QCD analysis of the *FSI* phases and *CP* asymmetry in inclusive decays of *B* particles

Yu. L. Dokshitser and N. G. Ural'tsev
B. P. Konstantinov Institute of Nuclear Physics, USSR Academy of Sciences

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The difference of the inclusive widths $\Gamma(\bar{b}\to\bar{s}+\text{charmless}) - \Gamma(b\to s+\text{charmless})$ is calculated in the leading-log approximation. In contradiction to earlier statements, the higher-order corrections do not preserve the *CP*-odd asymmetry, but decrease it by $\lesssim 15\%$. The asymmetry turns out to be about -10^{-2} and the total probability is $\simeq 2.5\times 10^{-3}$ for the parameters $|V_{ub}/V_{cb}| = 0.1$ and $\sin\alpha = 0.28$.

The problem of *CP* violation in beauty particles is currently very important. The strategies for the immediate search for these effects on the basis of the standard model (SM) in e^+e^- collisions and the size of the expected effects are basically clear[1] and focus on *CP*-odd effects in B^0 mesons arising because of B^0–$\bar{B}^0$ transitions. Effects for which B^0–$\bar{B}^0$ mixing is not important, for example, in decays of $B^\pm$ mesons or the Λ_b baryon, are also extremely interesting. Such decays are especially attractive for hadron collisions. However, in these processes the *CP*-odd asymmetries, except for the *CP*-odd difference in the phases of the elements of the Kobayashi-Maskawa (KM matrix, which determine the interference amplitudes, are also proportional to the sine of the difference in the *CP*-even phases (*FSI*) of these amplitudes (see, for example, Ref. 1 for a discussion). Since the *FSI* arise because of the strong final-state interaction, their magnitude is extremely uncertain.

The most promising decays are those corresponding to transitions $b\to u\bar{u}s$. In this case the "penguin" processes $b\to s+(c\bar{c},t\bar{t})_{virt}\to s+q\bar{q}$, which have a numerically similar suppression, can also contribute. The corresponding amplitude in the one-loop approximation literally has the *CP*-even phase

$$\tan\delta_p \simeq \pi/\ln(m_t^2, m_w^2/m_b^2).$$

Numerically $\delta_p \simeq 0.5$, but when the sizeable *c*-quark mass is taken into account, we have $\delta_p \simeq 0.1$.

There is a widely held notion that the other *FSI* phases, in particular, those related to large distances, are negligible, and only the phase δ_p is important for estimating these effects. One also encounters the opposite viewpoint that the *FSI* phases are large. In this case, however, it is impossible to theoretically predict not only the magnitude but also the sign of the effect.

We assume that *a priori* there is no reason to assume that the *FSI* phases are small, at least compared to δ_p. This pertains, in particular, to exclusive processes, where the answer depends on the actual final state formation dynamics, in particular, on its "hard" part. For example, for color-suppressed decays the "hard" part can contain hard gluon exchange, and in this case it is natural to expect that $FSI \sim \pi/2$. Moreover, even from the viewpoint of QCD the perturbation theory (PT) parameter for exclusive processes is $\alpha_s(m_b\mu_{hadr})$, rather than $\alpha_s(m_b^2)$; here "hybrid" logarithms can also arise. Meanwhile, from the theoretical point of view, the QCD language for quarks and gluons is adequate, the PT corrections are controlled by the parameters $\alpha_s(m_b^2)$, and nonperturbative effects must be small for the fairly heavy b quark. Therefore, here the estimate based on δ_p is a reasonable zeroth-order approximation.

In the present study we obtain an expression for the CP-odd difference of the inclusive widths of b decays in a state without heavy quarks. Numerically, the higher-order corrections only insignificantly (10–20%) decrease the asymmetry, contrary to the statement of Ref. 2, where strong cancellation of the effect was indicated.

The basic equation for the CP-odd difference of the widths is

$$\Delta\Gamma = \Gamma(\bar{b} \to \bar{s}q\bar{q}) - \Gamma(b \to sq\bar{q}) = -4\mathrm{Im}(\lambda_i\lambda_j^*)\sum_F \mathrm{Im}(A_i(b \to F)A_j^*(b \to F)), \quad (1)$$

where F are the final states, which are summed over, A_i are the decay amplitudes ignoring the KM factors, and λ_i are the corresponding KM factors. The unitarity condition for the amplitude A_i allows us to write (1) as

$$\Delta\Gamma = -2\mathrm{Im}(\lambda_i\lambda_j^*)\sum_F\sum_I A_i(b \to I)A^*(F \to I)\mathrm{Re}A_i(b \to F)$$

$$-A_j(b \to I)A^*(F \to I)\mathrm{Re}A_i(b \to F). \quad (2)$$

Here I are real intermediate states in decays $b \to F$, which lead to the FSI phases of the amplitudes A_i, j; $A(F \to I)$ are generated by the strong interaction.

Using the unitarity condition, it can be shown that in any graph, when all possible states F are summed over for fixed i,j, the second imaginary term in (1) vanishes. The unitarity of the Feynman diagrams therefore causes the requirement imposed by the CPT theorem to be satisfied already when all the cuts of each diagram are taken into account, and not only upon summing the contributions of all possible diagrams. Using these identities, instead of the width $\Delta\Gamma$ we shall calculate the difference in the widths $\Delta\Gamma_c = \Delta\Gamma(b \to sc\bar{c} + X) = -\Delta\Gamma$. This is convenient because here the suppression effect due to the c-quark mass is trivial and does not look like the accidental cancellation of the FSI phases of certain diagrams.

In perturbative QCD $\Delta\Gamma$ arises at order α_s and does not contain $\ln(m_t^2/m_b^2)$. In the spirit of the standard leading-log approximation (LLA), we calculate all corrections of the form $\alpha_s^{n+1}\ln^n(m_t^2/m_b^2)$. The result is the following: to calculate $\Delta\Gamma$, as in lowest order, it is sufficient to use for $A(F \to I)$ in (2) the one-gluon rescattering amplitude $c\bar{c}X \to q\bar{q}X'$, but the weak amplitudes $A_{i,j}(b \to F,I)$ must be determined by the renormalized effective Lagrangian for $q^2 = -m_b^2$. We note that this result cannot be obtained by simply assuming that the FSI phase is equal to the phase of the amplitude $b \to sq\bar{q}$, obtained by renormalization-group extrapolation to the Minkowski region $q^2 = +m_b^2$.

In order to prove the validity of this prescription, we shall consider all the possible states F and I, step by step, estimating the corresponding "strong" amplitudes $A(F\to I)$. It is important that, because of the reality of the states F and I, the $A(F\to I)$, which are expressed in terms of α_s normalized at $q^2 = -m_b^2$, do not contain large logarithms $\ln(m_t^2, m_w^2/m_b^2)$.[1)]

It is easy to see that for states I containing a $c\bar{c}$ pair the two terms in (2) cancel; this also follows naturally from CPT. In all other cases $A(F\to I)$ contains at least g_s. Systematic analysis of all types of states F and I shows that the minimum power of α_s is reached in one-gluon annihilation $c\bar{c}\to g_{virt}\to q\bar{q}$. Here a key fact is the absence of $gg+s$ and $ggg+s$ among the states I in the LLA. In fact, the "penguin" operators which appear due to the virtual momenta $q^2 \gg m_b^2$ have the form $\bar{b}\gamma_\mu(\lambda^a/2)s\nabla_\nu G^a_{\mu\nu}$, and from the equations of motion $\nabla_\nu G^2_{\nu\mu} = 0$ in the absence of a quark current. For states $gg+s$, in particular, this corresponds to cancellation of the graphs in Figs. 1a and 1b in the LLA for real gluons. The induced (with very small coefficient) operator $m_b\bar{b}_R(\sigma G)S_L$ also, obviously, does not contribute to (2) in this order in α_s.

In the end, the smallest (two-loop) correction to $\Delta\Gamma$ reduces to the substitution

$$\Delta\Gamma \sim \frac{\alpha_s}{3\pi}\mathrm{Im}\left(\ln\frac{m_t^2}{m_b^2} + i\pi\varsigma\left(\frac{m_c^2}{m_b^2}\right)\right)$$

$$\to \frac{\alpha_s}{3\pi}\mathrm{Im}\left(\ln\frac{m_t^2}{m_b^2} + i\pi\varsigma\left(1 - \frac{\alpha_s}{3\pi}(n_f+1)\ln\frac{m_t^2}{m_b^2} + \kappa(c_\pm)\right)\right), \qquad (3)$$

where n_f is the number of light quarks including the c quark, $\zeta \simeq 0.2$ is the phase space suppression factor, and $\kappa \simeq 2\alpha_s/4\pi\ln(m_w/m_b)^2$ is the "ordinary" correction due to the difference of $c\pm$ from 1 in H_{eff}. For $n_f = 3$ and $\alpha_s(m_b^2) = 0.18$ the first nontrivial

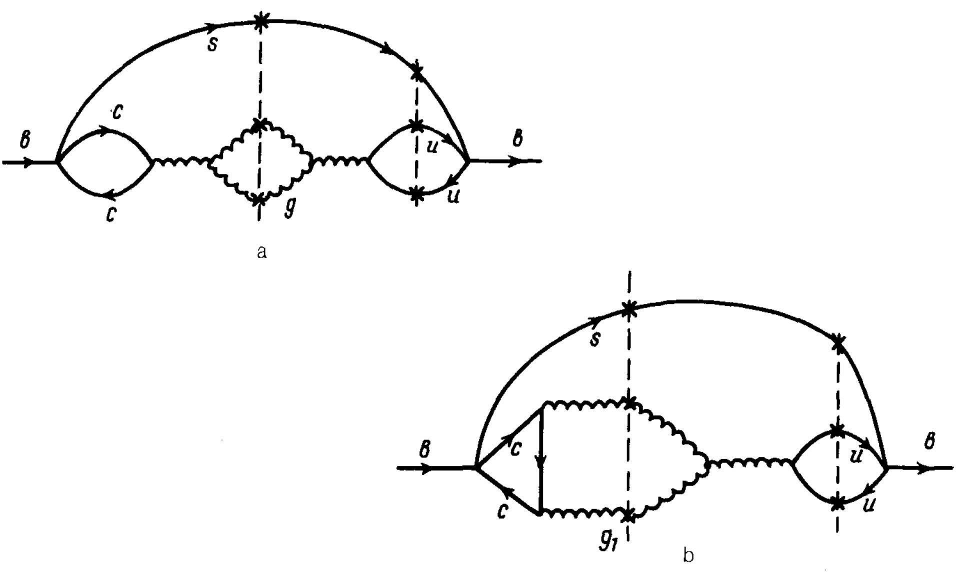

FIG. 1.

correction is -0.17, but it largely cancels with $\kappa \simeq 0.13$. Summation of all orders of the LLA hardly changes the answer at all—the total correction is literally about -2% for $\Lambda_{QCD} = 0.1$–0.3 GeV. The electroweak corrections to H_{eff} are also small even for $m_t \simeq 250$ GeV. Numerically, for the inclusive asymmetry we obtain

$$\frac{\Gamma(\bar{b} \to \bar{s} + \text{charmless}) - \Gamma(b \to s + \text{charmless})}{\Gamma(\bar{b} \to \bar{s} + \text{charmless}) + \Gamma(b \to s + \text{charmless})} \simeq -1.9\varsigma \left| \frac{V_{ub}}{V_{cb}} \right| \sin \alpha \simeq -10^{-2}, \quad (4)$$

where $\alpha = \arg(V_{cb}^* V_{cd} V_{ub} V_{ud}^*)$ is one of the angles of the unitarity triangle,[1] and $\alpha \simeq 0.28$ for $|V_{ub}/V_{cb}| = 0.1$. The total probability $\mathrm{Br}(b \to s + \text{charmless})$ here is $\simeq 2.5 \times 10^{-3}$.

This result completely disagrees with that of Ref. 2, where it was stated that the QCD corrections almost completely cancel this difference of the widths. The authors of that study actually took into account only (the gauge noninvariant!) part of the diagrams in Fig. 1a and ignored in Fig. 1b; also, the $c\bar{c}$ phase space factor for the corrections to the width was perhaps also left out in their numerical result.

Effects in decays $b \to qs\bar{s}$, which are proportional to the difference in the $u\bar{u}$ and $c\bar{c}$ phase spaces, are calculated in a similar manner. With the same assumptions for $b \to s\bar{s}s$ the asymmetry is only -9×10^{-3} for a probability of $\simeq 5 \times 10^{-4}$, and for $b \to d\bar{s}s$ they are $+7 \times 10^{-2}$ and 7×10^{-5}, respectively.

Our analysis also confirms the point of view that, in general, even for "penguin" processes in the limit $m_b^2 \gg m_c^2 \gg \Lambda_{QCD}^2$ it is impossible to state that the *FSI* phases are generated at short distances and are local. Nevertheless, for inclusive processes the parameter of perturbative QCD for the *FSI* complexity is $\alpha_s(m_b^2)$ and, at least parametrically, the inclusive *CP* asymmetries are amenable to perturbative analysis.

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[1)] For individual states F, I the amplitudes can contain infrared singularities, which vanish upon summation over states with arbitrary numbers of gluons. In any case, these singularities have no relation to the "ultraviolet" logarithms $\ln(m_t, m_w/m_b)^2$ of interest to us.

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