# New limitations on the violation of $T$-invariance in β decay

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(Submitted 8 October 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 9, 1065–1068 (10 November 1990)

Limitations at the level of $10^{-4}$ have been found on the imaginary parts of the weak-magnetism and weak-dipole constants in $\beta$ decay, which would violate $T$-invariance.

Severe limitations on certain $\beta$-decay constants which would violate $T$-invariance were recently found.[1] For the tensor, scalar, and pseudoscalar nucleon-lepton constants, these limitations are

$$\begin{aligned} &\mathrm{Im}(C_T + C_T') < 0.5\times10^{-3},\\ &\mathrm{Im}(C_s + C_s') < 4\times10^{-3},\\ &\mathrm{Im}(C_p + C_p') < 0.3 \end{aligned} \tag{1}$$

(the limitations reported in Ref. 1 refer directly to the corresponding quark-lepton constants, but their reformulation for nucleon-lepton constants is obvious.) In the present letter we derive limitations on the imaginary parts of the weak-magnetism and weak-dipole constants $g_m$ and $g_e$, which would violate $T$-invariance. So far, there is no direct experimental information on these parameters. In a recent proposal,[2] it was suggested that a precision of $10^{-2}$ could be achieved experimentally in measurements of the corresponding $T$-odd correlations in the $\beta$ decay of nuclei with $A = 8$.

We write the quark-lepton $\beta$-decay interaction of interest here in the form

$$\frac{G}{\sqrt{2}}\frac{1}{2m_p}[\bar{e}\gamma_\mu(1+\gamma_5)\nu\bar{u}(\tilde{g}_m + \tilde{g}_e\gamma_5)\sigma_{\mu\lambda}k_\lambda d + h.c.]. \tag{2}$$

Here $G$ is the Fermi weak-interaction constant, $m_p$ is the mass of the proton, and the operators $e$, $\nu$, $u$, and $d$ represent the corresponding lepton and quark fields. The tildes on the quark-lepton constants $\tilde{g}_{m,e}$ distinguish them from the corresponding nucleon-lepton constants.

In the spirit of Ref. 1, we transform $\beta$-decay interaction (2) into an effective diagonal quark-electron interaction by means of $W$ exchange (see the diagrams in Figs. 1 and 2). We will not attempt to derive this radiation correction systematically here; the corresponding derivation is model-dependent. In particular, we adopt the simple Feynman form $\delta_{\mu\nu}/(q^2 - M_W^2)$ for the $W$-Boson propagator in our calculations, with the expectation that the term $-q_\mu q_\nu/M_W^2$ in the numerator of this propagator will somehow cancel out in a more accurate calculation. We thus find an effective quark–electron interaction for which we can extract limitations from atomic experiments.

The quantity $\mathrm{Im}\tilde{g}_e$ is the obvious close analog of the parameter $q$ of the $T$-odd, $P$-

  

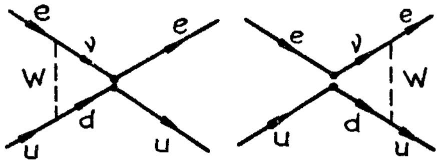


FIG. 1.

even quark–electron interaction

$$\frac{G}{\sqrt{2}}\frac{q}{2m_p}\bar{e}\gamma_\mu\gamma_5 e\bar{q}i\gamma_5\sigma_{\mu\nu}k_\nu q, \tag{3}$$

which was discussed in Ref. 1. Since $W$ exchange is at least as effective as the $Z$ exchange which was used in that previous paper, the ratio of the induced $T$-odd, $P$-odd quark–electron constant to $\mathrm{Im}\tilde{g}_e$ is

$$\sim \frac{\alpha}{\pi}\log\frac{\Lambda^2}{M_W^2}\frac{m_e}{m_p}, \tag{4}$$

and is roughly the same as the ratio of this constant to the parameter $q$ of interaction (3), which was discussed in Ref. 1. Here $\alpha = 1/137$; $M_W$ and $m_e$ are the masses of the $W$ and the electron, respectively; and $\Lambda$ is the cutoff parameter. To be as conservative as possible in our numerical estimates, we assume $\log\Lambda^2/M_W^2 \sim 1$ in them. We thus obtain the same estimate for $\mathrm{Im}\tilde{g}_e$ as for $q$:

$$\mathrm{Im}\tilde{g}_m < 10. \tag{5}$$

The same diagrams, with a different arrangement of the vector and axial vertices $\gamma_\mu$ and $\gamma_\mu\gamma_5$, lead to a similar estimate for the limitation on $\mathrm{Im}\tilde{g}_m$:

$$\mathrm{Im}\tilde{g}_m < 10. \tag{6}$$

Limitations (5) and (6) do not look particularly impressive. However, they can be improved substantially by switching from single-loop diagrams 1 and 2, which induce an effective $T$-odd, $P$-odd interaction, to a two-loop diagram of the type in Fig. 3, which induces an electric dipole moment of a quark. In evaluating these diagrams, we ignore the masses of the fermions. In order to induce the electric-dipole-moment structure $\gamma_5\sigma_{\mu\nu}F_{\mu\nu}$, which would alter the helicity, we thus need to put a vertex $(\tilde{g}_m + \tilde{g}_e\gamma_5)\sigma_{\mu\nu}k_\nu$, which also changes the helicity, on the lower line. In other words, leptons propagate in the closed loop, while the lower line refers to quarks. A simple

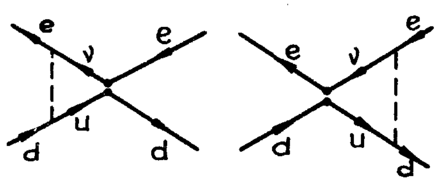


FIG. 2.

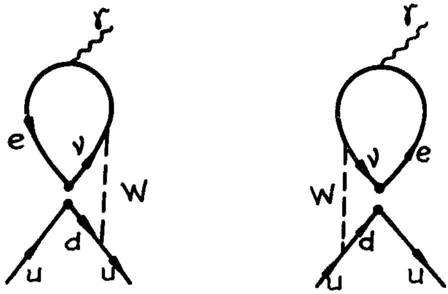


FIG. 3.

estimate yields the following result for the electric dipole moment of the quark which would be induced by these diagrams:

$$d/e \sim \frac{G\Lambda^2}{\sqrt{2}} \frac{\alpha}{a\pi^3} \frac{1}{m_p} \mathrm{Im}(\tilde{g}_m \pm \tilde{g}_e). \tag{7}$$

Here $\Lambda$ is a cutoff parameter, and we are again ignoring possible strengthening factors $\log\Lambda^2/M_W^2$. The two-loop diagrams contain as geometric suppression factors not only $1/\pi^3$ but also the small number $1/a$, which is close to $10^{-2}$, judging from experience in calculating such diagrams. The upper sign in the expression $\mathrm{Im}(\tilde{g}_m \pm \tilde{g}_e)$ refers to the electric dipole moment of the $u$ quark, and the lower sign to the dipole moment of the $d$ quark.

Simple dimensionality considerations show that the electric dipole moment of a neutron which would be induced by the dipole moment of a quark is of the same order of magnitude as the latter moment. In other words, we would find the same estimate, (7), for the electric dipole moment of the neutron. However, we do not have accurate knowledge of the relative weights of the $u$-quark and $d$-quark components of the dipole moment of the neutron or thus of the relative weights of $\mathrm{Im}\tilde{g}_m$ and $\mathrm{Im}\tilde{g}_e$ in the latter moment. We will nevertheless assume that there is no strong cancellation of the $\mathrm{Im}\tilde{g}_m$ and $\mathrm{Im}\tilde{g}_e$ components to the electric dipole moment of the neutron, $d_n$. Comparing estimate (7) with the recent experimental result[3]

$$d_n/e < 1{,}2\times10^{-25}\mathrm{cm}, \tag{8}$$

we then find the limitation

$$\mathrm{Im}\tilde{g}_{m,e} < 10^{-4} \tag{9}$$

even under the modest assumption $\Lambda \sim 100$ GeV. Dimensionality considerations show that the same limitations as in (9) for the quark-lepton $\beta$-decay constants also apply to the parameters of the nucleon weak current induced in this manner:

$$\mathrm{Im}g_{m,e} < 10^{-4}\ . \tag{10}$$

Possible contributions to the $T$-odd constants $\mathrm{Im}g_{m,e}$ of the nucleon weak current from the violation of $T$-invariance in hadron interactions would have to be much smaller than (10). Indeed, atomic experiments[4,5] yield limitations[6] at the level of $0.1G$

on the $T$-odd, $P$-odd nucleon–nucleon interaction. A crude estimate for the corresponding relative contribution of the hadron matrix elements yields $0.1Gm_\pi^2 \sim 10^{-8}$, where $m_\pi$ is the mass of the $\pi$ meson. For the $T$-odd, $P$-even nucleon–nucleon interaction, the limitations imposed in Ref. 1 are $10G$ and $10^{-4}G$ for the single-loop and two-loop approaches, respectively. These results correspond to relative admixtures at the levels of $10^{-6}$ and $10^{-11}$.

We note in conclusion that the explicit dependence of interaction (2) on the momenta and the dimensionality of 7 (as opposed to 6) of this operator are crucial for the strengthening of the limitations by five orders of magnitude in the switch from the single-loop approach to the two-loop approach. This strengthening does not occur for $T$-odd interactions without derivatives, i.e., for the constants $\mathrm{Im}C_{T,S,P}$.

I wish to thank L. De Braeckeleer for attracting my interest to this question.

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Translated by D. Parsons