Effect of compressed light on pumping of a crystal

R. G. Nazmitdinov and A. V. Chizhov
Joint Institute for Nuclear Research, Dubna

(Submitted 5 June 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 7, 993–996 (10 October 1990)

The dependence of quantum-fluctuation and statistical properties of polaritons on the initial conditions and interaction parameters has been established.

Recent progress in creating sources which can generate light in a compressed state[1,2] has made it possible to use the unique properties of light in solid-state spectroscopy.

In the present letter we will analyze the basic effects produced when a strong light in the compressed state (pumping) is directed on phonon excitations in a crystal.

A simple Hamiltonian of the interaction of monochromatic light with a crystal is[3]

$$H = \hbar\omega a^+ a + \sum_q \{\hbar\Omega_q b_q^+ b_q - i\hbar D_q [a^+(b_q + b_q^+) - a(b_q + b_q^+)]\}, \tag{1}$$

where a^+ (a) are the operators for creating (or annihilating) a photon, and b^+ (b) are the corresponding operators of the transverse phonons. For simplicity, we will restrict the analysis to an isotropic crystal, in which the frequencies Ω_q and the coupling constant D_q do not depend on the polarization. In terms of the mathematical structure, Hamiltonian (1) is a generalization of several standard models which are used in quantum optics to study the properties of the compressed state of light.[4]

Let us assume that at zero time t_0 the light is in the compressed state

$$|\alpha, \xi\rangle = D(\alpha)S(\xi)\,|0\rangle,$$

$$S(\xi) = \exp\{\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi a^{+2}\}, \quad D(\alpha) = \exp\{\alpha a^+ - \alpha^* a\}$$

with an average number of photons

$$\langle a^+ a\rangle_0 = \sinh^2|\xi| + |\alpha|^2,$$

while the phonons are in the state of thermal equilibrium which is described by the density matrix

$$\rho = \prod_q \frac{\langle b_q^+ b_q\rangle_0^{b_q^+ b_q}}{(1 + \langle b_q^+ b_q\rangle_0)^{1 + b_q^+ b_q}},$$

where

$$\langle b_q^+ b_q\rangle_0 = \frac{1}{\exp\{\hbar\Omega_q / k_B T\} - 1},$$

here k_B is the Boltzmann's constant, and T is the temperature.

We will analyze the evolution of the states of system (1), in particular, the possibility for the transition of phonons to the compressed state and the sub-Poisson state. In the sub-Poisson state fewer than the average number of photons $\langle n\rangle$ are dispersed $\langle(\Delta n)^2\rangle$. To determine the nature of the state, we must calculate the time-dependent dispersions of the generalized pulses $\langle(\Delta P_q)^2\rangle$ and coordinates $\langle(\Delta Q_q)^2\rangle$

$$\langle(\Delta P_q)^2\rangle \equiv \langle P_q^2\rangle - \langle P_q\rangle^2, \quad \langle(\Delta Q_q)^2\rangle \equiv \langle Q_q^2\rangle - \langle Q_q\rangle^2,$$
$$P_q(t) = i\,\frac{b_q^+(t) - b_q(t)}{2}, \quad Q_q(t) = \frac{b_q^+(t) + b_q(t)}{2}, \tag{2}$$

and also the second-order degree of coherence[4]

$$g^{(2)}(t, t+\tau) = \frac{\langle b_q^+(t) b_q^+(t+\tau) b_q(t+\tau) b_q(t)\rangle}{\langle b_q^+(t) b_q(t)\rangle \langle b_q^+(t+\tau) b_q(t+\tau)\rangle}\,. \tag{3}$$

Here the operators are considered in the Heisenberg representation, and the average is taken along with the initial-state statistical operator. The compressed state corresponds to the conditions

$$\langle(\Delta O)^2\rangle < \frac{1}{4},$$

where $O \equiv P_q$ or Q_q, and the sub-Poisson state corresponds to the conditions

$$g^{(2)}(t, t+\tau) < 1.$$

The explicit time dependence of the operators $b(b^+)$ is determined by means of a canonical transformation which reduces Hamiltonian (1) to the diagonal form:

$$b_q(t) = U^*\alpha(t) - V\alpha^+(t) + \sum_p (G_{qp}^*\beta_p(t) - F_{qp}\beta_p^+(t))$$
$$= u(t)a(0) + v(t)a^+(0) + \sum_p (g_{qp}(t) b_p(0) + f_{qp}(t) b_p^+(0)),$$

where the coefficients u, v, g_{qp}, and f_{qp} are functions which depend on the parameters of the interaction between the Hamiltonian and the time and can easily be determined by direct calculation. Since the dominant process in this case is the resonant interaction of light with the phonons,[5] we restrict the analysis in these calculations to the terms with $\mathbf{q} = \mathbf{k}$, where $\mathbf{k}$ is the wave vector of the pump field.

The results of calculations, shown in Fig. 1, indicate that compression occurs both in the generalized coordinate and in the generalized phonon momentum during the evolution of the system. In other words, the states in which fluctuations of the given quantities are suppressed in comparison with the vacuum state appear in the system at certain times, suggesting that a certain kind of ordering of the vibrational motion of the crystal ions occurs. Calculation of the correlation function (3) shows (see Fig. 2) that an interaction with compressed light changes the statistical properties of phonons. Starting with the disordered (super-Poisson) state, the system may undergo a transition to the sub-Poisson state at certain intervals of time in the course of

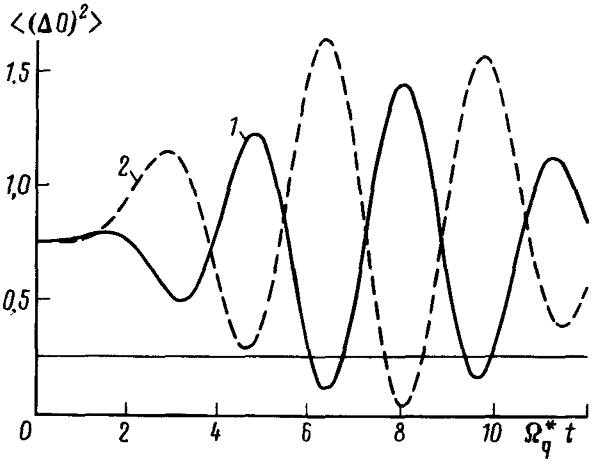

FIG. 1. Time evolution of the dispersions of the generalized coordinate (1) and the generalized pulse (2) for the parameters of the system $\omega = \Omega_q = 2.5$; $D_q = 0.5$, and the initial states of the fields $\langle a^+ a \rangle_0 = 101$ ($\xi = \text{Arsh}$, $\alpha = 10$); $\langle b_q^+ b_q \rangle_0 = 1$ ($\hbar\Omega_q / k_B T = \ln 2$).

the process. The compressed state, in this case, corresponds to the sub-Poisson behavior of the correlation function (3) (see Figs. 1 and 2). Since the fluctuation in the number of photons is very slight in the sub-Poisson state,[4] i.e., it is close to the Fock state, the system tends to the phonon condensate state (at the resonance frequency). A transition of the phonons to such a state due to the photon-phonon correlation can be traced from the behavior of the luminescence.

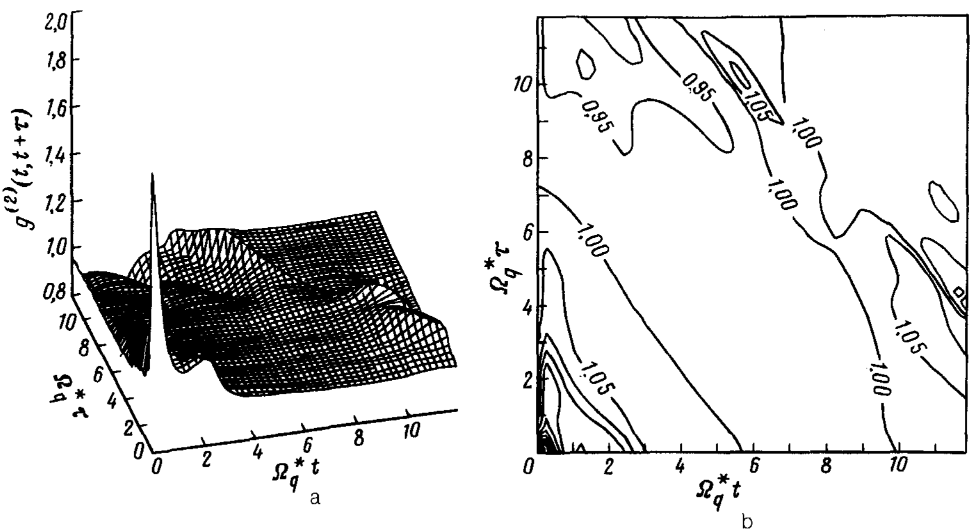

FIG. 2. Correlation phonon function versus (a) time and the time-delay and (b) its topogram for the same parameters as in Fig. 1.

We express our deep appreciation to A. S. Shumovsky for useful discussions and for support.

[1] R. E. Slusher *et al.*, Phys. Rev. Lett. **55**, 2409 (1985).
[2] L. Wu *et al.*, Phys. Lett. **7**, 2520 (1986).
[3] Yu. A. Il'inskiĭ and L. V. Keldysh, *Interaction of Electromagnetic Radiation With Matter*, Moscow State University, Moscow (1989).
[4] Ya. Perina, *Quantum Statistics of Linear and Nonlinear Optical Phenomena*, Mir, Moscow (1987); S. Ya. Kilin, *Quantum Optics. Fields and Their Detection*, Nauka i tekhnika, Minsk (1990).
[5] V. M. Agranovich and V. L. Ginzburg, *Crystal Optics, Spatial Dispersion, and Exciton Theory*, Nauka, Moscow (1965).

Translated by S. J. Amoretty