Higher-order corrections to the Chern-Simons term in scalar electrodynamics

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A nontrivial two-loop contribution to the Chern-Simons term appears in the spontaneous vilation of the gauge symmetry. There is no spontaneous breaking of parity.

In (2 + 1)-dimensional Abelian gauge theories the Chern-Simons (CS) term describes a change of the excitation statistics, which can serve as the basis for a mechanism of high-temperature superconductivity.[1] In Ref. 2 it was proved that in a fairly general case there is only a one-loop renormalization of the topological term, which requires transversality and analyticity of the photon Green's functions in the external momenta. A number of arguments in favor of such a statement for non-Abelian theories have been given in Ref. 3. In Ref. 4 it was shown by direct calculation that the two-loop contribution of massive fermions vanishes. The presence of massless charged particles violates the analyticity condition, which can lead to finite corrections. The two-loop contribution of massless scalars, computed approximately in Ref. 5, is nonzero. In all the studies quoted above it was assumed that scalar fields are not important for the one-loop analysis. However, it has recently been shown[6] that spontaneous violation of the gauge symmetry leads to one-loop renormalization of the CS term induced by the Higgs field. In the present study we analyze the two-loop corrections to the CS term in scalar electrodynamics for intact and broken gauge symmetry. In the former case, the massless scalar contribution is found analytically. In the latter it is shown that the contribution is nonzero even for massive scalars. This, in particular, implies the absence of spontaneous parity violation.[6]

We consider a $U(1)$ gauge theory with the Lagrangian

$$L = -\frac{1}{4e^2}F_{\mu\nu}^2 + \frac{1}{2}k\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda - \frac{1}{2\xi}(\partial_\mu A_\mu)^2$$

$$+|D_\mu\phi|^2 + h(\phi^*\phi - c^2/2)^2 + \sum_{i=1}^{n_s}(|D_\mu\chi_i|^2 - m^2\chi_i^*\chi_i), \quad (1)$$

where $D_\mu = \partial_\mu - iA_\mu$. For $c^2 < 0$ the gauge symmetry is unbroken and the polarization operator has the form

$$\Pi_{\mu\nu}(p) = (g_{\mu\nu}p^2 - p_\mu p_\nu)\Pi(p^2) + i\epsilon^{\mu\nu\lambda}p_\lambda\Pi_0(p^2). \quad (2)$$

The topological term enters into the effective action with the coefficient $k_{\text{eff}} = k + \Pi_0(0)$. If $m^2 \neq 0$, then $\Pi_0(0) = 0$ in all orders of perturbation theory.[2] We note that this statement remains valid even if a term $\propto M_A A_\mu^2$ is added to (1), since

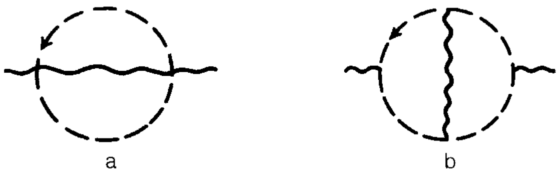

FIG. 1.

the transversality of the photon amplitudes is preserved. However, this addition crucially affects the contribution of massless particles to $\Pi_0(0)$.

Let us analytically calculate the contribution of χ scalars to $\Pi_0(0)$. The corresponding important diagrams are shown in Fig. 1. To analyze the various situations, we use the part of the propagator of the field A_μ proportional to the ϵ tensor in the form

$$D_{\mu\nu}(p) = \frac{\epsilon^{\mu\nu\lambda}\, p_\lambda}{M^2 - p^2} \, . \tag{3}$$

All the ϵ parts of the physically interesting propagators can be constructed by linear combinations of (3) for different M. In the calculations we use dimensional regularization[3] and integration by parts.[7] Diagram 1a gives

$$\Pi_0(p^2) = n_s \frac{4\Gamma(3-D)}{(4\pi)^D} \begin{cases} \dfrac{\Gamma^2(D/2-1)\Gamma(3-D/2)}{\Gamma(D/2+1)}(M^2)^{D-3}, & m^2 = p^2 = 0, \\ \dfrac{2\Gamma^2(2-D/2)}{D\Gamma(4-D)}(m^2)^{D-3}, & M^2 = p^2 = 0, \\ \dfrac{\Gamma^2(D/2-1)\Gamma(D/2)}{\Gamma(3D/2-2)}(-p^2)^{D-3}, & m^2 = M^2 = 0, \end{cases} \tag{4}$$

where D is the space dimension. Diagram 1b in the first two limits in (4) gives exactly the same expressions, but with the opposite sign, and for $m^2 = M^2 = 0$ we obtain

$$\Pi_0(p^2) = n_s \frac{4\Gamma(3-D)\Gamma^3(D/2-1)}{(4\pi)^D(D-1)} \left(\frac{\Gamma^2(2-D/2)\Gamma(D/2)}{\Gamma^2(D-1)\Gamma(3-D)} - \frac{1}{\Gamma(3D/2-2)} \right) (-p^2)^{D-3}. \tag{5}$$

Therefore, the contribution of massless scalars is nonzero only when there is a massless pole in the ϵ part of the propagator. When (4) and (5) are added, the ultraviolet divergences cancel and we have, regardless of the value of e^2 in (1),

$$k_{\text{eff}} = k\left(1 + \frac{\pi^2 - 4}{64\pi^2 k^2} n_s\right) = \frac{\alpha}{2\pi}\left(1 + \frac{2,30}{2\pi\alpha^2} n_s\right). \tag{6}$$

The second expression is given for a comparison with the result of Ref. 5, where numerical computation of a complicated integral gave -2.74, instead of 2.30. The difference in signs is apparently due to a difference in notation, and the difference between the absolute values is caused by numerical error.

Let us now consider the case $c^2 > 0$. Substituting in (1) $\phi = (c + \phi_1 + \phi_2)/\sqrt{2}$, we find the following Ward identity for the effective action Γ, from which the gauge-fixing term is omitted:

$$\partial_\mu \frac{\delta\Gamma}{\delta A_\mu(x)} + \phi_2(x)\frac{\delta\Gamma}{\delta\phi_1(x)} - (c + \phi_1(x))\frac{\delta\Gamma}{\delta\phi_2(x)} = 0. \quad (7)$$

We see that the purely photon Green's functions are no not transverse, so there is no reason to expect that higher-order corrections to the CS term are absent. (In the R_ξ gauge the Ward identities are more complicated and also lead to nontransversality.)

Let us calculate the contribution of χ scalars to $\Pi_0(0)$ in the two-loop approximation. The gauge propagator has the form (we do not write out the mixing of A_μ and ϕ_2)

$$iD_{\mu\nu}(p) = e^2 \frac{(g_{\mu\nu} - p_\mu p_\nu/p^2)(p^2 - M^2) + i\mu\epsilon_{\mu\nu\lambda}p_\lambda}{(M^2 - p^2)^2 - \mu^2 p^2} + \xi \frac{p_\mu p_\nu}{p^4}, \quad (8)$$

where $M = ec$, and $\mu = ke^2$. Since there is no massless pole in the ϵ part, the contribution of the diagrams in Fig. 1 vanishes independently of m^2. However, now there are additional diagrams, shown in Ref. 2, in which the solid line corresponds to the Higgs field. They are finite and give (the momentum integral is Euclidean)

$$\Pi_0(0) = -\frac{8}{3\pi}e^2 M^2 \mu n_s \int_0^1 dx \int \frac{d^3q}{(2\pi)^3} \frac{q^2(M^2 + q^2)((m^2 + x(1-x)q^2)^{1/2} - m)}{(M_H^2 + q^2)^2((M^2 + q^2)^2 + \mu^2 q^2)^2}, \quad (9)$$

where M_H is the Higgs boson mass. Taking into account the one-loop correction,[6] in the limit $e^2 \to \infty$, M_H^2, $m^2 \ll c^4/k^2$ we obtain

$$k_{\text{eff}} = k + \frac{2}{3\pi}\frac{k}{|k|} - \frac{1}{12\pi^2 k} n_s + \ldots, \quad (10)$$

where the ellipsis stands for the uncalculated two-loop contribution of the scalars ϕ_1 and ϕ_2. Since n_s is a free parameter, this unknown correction cannot cancel the contribution of the χ scalars.

On the basis of the one-loop calculation, the authors of Ref. 6 made the interesting suggestion that in the strong coupling regime ($e \gg c$) there is spontaneous breaking

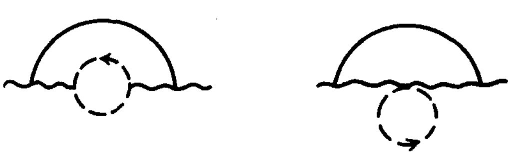

FIG. 2.

of parity—the small, bare topological term grows to macroscopic size with coefficient $\propto k/|k|$. We see from expression (10) that this is not true—there are higher radiative corrections which are unstable for $k \to 0$.

We have therefore shown that when the gauge symmetry breaks down spontaneously, the Coleman-Hill theorem does not apply and the Chern-Simons term is renormalized in higher orders of perturbation theory. Expression (9) determines the two-loop component of massive scalar fields $\chi_i(x)$. A curious feature is the smoothness of the limit $m^2 \to 0$. It is natural to assume that fermions also give a nonzero contribution that possesses this feature. Moreover, if the mass fermion terms do not conserve P-parity, then diagrams such as those shown in Fig. 2 will generate a topological term even for $k=0$. A separate study will be devoted to analysis of the structure of the fermion contribution.

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