Nonlinear electrical conductivity of ballistic quantum contacts

A. M. Zagoskin
Physicotechnical Institute of Low Temperatures, Academy of Sciences of the Ukrainian SSR

(Submitted 7 August 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 8, 1043–1045 (25 October 1990)

The second derivative of the current-voltage characteristic of a quantum ballistic contact has peaks at positions determined by the quantized electron energy levels near the contact.

Let us analyze the current-voltage characteristic of quantum ballistic contacts. A two-dimensional quantum ballistic contact[1] is a quasi-two-dimensional electron gas of the inversion layer of a GaAs–AlGaAs junction, at which (by virtue of a negative voltage V_g applied to gate electrodes deposited on the structure) regions inaccessible to the electrons are created. These inaccessible regions create a constriction, whose diameter can be continuously varied by varying V_g. The conductance of the contact, $G = I/V$, is quantized as a function of the contact diameter $d = d(V_g)$, taking on values which are multiples of $e^2/(\pi\hbar) = (12.9\ \text{k}\Omega)^{-1}$, where I is the current through the contact, and V is the potential difference across the banks of the contact.

A theory for the effect was derived in Refs. 2–4 in the limit of a linear response ($V \to 0$), for both a smooth contact and one with sharp edges. The effect of a finite voltage V on this quantization of the conductance was studied in Ref. 5, but the current-voltage characteristic of the system was not. The second derivative of this characteristic, d^2I/dV^2, can provide information about the quantized electron energy levels near the contact ("elastic contact-mode spectroscopy").

Let us consider the case of weak screening: $\Lambda \gg \max(\lambda_F, d)$, where Λ is the screen-

 0021-3640/90/200435-04$01.00

ing length. This case occurs at semiconductor contacts[6] and, especially, in two-dimensional GaAs–AlGaAs layers (since the screening by the two-dimensional electron gas is a power-law screening[7]), i.e., in precisely those systems in which we have $\lambda_F \sim d$, and quantum effects are important in the conductivity. As was shown in Ref. 6, a space-charge layer forms at a distance on the order of Λ from the contact in this case. The voltage across the banks, V, drops in this space-charge layer, while the voltage across the contact itself is small in comparison with V to the extent that the ratio L/Λ is small, where $L > d$ is the length of the contact. At distances $r \ll \Lambda$ from the contact, the electron wave functions thus have the same form as in the absence of a bias voltage across the contact, eV. The deviation of eV from zero affects only the electron momentum distribution, which is[6]

$$f_\pm(\vec{k}) = f_0(\epsilon_{\vec{k}} - \epsilon_F \pm eV/2 - e\varphi(0)), \tag{1}$$

where the plus sign refers to states of electrons which have arrived at the contact with a momentum $\hbar\vec{k}$ from the first bank (at which the potential is $V/2$), while the minus sign corresponds to electrons which have arrived from the second bank. In addition, $\varphi(0)$ is the potential inside the channel, and f_0 is a Fermi distribution. At $eV \ll \epsilon_F$, the current through the contact can thus be written in the form[4]

$$I(V) = (e/\pi\hbar)\sum_n \int d\epsilon T_n(\epsilon)[f_0(\epsilon - \tilde{\epsilon}_F - eV/2) - f_0(\epsilon - \tilde{\epsilon}_F + eV/2)], \tag{2}$$

where $T_n(\epsilon)$ is the transmission of conduction channel n for an electron with an energy ϵ in the limit $V \to 0$, and $\tilde{\epsilon}_F = \epsilon_F - e\varphi(0)$. The function $T_n(\epsilon)$ has a single step $[T_n(0) = 0,\ T_n(\infty) = 1]$ at $\epsilon = E_n$, where E_n is the nth energy level of the transverse motion of an electron in the contact. The width of the region over which the nth step is smeared at a contact of length L and diameter d is[4]

$$\delta\epsilon_n \simeq (\pi n/0,16L^2 d)(k(\epsilon)^{-2}\partial\epsilon/\partial k)|_{\epsilon=E_n}. \tag{3}$$

The number of steps which would be observed in the conductance $G(d)$ at absolute zero is

$$N = (2d/\lambda_F) \leq N_0 = 1,28\pi^2(L/\lambda_F). \tag{4}$$

If the constriction is sufficiently long we would have

$$T_n(\epsilon) \simeq \theta(\epsilon - E_n)F_n(\epsilon), \tag{5}$$

where $\theta(\epsilon)$ is the unit step function (Heaviside function), and $0 \leqslant F_n(\epsilon) \leqslant 1$ describes the details of $T_n(\epsilon)$. If the temperature is not too low, we would have $F_n(\epsilon) \approx 1$ (Ref. 4).

The second derivative of the current-voltage characteristic is (Fig 1)

$$d^2I/dV^2 = (e^3/4\pi\hbar)\int d\epsilon \left\{\frac{\partial}{\partial\epsilon}[-f_0(\epsilon - \tilde{\epsilon}_F - eV/2) - f_0(\epsilon - \tilde{\epsilon}_F + eV/2)]\right\}$$
$$\times \sum_n \frac{dT_n(\epsilon)}{d\epsilon} \simeq (e^3/4\pi\hbar)\sum_n\{\delta(eV/2 - (E_n - \tilde{\epsilon}_F)) - \delta(eV/2 - (\tilde{\epsilon}_F - E_n))\}. \tag{6}$$

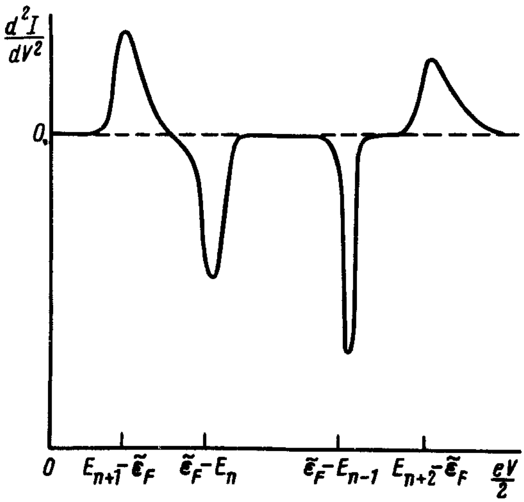

FIG. 1. Second derivative of the current-voltage characteristic versus the applied voltage.

This derivative has peaks at those values of the bias voltage across the contact, eV, at which a given level E_n falls in the band $[\tilde{\epsilon}_F - eV/2, \tilde{\epsilon}_F + eV/2]$ which contains the current states. The sign of a peak is positive if $E_n > \tilde{\epsilon}_F$, and negative if $E_n < \tilde{\epsilon}_F$. The functional dependence $d^2I/dV^2(V)$ can be used to determine the energies of the transverse modes, E_n. It thus bears information about the diameter of the contact and the potential relief over its cross section. The condition for the existence of the effect, $eV > \Delta E$ (ΔE is the distance between levels in the channel), must be compatible with the condition for the applicability of expression (2), $eV \ll \epsilon_F$; with the condition for a ballistic nature of the contact, $l\varphi \gg L, d$ ($l\varphi$ is the phase disruption length of an electron in the system); and with the condition under which quantization of the conductance can be observed, (4). The first two of these requirements lead to the condition $d^2 \gg \lambda_F^2$; along with the third and fourth requirements, they impose the following limitation on the shape and dimensions of the contact:

$$l_\varphi > 1,28\pi^2 L > d > \lambda_F. \qquad (7)$$

To round out the discussion, we now consider the case of complete screening ($\Lambda \ll d$). (In such systems we can set $\lambda_F \ll d$.) The voltage drop occurs in a region of size $\sim d$ near the edges of the constriction. The electric field penetrates a certain distance, also $\sim d \ll L$, into the constriction. Consequently, under the condition $eV \ll \sqrt{\epsilon_F \Delta\epsilon}$ [here $\Delta\epsilon \equiv \hbar^2/(m^* d^2) \simeq \Delta E$], we can calculate the current for a stepped potential profile ($\varphi = \pm V/2$ at the banks of the contact; $\varphi = 0$ in the constriction) in a first approximation. The deviations of $\varphi(\vec{r})$ from this dependence near the ends of the constriction contribute small corrections on the order of the parameter $eV/\sqrt{\epsilon_F \Delta\epsilon}$ (Ref. 8). An expression for the current can be derived by the method of Ref. 4:

$$I(V) = (e/\pi\hbar) \int d\epsilon f_0(\epsilon - \epsilon_F) \sum_n [T_n^+(\epsilon, eV) - T_n^-(\epsilon, eV)]. \qquad (8)$$

Here $T_n^\pm(\epsilon, eV) \simeq \theta(\epsilon \pm eV/2 - E_n) F_n^\pm(\epsilon)$, and the functions $T_n^\pm$ and $F_n^\pm$ have the same meaning as in (5). The functional dependence $d^2I/dV^2(V)$ determined by (8) is similar to that of the second derivative of the current-voltage characteristic in the weak-screening limit, (6), having peaks at the same bias voltages eV. The requirements in this case are more stringent, however: The condition $l_\varphi \gg d \gg \lambda_F$ must hold (instead of $l_\varphi \gg d$, $d^2 \gg \lambda_F^2$).

The results remain valid in the case of three-dimensional quantum ballistic contacts. In this case the peaks on the second derivative of the current-voltage characteristic are determined by the energies $E_{\min}$ of the transverse electron modes, which depend on the dimensions and shape of the cross section of the microconstriction. One might thus expect to observe the effect at ballistic semiconductor point contacts which satisfy restrictions (7).

I wish to thank I. O. Kulik for interest in this study and for a stimulating discussion.

[1]D. A. Wharam *et al.*, J. Phys. C **21**, L209 (1988); C. G. Smith *et al.*, J. Phys. C **21**, L893 (1988).
[2]L. I. Glazman *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 218 (1988) [JETP Lett. **48**, 238 (1988)].
[3]A. Szafer and A. D. Stone, Phys. Rev. Lett. **62**, 300 (1989).
[4]A. M. Zagoskin and I. O. Kulik, Fiz. Nizk. Temp. **16**, 911 (1990) [Sov. J. Low Temp. Phys.].
[5]L. I. Glazman and A. V. Khaetskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **48**, 546 (1988) [JETP Lett. **48**, 591 (1988)].
[6]I. F. Itskowich, I. O. Kulik, and R. I. Shekhter, Solid State Commun., 1984, p. 421; I. F. Itskovich and R. I. Shekhter, Fiz. Nizk, Temp. **11**, 373 (1985) [Sov. J. Low Temp. Phys. **11**, 202 (1985)].
[7]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
[8]L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*, Pergamon, New York, 1977.

Translated by D. Parsons