Transition temperature of a superconductor-ferromagnet superlattice

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A method is proposed for calculating the transition temperature T_c^* of a superconductor-ferromagnet (*SF*) superlattice. It is shown that T_c^* of an *SF* superlattice oscillates with the thickness of the *F* layer and with the strength of the exchange field. States with a phase order parameter, which varies from layer to layer, are possible in this superlattice.

Extremely interesting targets for studying the mutual effects of superconductivity and magnetism are artificial superlattices with alternating layers of a ferromagnet (*F*) and a superconductor (*S*).[1–3] These structures have several unique properties; e.g., their transition temperature T_c^* is a nonmonotonic function of the thickness of the *F* layers, d_n.

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The transition temperature T_c^* of an SF superlattice has previously been calculated by numerical methods on the basis of generalized Usadel equations,[4] but only for particular thicknesses of the F and S layers and for a particular exchange field I of the ferromagnet.[5] In the present letter, we derive $T_c^*(d_n,I)$ analytically. To the best of our knowledge, this is the first analytic study of short-period SN superlattices.

We assume that the conditions of the dirty limit hold for the materials making up the superlattice, that the transition temperature of the ferromagnet is zero, and that its exchange field satisfies $I \gg T_c$, where T_c is the transition temperature of the bulk superconductor. The values of I of ferromagnets are typically in the interval 10^2–10^3 K. The last of these conditions thus holds in essentially all cases, at least for conventional superconductors.

We place the origin of coordinates at one of the SF interfaces, and we direct the X axis perpendicular to the interface. Under the assumptions listed above, the Usadel equations near the transition temperature of the superlattice, T_c^*, can be written in the form

$$\pi T_{c0}(\xi_n^*)^2 \frac{d^2}{dx^2} F_n^{\pm} \mp i I F_n^{\mp} = 0, \quad -d_n < x < 0, \tag{1a}$$

$$\pi T_{c0}(\xi_s^*)^2 \frac{d^2}{dx^2} F_s^{\pm} - |\omega| F_s^{\pm} = 2\Delta\delta^{\pm}, \quad 0 < x < d_s, \tag{1b}$$

$$\Delta \ln\left(\frac{T_c^*}{T_c}\right) - \pi T_c^* \sum_{\omega>0} \left(\frac{2\Delta}{\omega} - F_s^+\right) = 0. \tag{1c}$$

Here $\delta^+ = 1$; $\delta^- = 0$; $\xi_{n,s}^* = (D_{n,s}/2\pi T_c)^{1/2}$ and $D_{n,s}$ are the coherence lengths and diffusion coefficients of the F and S metals; and $\omega = \pi T(2n+1)$ are the Matsubara frequencies. The functions $F_{s,n}^*$ are related to the Usadel functions by the simple relation

$$F_{s,n}^{\pm} = F_{s,n}(\omega) \pm F_{s,n}(-\omega). \tag{2}$$

We must supplement system equations (1) with boundary conditions at the SF interface[6] (at the point $x = 0$):

$$F_s^{\pm}(0) = F_n^{\pm}(0), \quad \sigma_s \frac{d}{dx} F_s^{\pm}(0) = \sigma_n \frac{d}{dx} F_n^{\pm}(0), \tag{3}$$

where $\sigma_{s,n}$ are the conductivities of the S and F layers in their normal state. By virtue of the translational invariance of the superlattice, we can assume that the functions $F^{\pm}$ at points separated from each other by a lattice constant $D = d_n + d_s$ can differ only by a constant phase factor: $F^{\pm}(x+D) = F^{\pm}(x)\exp(i\varphi)$. We restrict the discussion below to the most interesting cases, $\varphi = 0$ (the 0 phase) and $\varphi = \pi$ (the π phase), since numerical calculations[5] show that the region of intermediate values of φ is extremely narrow. Under this restriction, we can reduce the problem to one of solving Eqs. (1) on the interval $-(d_n/2) \leqslant x \leqslant d_s/2$ with boundary conditions at the end points of the interval:

$$\frac{d}{dx}F^{\pm}_{s0,s\pi}(d_s/2)=0,\ \ \frac{d}{dx}F^{\pm}_{n0}(-d_n/2)=0,\ \ F^{\pm}_{n\pi}(-d_n/2)=0, \tag{4}$$

where the new subscripts 0 and π show that the corresponding functions correspond to the 0 and π phases, respectively.

Boundary-value problems (1), (3), (4) for the functions $F^{\pm}_{n0,n\pi}$ and $F^{-}_{s0,s\pi}$ can be solved analytically. Using the solutions found as a result, we can reduce the problem to one of solving equations for $F^{+}_{s0,s\pi}$ in a superconductor with a boundary condition

$$\xi_s^*\frac{d}{dx}F^{+}_{s0,s\pi}=F^{+}_{s0,s\pi}\gamma\frac{a_s(a_{0,\pi}+a^*_{0,\pi})+2\gamma a^*_{0,\pi}a_{0,\pi}}{2a_s+\gamma(a_{0,\pi}+a^*_{0,\pi})},$$

$$a_s=\Omega\tanh(\Omega d_s/2\xi_s^*),\ \ a_0=\alpha\tanh(\alpha d_n/2\xi_n^*),\ \ a_\pi=\alpha\coth(\alpha d_n/2\xi_n^*), \tag{5}$$

at $x=0$, where $\Omega=(\omega/\pi T_c)^{1/2}$, $\alpha=(ih)^{1/2}$, $h=(I/\pi T_c)$, and $\gamma=(\sigma_n\xi_s^*)/(\sigma_s\xi_n^*)$.

The solution of Eqs. (1b) and (1c) under boundary conditions (4) and (5) simplifies substantially in the limits of small and large values of the parameter γ. In these approximations, boundary conditions (5) are independent of ω, and a general solution of the problem can be written in the form

$$F^{+}_{s0,s\pi}=\frac{2\Delta}{|\omega|+\rho_{0,\pi}},\ \ \Delta=B\cos[q_{0,\pi}(x-d_s/2)], \tag{6}$$

where $\rho_{0,\pi}=\pi T_c(q_{0,\pi}\xi_s^*)^2$, and the parameter $q_{0,\pi}$ is found from (5). Substituting (6) into the self-consistency equation, we find a simple equation for determining the transition temperature in the cases of the 0 phase (T^*_{c0}) and the π phase ($T^*_{c\pi}$):

$$\ln(T_c^*/T_c)=\Psi(1/2)-\Psi(1/2+\rho_{0,\pi}/2\pi T_c^*). \tag{7}$$

The transition temperature of the superlattice, T_c^*, is the larger of T^*_{c0}, $T^*_{c\pi}$.

SMALL-γ APPROXIMATION

If the parameter γ is sufficiently small,

$$\gamma\ll\begin{cases}(I/T_c)^{1/2}, & d_n>\xi_n^*,\\ min\{d_n/\xi_n^*,\ (\xi_n^*/d_n)(T_c/I)^{1/2}\}, & d_n<\xi_n^*,\end{cases} \tag{8}$$

we find the following expression for $\rho_{0,\pi}$ from (5) and (6):

$$(\frac{\rho_{0,\pi}}{\pi T_c})^{1/2}\tan\left[\frac{d_s}{2\xi_s^*}(\frac{\rho_{0,\pi}}{\pi T_c})^{1/2}\right]=\gamma(a_{0,\pi}+a^*_{0,\pi})/2=\gamma Z_{0,\pi}. \tag{9}$$

To determine the regions in which the 0 and π phases exist, it is sufficient to determine the sign of the difference between the coefficients Z_0 and Z_π in (9):

$$Z_0 - Z_\pi = \left[\frac{2I}{\pi T_c}\right]^{1/2} \frac{\sinh(y)\cos(y) + \cosh(y)\sin(y)}{\sinh^2(y)\cos^2(y) + \cosh^2(y)\sin^2(y)}, \quad y = \frac{d_n}{\xi_n^*}\left[\frac{2I}{\pi T_c}\right]^{1/2} \tag{10}$$

It follows from (10) that at small thicknesses of the N layer the relation $Z_\pi > Z_0$ holds, so we have $\rho_\pi > \rho_0$. In other words, the formation of the 0 phase is always preferable in the superlattice. With increasing thickness of the N layers, the situation changes, and in intervals of thickness

$$(\xi_n^*/h)(-\theta + \pi k) < d_n < (\xi_n^*/h)(-\theta + \pi(k+1)), \tag{11}$$

where $\theta = \arctan(\tanh(y)) \approx \pi/4$, and $k = (2n+1)$ is an odd integer, the π phase exists. At even integer values of k, expression (11) determines the region in which the 0 phase exists.

LARGE-γ APPROXIMATION

Reversing the inequality sign in (8), we find from (5) and (6)

$$\left(\frac{\rho_{0,\pi}}{\pi T_c}\right)^{1/2} \tan\left[\frac{d_s}{2\xi_s^*}\left(\frac{\rho_{0,\pi}}{\pi T_c}\right)^{1/2}\right] = \gamma \frac{2a_{0,\pi}^* a_{0,\pi}}{(a_{0,\pi} + a_{0,\pi}^*)} = \gamma Z_{0,\pi}^*. \tag{12}$$

Since the right side of (12) is substantially greater than unity in this approximation, we can write the following expression, in the first approximation in γ^{-1}, on the basis of (12):

$$(\rho_\pi/\pi T_c)^{1/2} = \pi(\xi_s^*/d_s)(1 - \gamma^{-1} Z_{0,\pi}^*). \tag{13}$$

From (5), (6), and (13) we find

$$\left(\frac{\rho_\pi}{\pi T_c}\right)^{1/2} - \left(\frac{\rho_0}{\pi T_c}\right)^{1/2} = \frac{\pi y}{2\gamma\xi_n^*} \frac{\sinh(y)\cos(y) - \cosh(y)\sin(y)}{\sinh^2(y)\cos^2(y) + \cosh^2(y)\sin^2(y)}. \tag{14}$$

It follows from (14) that all small thicknesses of the N layer the relation $\rho_\pi > \rho_0$ holds, and the 0 phase exists in the superlattice. The transition to the π phase with increasing d_n, however, occurs at smaller thicknesses of the normal regions than at small values of γ. This phase turns out to be preferable under the conditions

$$(\xi_n^*/h)(\theta + \pi k) < d_n < (\xi_n^*/h)(\theta + \pi(k+1)), \tag{15}$$

where $k = 2n + 1$ is an even integer. For odd integer values of k, expression (15) determines the region in which the zero phase exists.

The alternation of 0 and π phases with increasing value of the parameter d_n or I leads to a nonmonotonic dependence of the superlattice transition temperature $T_c^*(d_n, I)$, determined by (7), (9), and (12).

A ground state with an order-parameter phase which varies from layer to layer can thus exist in an SF superlattice. It is important to note that under the condition $d_n \ll \xi_n^* h$ the formation of the zero phase is always preferable, and the transition to the

π phase with increasing d_n (or with decreasing I) occurs at values of d_n which are smaller, the larger the parameter γ.

This approach opens up some possibilities for a theoretical analysis of the Josephson effect in weak links with a ferromagnetic interlayer, in which we might also expect an oscillatory dependence of the critical current on d_n and I.

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