Decay of nonsingular vortex in superfluid ^{3}He-A near the transition to the A_1 phase

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Vortices in ^{3}He-A in a strong magnetic field near the transition to the A_1 phase are discussed. It is shown that a nonsingular vortex with two quanta of circulation splits into two nonsingular single-quantum Mermin-Ho vortices. The distance between the Mermin-Ho vortices in a vortex pair diverges near the transition to the A_1 phase. The possibility for observing this effect in experiments with ultrasound is discussed.

A substantial body of experimental data on the nature of quantized vortices in the superfluid phases of ^{3}He has been compiled over the past decade.[1–3] It has been shown that nonsingular quantized vortices exist in ^{3}He-A. In these vortices, a singularity at the vortex axis dissipates as a result of the topologically nontrivial texture of the orbital vector $\vec{l}$ within the soft vortex core. Experiments with ultrasound have revealed

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the existence of nonsingular vortices of two types.[4] The substantial difference between the soft cores of these vortices is determined by the tiny spin-orbit (dipole) interaction between the orbital vector $\vec{l}$ and the spin vector $\vec{d}$ of the matrix order parameter. In weak magnetic fields $H < H_d$, where $H_d \sim 2$–5 mT, a vortex occurs. In this vortex the vector $\vec{d}$ is linked by the dipole interaction with the vector $\vec{l}$ (i.e., it is dipole-locked), so it reproduces the nontrivial topology of this vector. In a vortex which occurs in stronger fields, $H > H_d$, the vector $\vec{d}$ detaches from the vector $\vec{l}$ (becomes dipole-unlocked), and its topology in the vortex becomes trivial. The latter vortex, with two quanta of circulation and a soft core on the order of the dipole length[5] $\xi_d \sim 10\ \mu$m (a so-called Seppälä-Volovik vortex) has been the subject of very active research in MNR experiments.[6]

In the present letter we show that in very strong fields, near the second-order phase transition to the A_1 phase, where the nature of the dipole forces changes, there should be a substantial restructuring of a Seppälä-Volovik vortex. This vortex will split into two Mermin-Ho textures. Each of these nonsingular textures is a combination of a vortex with one quantum of circulation and a disclination in the field of the vector $\vec{l}$; it has a dipole-unlocked soft core. The distance between these cores becomes infinite as the A_1 phase is approached.

The order parameter of the A phase, the 3×3 matrix $A_{\alpha i}$, is distorted by a magnetic field. It contains two different complex gaps, $\Delta_\uparrow$ and $\Delta_\downarrow$, for each of two spin components, along ($\uparrow$) and opposite ($\downarrow$) the magnetic field.[7,8] This distorted state is frequently called the "A_2 phase" to distinguish it from the A_1 phase, in which one of the gaps is not present ($\Delta_\uparrow = 0$). The equilibrium order parameter for the A_2 and A_1 phases is directed along the z axis in a magnetic field and is given by

$$A_{\alpha i} = \frac{1}{2}(\hat{e}_1 + i\hat{e}_2)_i[\Delta_\uparrow(\hat{x} + i\hat{y})_\alpha + \Delta_\downarrow(\hat{x} - i\hat{y})_\alpha] \ , \qquad (1)$$

where the vectors with Latin indices, $\hat{e}_1$ and $\hat{e}_2$, are orbital vectors. Their vector product is the vector $\vec{l} = \hat{e}_1 \times \hat{e}_2$. The Greek indices correspond to the spin degrees of freedom. In the A_1 phase, which exists in the interval $T_{c2} < T < T_{c1}$, there is only one gap, $\Delta_\downarrow \neq 0$. Below T_{c2}, in the A_2 phase, both gaps are present, and $|\Delta_\uparrow|$ varies continuously from 0 at T_{c2} to $|\Delta_\downarrow|$ far from T_{c2}, i.e., under the condition $T_{c2} - T \gg T_{c1} - T_{c2}$, where the magnetic distortion of the order parameter is small, and the A-phase structure with $|\Delta_\uparrow| \cong |\Delta_\downarrow| = \Delta_A$ is restored.

Introducing the common phase and the relative phase of the gaps, $\Delta_\downarrow = |\Delta_\downarrow| e^{i(\Phi+\alpha)}$ and $\Delta_\uparrow = |\Delta_\uparrow| e^{i(\Phi-\alpha)}$, we can write the spinor part of the order parameter in terms of a pair of orthogonal unit vectors:

$$\hat{d}_1 = \hat{x}\cos\alpha - \hat{y}\sin\alpha \ , \quad \hat{d}_2 = \hat{y}\cos\alpha + \hat{x}\sin\alpha \ . \qquad (2)$$

These vectors are oriented perpendicular to the magnetic field. They describe a biaxial magnetic anisotropy of the A_2 phase (in contrast with the uniaxial anisotropy of the pure A phase):

$$A_{\alpha i} = \frac{1}{2}e^{i\Phi}(\hat{e}_1 + i\hat{e}_2)_i[(|\Delta_\uparrow| + |\Delta_\downarrow|)\hat{d}_{1\alpha} + i(|\Delta_\downarrow| - |\Delta_\uparrow|)\hat{d}_{2\alpha}] \ . \qquad (3)$$

The dipole energies for the A_2 and A_1 phases are, in the notation of the review in Ref. 1,

$$F_d = g_d(A^*_{ij}A_{ji} + A^*_{ii}A_{jj})$$

$$= -\frac{1}{2}g_d[(|\Delta_\uparrow| + |\Delta_\downarrow|)^2(\hat{d}_1\cdot\vec{l})^2 + (|\Delta_\downarrow| - |\Delta_\uparrow|)^2(\hat{d}_2\cdot\vec{l})^2]$$

$$= -\frac{1}{2}g_d(|\Delta_\uparrow| + |\Delta_\downarrow|)^2\sin^2\eta + 2g_d|\Delta_\uparrow||\Delta_\downarrow|\sin^2\eta\ \sin^2\gamma\ , \tag{4}$$

where η and γ are the polar and azimuthal angles of the vector $\vec{l}$:

$$\vec{l} = \hat{z}\cos\eta + \sin\eta\ \ (\hat{d}_1\cos\gamma + \hat{d}_2\sin\gamma)\ . \tag{5}$$

It follows from Eq. (4) that very close to T_{c2}, with $|\Delta_\uparrow| \ll |\Delta_\downarrow|$, there are two very different length scales of the dipole interaction. The interaction which puts the vector $\vec{l}$ in the plane perpendicular to the magnetic field ($\eta = \pi/2$) is substantially stronger than the interaction which orients the vector $\vec{l}$ along the spin vector $\vec{d}_1$ in this plane ($\gamma = 0$).

Correspondingly, there are two dipole lengths, ξ_{d1} and ξ_{d2}, which can be found from (4) and the gradient energy

$$F_G = \gamma_0(\partial_i A^*_{\alpha j}\partial_i A_{\alpha j} + \partial_i A^*_{\alpha i}\partial_j A_{\alpha j} + \partial_i A^*_{\alpha j}\partial_j A_{\alpha i})\ . \tag{6}$$

Using the notation[1] $\xi_d = (\gamma_0/g_d)^{1/2}$, we write

$$\xi_{d1} = \xi_d\sqrt{\frac{2}{1+\beta}}\ ,\ \ \xi_{d2} = \frac{\xi_d}{\sqrt{\beta}}\ ,\ \text{where}\ \ \beta = \frac{2|\Delta_\uparrow||\Delta_\downarrow|}{|\Delta_\uparrow|^2 + |\Delta_\downarrow|^2}\ . \tag{7}$$

The quantity ξ_{d2} diverges as the A_1 phase is approached ($\beta\to 0$), while ξ_{d1} increases, from ξ_d in the A phase (far from T_{c2}, with $\beta \approx 1$) to $\sqrt{2}\xi_d$ in the A_1 phase ($\beta = 0$).

The structure of the soft vortex core in the A_2 phase forms as a result of a competition between the gradient energy and the dipole energy. We write these energies in terms of the slow hydrodynamic variables; the superfluid velocity $\vec{v}_s = (\hbar/2m_3)\vec{v}$, $\vec{v} = \vec{\nabla}\Phi + e_{1i}\nabla e_{2i}$ and the vector $\vec{l}$:

$$F_G + F_d = \frac{1}{2}\rho_{s\|}\left(\frac{\hbar}{2m_3}\right)^2\{2\vec{v}^2 - (\vec{v}\cdot\vec{l})^2 + \frac{1}{2}(\partial_i\vec{l})^2 + ((\vec{l}\cdot\vec{\nabla})\vec{l})^2$$

$$+\vec{v}\cdot(\vec{\nabla}\times\vec{l}) - 2(\vec{l}\cdot\vec{v})(\vec{l}\cdot(\vec{\nabla}\times\vec{l})) - \xi_{d1}^{-2}\sin^2\eta + \xi_{d2}^{-2}\sin^2\eta\sin^2\gamma\}\ , \tag{8}$$

where the longitudinal component of the density tensor of the superfluid component is

$$\rho_{s\|} = 2\gamma_0(|\Delta_\uparrow|^2 + |\Delta_\downarrow|^2)\left(\frac{2m_3}{\hbar}\right)^2\ . \tag{9}$$

Far from T_{c2}, the two-quantum Seppälä-Volovik vortices have a well-defined two-core structure.[6] Zotos and Maki[9] have interpreted this structure as a bound pair of two analytic (i.e., nonsingular) single-quantum Mermin-Ho vortices.[10] Outside the cores of the Mermin-Ho vortices, the vector $\vec{l}$ is essentially fixed in the plane perpendicular to $\vec{H}$, while inside the cores $\vec{l}$ swings out of the plane and traces out half a unit sphere in each of the two cores. These Mermin-Ho vortices have an identical vortex index, $N_1 = N_2 = 1$, but opposite indices of the planar vector field $\vec{l}$ outside the cores ($p_1 = -p_2 = 1$). The cores of the Mermin-Ho vortices making up a Seppälä-Volovik vortex are isolated from each other poorly, however, since the distance between the cores is comparable in magnitude to the core itself, i.e., $\sim\xi_d$. The situation may change near T_{c2}. Here the size of the core of a Mermin-Ho vortex, within which the vector $\vec{l}$ swings out of the plane, is determined by the scale ξ_{d1}, while the distance (R) between the Mermin-Ho vortices is determined by the planar distribution of $\vec{l}$ outside the cores, and its size should be $\xi_{d2} > \xi_{d1}$. If there is a hydrodynamic repulsion between the Mermin-Ho vortices making up a Seppälä-Volovik vortex, the distance between the Mermin-Ho vortices, R, increases until it reaches the second dipole length, ξ_{d2}. Beyond this point, the dipole interaction between $\vec{l}$ and $\vec{d}_1$ in the plane prevents a further increase in R. As T_{c2} is approached, we can thus expect a Seppälä-Volovik vortex to decay into two Mermin-Ho textures (Fig. 1). An indication of this behavior emerges from a calculation[11] of the structure of a Seppälä-Volovik vortex near the transition to the A_1 phase.

The hydrodynamic interaction of Mermin-Ho vortices is the sum of the "Coulomb" repulsion of the like vortex charges N and the "Coulomb" attraction of the opposite disclination charges p. We can show that the repulsion is dominant at least in the limit $\xi_{d2} \gg \xi_{d1}$, i.e., under the condition $T_{c2} - T \ll T_{c1} - T_{c2}$. To prove this point, we consider a pair of Mermin-Ho vortices separated by a distance $\xi_{d1} \ll R \ll \xi_{d2}$. In this case we can, in the calculation of the interaction energy, ignore the smaller dipole energy and assume that the vector $\vec{l}$ is free to rotate in the plane outside the cores. On the other hand, we could ignore the hydrodynamic energy within the cores, since the primary logarithmic component comes from the "Coulomb" region outside the cores.

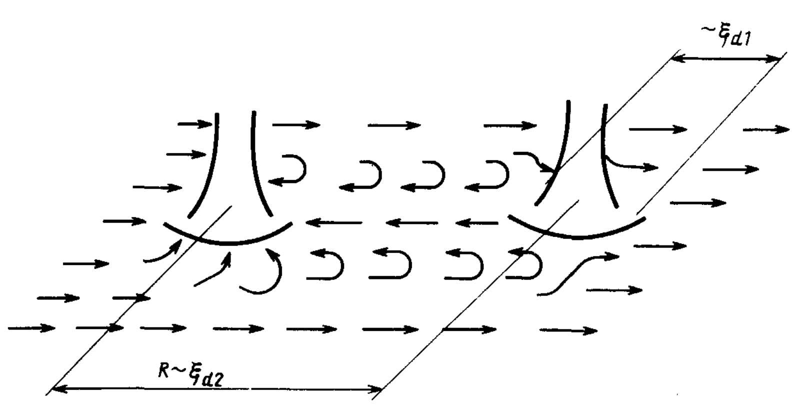

FIG. 1. Schematic diagram of the structure of a nonsingular vortex with two quanta of circulation as the second-order phase transition to the A_1 phase is approached. The distance $R\sim\xi_{d2}$ between the two Mermin-Ho vortices making up a vortex pair diverges, while the size of the cores of the Mermin-Ho vortices, in which the vorticity $\vec{\nabla}\times\vec{v}_s$, is concentrated, is on the order of ξ_{d1} and tends toward a constant limit.

There are three regions which make logarithmic contributions to the interaction of the vortices: (1) The region of distances $\xi_{d1} < r < R$ from the axis of the first Mermin-Ho vortex, which has charges $N_1 = p_1 = 1$; (2) the same region of distances, from the axis of the second Mermin-Ho vortex; and (3) the region outside the vortex pair, at a distance $R < r < \xi_{d2}$ from the center of the Seppälä-Volovik vortex. In the first two regions, the order parameter can be chosen in the following form, where z, r, ϕ are cylindrical coordinates, with the z axis running along the axis of each of the Mermin-Ho vortices:

$$\Phi = \phi \ , \ \vec{e}_2 = \hat{z} \ , \ \vec{v} = \frac{\hat{\phi}}{r} \ , \ \vec{l} = \hat{x} \cos p(\phi - \phi_0) + \hat{y} \sin p(\phi - \phi_0) \ , \tag{10}$$

where $p = 1$ for the first Mermin-Ho vortex in the pair, and $p = -1$ for the second. The logarithmic gradient energy from these two regions does not depend on the parameters p and ϕ_0 in Eq. (10):

$$F_1 = \int_{region\ 1} d^2r \ F_G = F_2 = \frac{5}{2}\pi\rho_{s\|}\left(\frac{\hbar}{2m_3}\right)^2 \ \ln\frac{R}{\xi_{d1}} \ . \tag{11}$$

In the third region, the order parameter corresponds to the asymptotic form of a vortex with $N = 2, p = 0$. The energy in this region, calculated by the same method as in Ref. 12, is

$$F_3 = \int_{region\ 3} d^2r \ F_G \cong 5,2\pi\rho_{s\|}\left(\frac{\hbar}{2m_3}\right)^2 \ \ln\frac{\xi_{d2}}{R} \ . \tag{12}$$

As a result, the total Coulomb interaction of the Mermin-Ho vortices,

$$F_{int} = F_1 + F_2 + F_3 = -0,2\pi\rho_{s\|}\left(\frac{\hbar}{2m_3}\right)^2 \ \ln R \tag{13}$$

is repulsive. Since the only factor placing a limit on the equilibrium size R of Mermin-Ho vortices is the second dipole energy, this limiting size is on the order of ξ_{d2}. Away from this size, the vector $\vec{l}$ is essentially uniform and is directed along $\vec{d}_1$ (Fig. 1).

For the splitting of a Seppälä-Volovik vortex in experiments with ultrasound, whose attenuation α is sensitive to the orientation of the vector $\vec{l}$, it is necessary (first) to fix the direction of the vector $\vec{l}$ away from the vortex. This can be done by directing the field $\vec{H}$ slightly at an angle with respect to the rotation axis, $\hat{z}$. The equilibrium orientation $\vec{l}_0$ of the vector $\vec{l}$ away from Seppälä-Volovik vortices is then along the direction of the vector product $\hat{z}\times\vec{H}$. Second, it is necessary to use two mutually orthogonal ultrasonic propagation directions $\hat{q}$: along $\hat{z}$ and along $\hat{z}\times\vec{l}_0$. It then becomes possible to observe the different temperature dependences of these two length scales, $\xi_{d1}(T)$ and $\xi_{d2}(T)$, which are characteristic of a split Seppälä-Volovik vortex, as the A_1 phase is approached. The change in attenuation, $\delta\alpha$, during the rotation is proportional to the volume in which the vector $\vec{l}$ deviates from the plane perpendicular to $\hat{q}$ (Ref. 4, for example). If $\hat{q} = \hat{z}$, then this is the volume of the cores of the Mermin-Ho vortices, since only there does $\vec{l}$ swing out of the plane perpendicular to $\hat{q}$. If, on the other hand, $\hat{q} = \hat{z}\times\vec{l}_0$, then this volume is proportional to the square of the distance

($R \sim \xi_{d2}$) between the Mermin-Ho vortices, and it becomes infinite as T_{c2} is approached, in contrast with the behavior in the case $\hat{q} = \hat{z}$:

$$\delta\alpha(\hat{q} = \hat{z}) \sim \alpha n_V \xi_{d1}^2 \ , \ \delta\alpha(\hat{q} = \hat{z} \times \vec{l}_0) \sim \alpha n_V \xi_{d2}^2 \ , \qquad (14$$

where n_V is the density of vortices.

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