma(A^1, L) = \sigma_{\perp}^{AB} + L(F_A - F_B) + a|F_B|\xi \exp(-b\frac{L}{\xi}) \ , \tag{1}$$

$$\sigma(A^2, L) = \sigma_{\parallel}^{AB} + L(F_A - F_B) + c|F_B|\xi\frac{\xi}{L} \ . \tag{2}$$

The dimensionless parameters a,b, and c are on the order of unity; the parameter c was found in Ref. 7. A minimization with respect to L leads to the equilibrium layer thicknesses

$$L_1 \sim \xi \ln \frac{|F_B|}{F_A - F_B} \quad , \quad L_2 \sim \xi\sqrt{\frac{|F_B|}{F_A - F_B}} \tag{3}$$

and energies

$$\sigma(A^1) = \sigma_{\perp}^{AB} + b^{-1}\xi(F_A - F_B)\ln\frac{|F_B|}{F_A - F_B} \ , \tag{4}$$

$$\sigma(A^2) = \sigma_{\parallel}^{AB} + 2\xi\sqrt{c|F_b|(F_A - F_B)} \ . \tag{5}$$

Since we have $\sigma_{\parallel}^{AB} < \sigma_{\perp}^{AB}$, structure A^2 becomes preferable if the transition to the A phase is close enough, while it follows from the numerical analysis of Ref. 4 that structure A^1 is preferable far from the AB transition but near T_c. Elsewhere on the phase diagram of ^{3}He, the situation is not yet clear.

We turn now to layers with a planar phase. For the P^1 and P^2 layers, with a well-developed planar phase, the first-order phase transition between the B and P phases must be close. This is the situation at low pressures in a strong magnetic field $H > 2$ mT, where the transition from the B phase to the A phase is very close to the first-order phase transition from B to P, but it occurs earlier (see Fig. 1 in Ref. 14). In this region, near the AB transition, we thus have $F_P - F_B \ll |F_B|$, so everything that was said regarding the A phase can be extended to the planar phase. The vector $\vec{l}$ in the

planar phase (and also in a B phase distorted by a magnetic field) appears in the matrix order parameter for the P and B phases in the following form (the field is along the z axis):

$$A^P_{\alpha i} = \Delta_P(\hat{x}_\alpha \hat{m}_i + \hat{y}_\alpha \hat{n}_i) \quad , A^B_{\alpha i} = \Delta_1(\hat{x}_\alpha \hat{m}_i + \hat{y}_\alpha \hat{n}_i) \; + \Delta_2 \hat{z}_\alpha \vec{l}_i \; , \tag{6}$$

where $\hat{m}$ and $\hat{n}$ are two unit vectors of an orbital coordinate system with $\vec{l} = \hat{m} \times \hat{n}$.

As in the case of the A phase, there are interfaces of two types between the B and P phases, with $\sigma^{PB}_{\parallel} < \sigma^{PB}_{\perp}$, at least if we are not too far from T_c. Similarly, the energies of the two corresponding structures, P^1 and P^2 in Figs. 1a and 1b, are

$$\sigma(P^1) = \sigma^{PB}_{\perp} + b^{-1}\xi(F_P - F_B)\ln\frac{|F_B|}{F_P - F_B} \; , \tag{7}$$

$$\sigma(P^2) = \sigma^{PB}_{\parallel} + 2\xi\sqrt{c|F_b|(F_P - F_B)} \; . \tag{8}$$

Again, if we are sufficiently close to the AB boundary, structure P^2 should become preferable to the higher-symmetry structure P^1.

How do the surface states found previously through a numerical analysis near T_c relate to those found here near T_{AB}? The state with A phase at the boundary found by Thuneberg[4] corresponds precisely to structure A^1, since the numerical solution revealed a logarithmic growth of the thickness of the A-phase layer toward T_{AB} (Ref. 6), in agreement with expression (3) for L_1. The most symmetric solution with a planar phase at the surface,[4] which has the lowest energy of any of the states near T_c, may in general not correspond to structure P^1 with the same symmetry, since there is no common range of applicability. The situation is that the exponential repulsion of the PB boundary away from the vessel surface which necessarily occurs at large values $L \gg \xi$ may give way to an attraction at $L \sim \xi$. There is thus the possibility in principle of another bound state of the PB boundary at the wall, $\tilde{P}^1$, with the same symmetry, but of small size $L \sim \xi$ (Fig. 1e). Into which of these two structures, P^1 or $\tilde{P}^1$, the numerical solution converts remains an open question. The state found in Ref. 6 in a modified phenomenological Ginzburg–Landau theory, with a poorly defined A phase at the boundary and with the same symmetry as A^1, quite probably corresponds to structure $\tilde{A}^1$ (i.e., a structure with an AB boundary glued to the vessel surface; Fig. 1f). The metastable state of class 12, which was found in Ref. 5, corresponds to structure $\tilde{P}^2$, in which the PB boundary with a tangential $\vec{l}$ is glued to the surface of the vessel (Fig. 1h). Because of the disruption of the boundary conditions on the vector l, there is a pronounced deformation of the order parameter of the P phase, with the result that this phase converts into a polar phase at the vessel surface.

In summary, we now have solutions (numerical or analytic) for six or seven surface structures: A^1, P^1, A^2, P^2, $\tilde{A}^1$, $\tilde{P}^2$, and possibly $\tilde{P}^1$.

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