# Quark and lepton masses in a model with a discrete symmetry

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A model with $n = 2^p$ $(p = 14)$ heavy Higgs doublets and a $Z_n$ symmetry is studied. After spontaneous breaking of the $Z_n$ symmetry, realistic mass matrices for the quarks and leptons are easily reproduced in the model and the moduli of all the Yukawa constants are equal.

The observed fermion (quark and lepton) mass hierarchy plus the limit[1] on the $t$-quark mass, $m_t \geqslant 77$ GeV, has no natural explanation in the standard model (SM). Various models, in which this hierarchy can arise naturally, have been proposed.[2] One of them is a model with a discrete $Z_n$ symmetry, which prohibits mass terms for all the

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fermions except the $t$ quark. In Ref. 3 the author studied a model of this type with $n=2^p$, $p=9$, where all the fermion masses (and quark mixing angles) were reproduced for all the Yukawa constants strictly equal in magnitude due to the choice of the small parameters characterizing the spontaneous breaking (SB) of the symmetry $Z_{(2^p)}$ to $Z_{(2^q)}$, where $q=0,...,p-1$. We can go further and require that the constants responsible for the breaking $Z_{(2^p)} \to Z_{(2^q)}$ would be identical for all $q$. In this natural scenario of $Z_n$ symmetry breaking it is possible to reproduce 12 physical quantities (3 lepton and 5 quark masses, 3 quark mixing angles, and the $CP$ phase) with a minimal number of fitted parameters.

In the model there are $1+n(n=2^p, p=14)$ Higgs doublets $\phi$, $\varphi_i$ $(i=1,...,n)$ which have the potential

$$V(\phi,\varphi_i) = \kappa(|\phi|^2 - v^2)^2 + m^2 \sum_{i=1}^{n} |\varphi_i - a\phi|^2. \tag{1}$$

The $Z_n$ symmetry is realized as the group of cyclic permutations of the doublets $\varphi_i$:

$$S = \begin{bmatrix} 123..n \\ 234..1 \end{bmatrix}.$$

The "Fourier transforms" of the doublets

$$\phi_k = \frac{1}{\sqrt{n}} \sum_{m=1}^{n} \exp\left(i\frac{2\pi k(m-1)}{n}\right) \varphi_m \tag{2}$$

have charges $k$ under $Z_n$ transformations: $S$: $\phi_k \to e^{i2\pi k/n}\phi_k$. The $Z_n$ symmetry fixes the form of the Yukawa couplings. For example, if the quark fields $Q_{Lj} \equiv \binom{u}{d}$, $d_{Rj}$ have $Z_n$ charges $q_i$, $d_j$, the Yukawa couplings of the $d$ quarks have the form

$$\mathcal{L}^d_{\text{Yuk}} = -\lambda^d_{ij} \bar{Q}_{Li} d_{Rj} \phi_{(q_i - d_j)}, \tag{3}$$

and similarly for the $u$ quarks (charges $q_j$, $u_j$, Yukawa constants $\lambda^u_{ij}$) and leptons (charges $l_j$,$e_j$, Yukawa constants $\lambda^e_{ij}$). The $Z_n$ charges of the quarks are chosen in such a way that $q_3 - u_3 = 0$, so the $t$ quark can also have Yukawa coupling with the $Z_n$-singlet doublet $\phi$:

$$\mathcal{L}^t_{\text{Yuk}} = -\bar{Q}_{L3} u_{R3} (\lambda^u_{33} \phi^c_0 + \eta^u \phi^c). \tag{4}$$

The $SU(2) \otimes U(1)$ symmetry breaks down to $U(1)$ in the vacuum:

$$<\varphi> = \begin{bmatrix} 0 \\ \nu \end{bmatrix}, \quad <\phi_k> = \begin{bmatrix} 0 \\ \nu_p \end{bmatrix} \delta_{k,0}, \tag{5}$$

where $v_p \equiv bv$, $b \equiv (n)^{-1/2}a$. As a result, we obtain the masses $m^2_W = \frac{1}{2}g^2_2 v^2(1+b^2)$, $m^2_Z = \frac{1}{2}(g^2_2 + g'^2)v^2(1+b^2)$ for the $W$ and $Z$ bosons and $m_t = [b\lambda^u_{33} + \eta^u]v$ for the $t$ quark.

Among the scalar particles in the physical sector are an "almost standard" Higgs of mass $m^2_H = 4\kappa v^2$ and $n$ charged and $+2n$ neutral scalars of mass $m$.

The $u$, $d$, $s$, $c$, and $b$ quarks and leptons acquire masses after the spontaneous breaking of the $Z_n$ symmetry. In order to break the $Z_{(2^p)}$ symmetry to $Z_{(2^q)}$, we introduce auxiliary scalar fields—electroweak singlets $\chi_{q,1}$, $\chi_{q,2},...,\chi_{q,l}$ $(l = 2^{p-q})$ which form an $l$-dimensional representation of $Z_n$, $:S:\chi_{q,1} \to \chi_{q,2},...,\chi_{q,l} \to \chi_{q,1}$, and which have the potential

$$V_q = \alpha[\sum_{i=1}^{l} \chi_{q,i}^2 - \mu_q^2]^2 + \sum_{j=1}^{l/2} \alpha_{q,j}[\sum_{i=1}^{l} \chi_{q,i}^2 \chi_{q,i+j}^2], \tag{6}$$

which spontaneously breaks $Z_{(2^p)}$ down to $Z_{(2^q)}$: $\langle \chi_{q,i} \rangle = \mu_q \delta_{i,1}$. The symmetry-breaking parameter is $z_q \equiv 2^{q-p} \beta_q \mu_q^2 / m^2$. This breaking of $Z_n$ is transferred to the sector of doublets $\phi_k$ by the interaction

$$V_q' = \beta_q \sum_{i=1}^{l} \chi_{q,i}^2 \sum_{j=0}^{2^q-1} [|\varphi_{i+jl}|^2 - a^2 \nu^2]. \tag{7}$$

As a result, the doublets $\phi_k$ acquire vacuum expectation values. We shall consider the simplest case, where $\beta_q = \beta$, $\mu_q = \mu$, i.e., $z_q = 2^{p-1-q} z$, where $z \equiv z_{p-1}$. For any $\phi_k$, where $k$ can be written as $k = 2^q (2j+1)$, $j = 0,1,...$, we define the corresponding vacuum expectation value as $\nu_q \equiv \langle \phi_k \rangle$. After minimizing the scalar potential, we obtain expressions for $\nu_q$, $q = 0,...,p-1$:

$$\nu_q = -\nu_p 2^{1-p} \left\{ \frac{pz}{1+2pz} + \sum_{s=1}^{q} 2^{s-1} \frac{(p-s)z}{1+2(p-s)z} - 2^q \frac{(p-q-1)z}{1+2(p-q-1)z} \right\}. \tag{8}$$

For $z \ll 1$ expression (8) reduces to $\nu_q = -\nu_p 2^{-p} (2^{q+1} - 1) z$. We take the matrices of Yukawa constants $\hat{\lambda}_{u,d,e}$ to be identical and hermitian,with the moduli of all the elements equal. By phase rotations of the fermions they can be brought to the form

$$\hat{\lambda}_{u,d,e} = \lambda \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & \exp(i\delta) \\ 1 & \exp(-i\delta) & 1 \end{bmatrix}. \tag{9}$$

To obtain the fermion mass matrices, it is necessary to choose the $Z_n$ charges of the quarks and leptons. The following choice of charges is suitable:

$$q_j = \begin{bmatrix} 2^6 \\ 2^9 \\ 2^{14} \end{bmatrix}, \quad u_j = -3 \begin{bmatrix} 2^3 \\ 2^9 \\ 2^{14} \end{bmatrix}, \quad d_j = - \begin{bmatrix} 2^4 \\ 2^8 \\ 2^{13} \end{bmatrix},$$

$$l_j = \begin{bmatrix} 2^5 \\ 2^7 \\ 2^{12} \end{bmatrix}, e_j = -5 \begin{bmatrix} 2^1 \\ 2^7 \\ 2^{13} \end{bmatrix}. \tag{10}$$

Substituting these charges into the Yukawa couplings (3), we find the fermion mass matrices:

$$M_e = \lambda \begin{bmatrix} \nu_1 & \nu_5 & \nu_5 \\ \nu_1 & \nu_8 & e^{i\delta}\nu_7 \\ \nu_1 & e^{-i\delta}\nu_7 & \nu_{12} \end{bmatrix}, \quad M_d = \lambda \begin{bmatrix} \nu_4 & \nu_6 & \nu_6 \\ \nu_4 & \nu_8 & e^{i\delta}\nu_9 \\ \nu_4 & e^{-i\delta}\nu_8 & \nu_{13} \end{bmatrix}, \tag{11}$$

$$M_u = \lambda \begin{bmatrix} \nu_3 & \nu_6 & \nu_6 \\ \nu_3 & \nu_{11} & e^{i\delta}\nu_9 \\ \nu_3 & e^{-i\delta}\nu_9 & \nu_{14} \end{bmatrix} + \eta \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \nu \end{bmatrix}.$$

For definiteness we assume $m_t \simeq 140$ GeV. Diagonalization of the mass matrix (11) for

$$\nu_p = \nu \simeq 130 \text{ GeV} \quad \lambda \simeq 2; \quad \eta \simeq -1; \quad z = 0.035; \quad \delta \simeq 0.1$$

gives the elements of the quark mixing matrix:

$$|V_{us}| = 0.2143, \quad |V_{cb}| = 0.0582, \quad |V_{ub}| = 0.0055$$

and the fermion masses on the scale $\mu \simeq m_W$, which by means of the standard formulas we can write as (masses given in MeV)

$$\begin{aligned} &m_e = 0.5117; && m_\mu = 105.6, && m_\tau = 1784 \\ &m_u(\text{ 1 Gev }) = 4.73; && m_c^0 = 1.48 \cdot 10^3, && m_t^0 = 140 \cdot 10^3, \\ &m_d(\text{ 1 Gev }) = 7.45; && m_s(1 \text{ Gev }) = 172, && m_b^0 = 5.04 \cdot 10^3, \end{aligned}$$

where we have used $\Lambda_{QCD} \simeq 100$ MeV. These results are in good agreement with those in the literature[4] (see, for example, Ref. 5 for the quark masses).

The smallness of the parameter $z = \beta\mu^2/(2m^2)$ is related to the fact that the scale $m$, which characterizes the masses of the additional (compared to the SM) Higgs scalars, is large. An analysis of various flavor-changing neutral current processes similar to that carried out in Ref. 3 shows that the model is consistent with experiment ($K$–$\overline{K}$ mixing and so on) for $m \geqslant 10$ TeV. The scalars $\chi_{q,i}$ necessary for spontaneous breaking of the discrete $Z_n$ symmetry have masses $\simeq \mu \simeq \sqrt{z}m \leqslant 500$ GeV. They do not interact with the vector and spinor fields and are therefore invisible in experiments.

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[1]F. Abe *et al.*, Phys. Rev. Lett. **64**, 142 (1990).
[2]S. Weinberg, Phys. Rev. Lett. **29**, 388 (1972); H. Georgi and S. L. Glashow, Phys. Rev. D **6**, 2977 (1972); 7, 2457 (1973); R. N. Mohapatra, Phys. Rev. D **9**, 3461 (1974); S. M. Barr and A. Zee, Phys. Rev. D **15**, 2652 (1977); **17**, 1854 (1978).
[3]N. I. Polyakov, Preprint ITEP-90-90, ITEP, Moscow (1990) [in Russian].
[4]Particle data Group, Phys. Lett. **204B**, 1 (1988).
[5]J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

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