Optical bistability of atoms near a material object

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A mirrorless optical bistability in a system of two-level atoms near a material object is analyzed. The conditions for the occurrence of this bistability in this case may differ substantially, by several orders of magnitude, from the conditions in the absence of a solid object.

1. Optical bistability has been the subject of very active research over the past decade because of the promising outlook for practical applications in optical computing and also from a purely scientific standpoint, since this is an interesting physical phenomenon.[1] Particularly attractive is mirrorless optical bistability (MLOB), which has been predicted and observed in a variety of physical systems, including a set of two-level atoms.[1]

In the present letter we derive a theory for MLOB in a system of two-level atoms near a material object.

2. We consider a set of radiators (atoms, molecules, nuclei) near a material object. These radiators are interacting in a radiative fashion with a resonant electromagnetic field. For definiteness, we will discuss atomic transitions. We will describe this system from a purely quantum mechanical standpoint, i.e., we will assume that both the atoms and the electromagnetic field are quantized.

The total Hamiltonian consists of three terms:

$$\hat{H} = \hat{H}_a + \hat{H}_f + \hat{H}_{int}, \tag{1}$$

where $\hat{H}_a$, $\hat{H}_f$, and $\hat{H}_{\text{int}}$ are the Hamiltonians of the atomic subsystem, of the free electromagnetic field, and of the interaction of the atomic subsystem with the electromagnetic field, respectively. The first and third of these Hamiltonians are given by

$$\hat{H}_a = \frac{1}{2}\hbar\omega_0 \sum_{k=1}^{N} \hat{\sigma}_{k3}. \tag{2.1}$$

$$\hat{H}_{int} = -\frac{1}{c}\int \hat{\vec{j}}(\vec{r},t)\hat{\vec{A}}(\vec{r},t)d\vec{r}. \tag{2.2}$$

The frequency ω_0 is the resonant frequency of the atomic transition, the operator $\hat{\vec{A}}(\vec{r},t)$ represents the vector potential of the electromagnetic field, and the operator $\hat{\vec{j}}(\vec{r},t)$ represents the current density of the atomic transition (we assume below that the atoms are point atoms) and is given by

$$\hat{\vec{j}}(\vec{r},t)=\sum_{k=1}^{N}[\vec{j}_k\hat{\sigma}_{k+}(t)+\vec{j}_k^*\hat{\sigma}_{k-}]\delta(\vec{r}-\vec{r}_k), \tag{3}$$

where σ_3, $\sigma_\pm$ are the Pauli spin operators, $\vec{j}_k$ is the matrix element of the positive-frequency part of the current-density operator for a transition of atom K, and $\vec{r}_k$ is its radius vector.

Corresponding to Hamiltonian (1) is the following system of Heisenberg equations for the operators:

$$\frac{d\hat{\sigma}_{k+}}{dt}=i\omega_0\hat{\sigma}_{k+}+\frac{i}{\hbar c}\vec{j}_k^*\hat{\vec{A}}(\vec{r}_k,\ t)\hat{\sigma}_{k3}, \tag{4.1}$$

$$\frac{d\hat{\sigma}_{k3}}{dt}=\frac{2i}{\hbar c}(\vec{j}_k\hat{\sigma}_{k+}-\vec{j}_k^*\hat{\sigma}_{k-})\hat{\vec{A}}(\vec{r}_k,\ t), \tag{4.2}$$

$$\vec{\text{curl}}\,\vec{\text{curl}}\,\hat{\vec{A}}(\vec{r},t)+\frac{d^2}{c^2dt^2}\int\int_{-\infty}^{t}\epsilon(\vec{r},\vec{r}\,';\ t-t')\hat{\vec{A}}(\vec{r}\,',t')d\vec{r}\,'dt'=\frac{4\pi}{c}\hat{\vec{j}}(\vec{r},t). \tag{4.3}$$

As we know, the solution of the last of these equations can be written as a sum of the solution of the homogeneous version of Eq. (4.3), i.e., $\hat{\vec{A}}_0(\vec{r},t)$, and a stimulated solution expressed in terms of the photon propagator in the medium, $\vec{D}(\vec{r},\vec{r}';t-t')$:

$$\hat{\vec{A}}(\vec{r},t)=\hat{\vec{A}}_0(\vec{r},t)-\frac{1}{\hbar c}\int\int_{-\infty}^{t}\vec{D}(\vec{r},\vec{r}';\ t-t')\hat{\vec{j}}(\vec{r}')d\vec{r}\,'dt'. \tag{5}$$

From system (4) we find equations for the difference between the populations of the atomic levels, Δ_k, and for the coherence between them, $\rho_k=\langle\sigma_{k-}\rangle\exp(+i\omega t)$:

$$\frac{d\rho_k}{dt}=-(\gamma_k-i\tilde{\Omega}_k)\rho_k-i[G_k-\sum_{m\neq k}{}'\Gamma_{km}\rho_m]\Delta_k, \tag{6.1}$$

$$\frac{d\Delta_k}{dt}=-2\gamma_k(I+\Delta_k)-4\text{Im}[G_k-\sum_{m\neq k}{}'\Gamma_{km}\rho_m)\rho_k^*]. \tag{6.2}$$

Here $G_k=\vec{E}_k\vec{d}_k/2\hbar$, $\vec{E}_k$ is the electric field of the incident light wave at the position of atom k; $\vec{d}_k$ is the dipole moment of the optical transition of this atom; ω is the frequency of the incident light; $\tilde{\Omega}_k=\omega-\omega_0-\delta_k$;$\delta_k=\text{Re}(\Gamma_{kk})$;$_{\gamma k}=\text{Im}(\Gamma_{kk})$; and

$$\Gamma_{km}=-(\frac{\omega_0}{\hbar c})^2\vec{d}_k\vec{D}(\vec{r}_k,\vec{r}_m;\ \omega_0)\vec{d}_m^*.$$

Since Maxwell's equations are linear, a Fourier component of the photon propagator, $\vec{D}(\vec{r}_k,\vec{r}_m;\omega)$, consists of a free-space part $\vec{D}^{v}(\vec{r}_k,\vec{r}_m;\omega)$ and a part which stems from the presence of the material object, $\vec{D}^{b}(\vec{r}_k,\vec{r}_m;\omega)$. For free space we have[2] $\text{Im}\vec{D}^{v}_{ii}(R,R;\omega_0)=-2\hbar\omega_0/c$, which gives rise to a component of the radiative damping of an isolated atom, γ_0, in γ_k.

The second terms in square brackets in (6) are proportional to the polarization electric field of all the other atoms (the Lorentz field). The nonlinearity which they cause is responsible for the bistability of the steady-state solution of system (6). In other words, the bistability results from an interaction of atoms with each other through a dipole reradiation field.

3. Let us examine in more detail the case in which the atoms are at a uniform distance R from the flat surface of a solid with a dielectric constant ϵ. We assume that the light wave is incident along the normal to the surface. In this case the dynamics is the same for all the atoms, so we can omit the subscripts on Δ, γ, and ρ in (6). In the steady state, we find from (6) the following relation between the normalized field amplitudes—that of the initial field, Y, and that of the total field, X—which are acting on the atoms:

$$|Y|^2 = |X|^2[(1+\Omega^2+|X|^2+N_1)^2+N_2^2]/(1+\Omega^2+|X|^2)^2. \qquad (7)$$

Here $N_1 = 2(C' - C''\Omega)$; $N_2 = 2(C'' + C'\Omega)$; $C = C' + iC'' = i\sum'_{m,k\neq m}\Gamma_{km}/(2\gamma)$; $\Omega = \infty\Omega/\gamma = \gamma_0 + \gamma_b$; $Y = \sqrt{2}\,G/\gamma$; $X = Y + Y_L$; and $Y_L = i2\sqrt{2}C\rho$.

Expression (7) has the same form as that derived in Ref. 3, in which the bistability of an ensemble of atoms in vacuum was studied.

The second part of the damping constant, γ_b, stems from the effect of the material object. For the most interesting case, $R \ll \lambda\!\!\!^{-}$, it is given by

$$\gamma_b = \frac{1}{4}\left(\frac{\lambda\!\!\!^{-}}{R}\right)^3 \mathrm{Im}\left(\frac{\epsilon-1}{\epsilon+1}\right)\gamma_0. \qquad (8)$$

If there is no solid object we have

$$C = C_0 = \pi n_s \lambda\!\!\!^{-2}, \qquad (9)$$

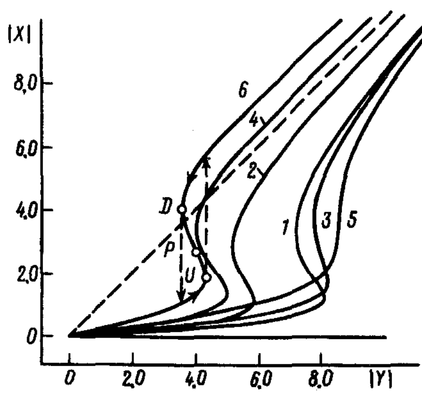

FIG. 1. Behavior of the fields acting on an atom, $|X|$, and of the field of the light wave, $|Y|$, for various values of C', C'', and Ω: 1—$\Omega = 0$, $C' = 7$, $C'' = 0$; 2—$\Omega = 0$, $C' = 2$, $C'' = 5$; 3—$\Omega = 1$, $C' = 7$, $C'' = 0$; 4—$\Omega = 1$, $C' = 2$, $C'' = 5$; 5—$\Omega = 2$, $C' = 7$, $C'' = 0$; 6—$\Omega = 2$, $C' = 2$, $C'' = 5$.

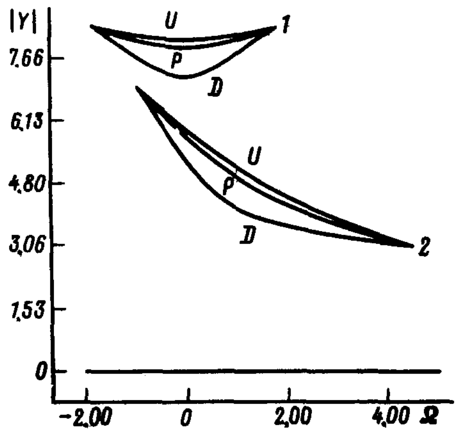

FIG. 2. Dependence of the bifurcation points and of the inflection points—U, D, and P, respectively (see also Fig. 1)—on Ω for various values of C' and C'': 1—$(C',C'') = (7,0)$; 2—$(C',C'') = (2,5)$. The extrema (U, D, and P) of curve 5 in Fig. 1 cannot be distinguished from each other. They correspond to the point at which the curves of family 2 meet at the right in this figure. To the right of this point (at larger values of Ω) the bistability disappears. Correspondingly, the bistability disappears to the left of the other point at which the curves of the family meet.

where n_s is the surface density of atoms. In this case the bistability described by expression (7) is of a purely absorption nature.

The material object gives rise to an additional component in $C(R \ll \lambda)$:

$$C = C_0[1 + \frac{1-\epsilon}{1+\epsilon}(-0,5 - \nu + \ln(\lambda/2R) + i\pi/2)]\gamma_0/\gamma, \qquad (10)$$

where $\nu = 0.5771...$ is the Euler constant.[4]

4. A qualitative consequence of the presence of the material object is that the mirrorless optical bistability (MLOB) becomes a mixed absorption-dispersion bistability, because C acquires an imaginary part. As a result, the threshold for the occurrence of the MLOB along the light intensity scale decreases (Fig. 1).

Other qualitative differences between this MLOB and a purely absorption instability are that the region in which the bistability exists is wider and that the MLOB is asymmetric with respect to Ω (Fig. 2).

The conditions which must be met for the occurrence of MLOB can be expected to be substantially more relaxed in the case $\mathrm{Re}(1+\epsilon) \sim 0$ [see (10)], in which case surface polaritons (for example) may be excited. In this case the MLOB can be observed at atomic densities several orders of magnitude lower than in the absence of a solid.

[1]H. Gibbs, *Optical Bistability*, Academic Press, Orlando, 1985.
[2]L. D. Landau and E. M. Lifshitz, *Statistical Physics*, Pergamon, New York, 1980.

[3]S. S. Hassan and R. K. Bullough, in *Optical Bistability* (ed. Ch. Bowden, M. Giftan, and H. Robf), Plenum Press, New York, 1981.
[4]M. Abramowitz and I. A. Stegun (editors), *Handbook of Mathematical Functions*, Dover, New York, 1064.

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