On the renormalization of nonrenormalizable supersymmetric interactions

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It is shown that in four-dimensional spacetime the use of a special form of the diagrammatic technique makes it possible to renormalize a nonrenormalizable interaction of the form $\Delta L = h/M^2 \int d^2\theta (X,Y)^4 + h.c.$, where X and Y are superquark fields. The nonrenormalizable interaction ΔL corresponds to a special solution of the renormalization group equations of the renormalizable theory with superfield Φ and Lagrangian $\Delta L_1 = \int \Phi^+ \Phi d^4\theta + \int d^2\theta [hXY\Phi - (M\Phi^2/4)] + h.c.$

It is well known that the requirement of renormalizability greatly restricts the arbitrariness in the choice of Lagrangian, which leads to the large predictive power of renormalizable theories. It is natural to raise the question of whether any meaning can be attached to nonrenormalizable theories. In Ref. 1 it was shown that the use of the $1/N$ expansion allows the renormalization of the three-dimensional nonlinear σ model. In Refs. 2 and 3 the $1/N$ expansion was used to renormalize three-dimensional four-fermion models. Of course, the four-dimensional case is of greatest interest.

In the present study we show that in four-dimensional spacetime the use of a special form of the diagrammatic technique allows the renormalization of the nonrenormalizable supersymmetric interaction of the form $\Delta L = h/M \int d^2\theta (XY)^4 + h.c.$, where X and Y are superquark fields. The nonrenormalizable interaction ΔL corresponds to a special solution of the renormalization group equations of the renormalizable theory with superfield Φ and Lagrangian $\Delta L_Z = Z \int \Phi^+ \Phi d^4\theta + \int d^2\theta [hXY\Phi - (M\Phi^2/4)] + h.c.$

The Lagrangian of supersymmetric quantum chromodynamics has the form[4]

$$L = \int d^4\theta [X_{ai}(\exp(gV^i))^a_b X^b_i + Y_{ai}(\exp(-gV^i))^a_b Y^b_i] - \int d^2\theta m_i X^b_i Y_{bi}$$

$$+\frac{1}{64}\frac{\mathrm{tr}}{C_2(G)}\int d^2\theta W^\alpha W_\alpha + h.c. \tag{1}$$

The model contains a gauge supermultiplet $W_\alpha \sim (\lambda_\alpha, F_{\mu\nu})$ in the adjoint representation of the gauge group $SU(N)$ and M quark chiral superfields

$$X^b_i = \varphi^b_i + \sqrt{2}\theta\psi^b_i + \theta\theta F^b_i,$$

$$Y_{bi} = \bar{\varphi}^b_i + \sqrt{2}\theta\bar{\psi}_{bi} + \theta\theta \bar{F}^b_i$$

in the fundamental representation. We add to L the nonrenormalizable Lagrangian

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FIG. 1. Order-h^2 correction to the propagator of the field Φ.

ΔL, which after the introduction of an additional superfield Φ, is written in the form $\Delta L = \int d^2\theta[hXY\Phi + (M\Phi^2/4) + h.c., XY = X_i^b Y_{bi}$. The Lagrangian ΔL is the $Z \to 0$ limit of the renormalizable Lagrangian $L_Z = \Delta L_Z + L$. We need to show that for the Lagrangian $L + \Delta L$ the radiative corrections to the propagator of the field Φ are ultraviolet finite. Since we are interested in the ultraviolet behavior, the masses are not important. The order-h^2 correction to the propagator of the field Ψ is given by the graph shown in Fig. 1, and is proportional to the correlator

$$K(p^2) = -i\int e^{ipx} < 0|T((\varphi_i^b(x)\varphi_{bi}(x)), \quad (\varphi_i^b(0)\bar{\varphi}_{bi}(0))^*|0 > d^4x. \tag{2}$$

The Källén-Lehmann representation is valid for $K(p^2)$:

$$K(p^2) = -\int_0^\infty \frac{\rho(t)dt}{t + p^2 - i\epsilon}, \quad \rho(t) \geq 0. \tag{3}$$

In the case of free quarks $\rho(t) = \text{const}$ and the integral (3) is logarithmically divergent, which indicates the appearance of the logarithmically divergent counterterm $\Delta Z \int \Phi^+ \Phi d^4\theta$. However, the situation changes fundamentally when strong interactions are included. The spectral density $\rho(t)$ satisfies the renormalization group equation

$$\begin{gathered}(\mu\frac{d}{d\mu} + \beta(g)\frac{d}{dg} + 2\gamma(g))\rho = 0,\\ \beta(g) = -\beta_0 g^3 + O(g^5), \quad \gamma(g) = -\gamma_0 g^2 + O(g^4),\\ \beta_0 = \frac{1}{16\pi^2}(3N - M), \qquad \gamma_0 = \frac{1}{8\pi^2}\frac{N^2-1}{N}.\end{gathered} \tag{4}$$

The solution of the renormalization group equation is

$$\rho(t) = c(\ln t)^{-\gamma_0/\beta_0}(1 + O(\ln\ln t/\ln^2 t)), \tag{5}$$

where c is some number. It follows from solution (5) that for $\gamma_0/\beta_0 > 1$ the integral in the Källén-Lehmann representation (3) is ultraviolet finite. For $N = 3$ the integral is finite for $M = 4,5,6,7,8$. We have therefore found that in the h^2 approximation the inclusion of strong interactions leads to ultraviolet convergence. In our case we can develop a special form of the diagrammatic technique like the $1/N$ expansion[5] using as the bare propagator of the field Φ propagator $(1/p^2)K^{-1}(p^2)$ generated by the graph in Fig. 1. In this case it is possible to show that in higher orders of the modified perturbation theory the radiative corrections to the propagator of the field Φ are finite. This can be understood using the renormalization group equations for the renormalizable Lagrangian $L + \Delta L_1$. In the one-loop approximation the equations have the form

$$\mu \frac{d\bar{g}}{d\mu} = -\beta_0 \bar{g}^3,$$

$$\mu \frac{d\bar{h}}{d\mu} = a\bar{h}^3 - \gamma_0 \bar{g}^2 \bar{h}, \quad a = \frac{1}{16\pi^2}(2 + MN). \tag{6}$$

For the special solution

$$\bar{h}^2 = k\bar{g}^2, k = \frac{1}{a}(\gamma_0 - \beta_0)$$

the propagator of the field Φ has ultraviolet asymptotic behavior of the form

$$D(p^2) \sim \frac{1}{p^2}(\ln p^2)^d, \quad d = (\frac{\gamma_0}{\beta_0} - 1)\frac{NM}{2 + NM} \quad .$$

For $h^2 < kg^2$ the asymptotic limit of the propagator coincides with the free propagator $1/p^2$. It follows from the equal-time commutation relations that for the Lagrangian $L + \Delta L_Z$ the asymptotic behavior of the propagator of the field Φ is $1/Zp^2$. We thus find that the special solution corresponds to the limit $Z \to 0$ when the regularization is removed. Therefore, the special solution corresponds to the absence of the kinetic term for the superfield in the Lagrangian, i.e., to the nonrenormalizable Lagrangian $L + \Delta L$. This can also be verified by writing out the Schwinger equation for the propagator of the field Φ.

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[1] I. Ya. Arefieva, Ann. Phys. (N.Y.) **117**, 393 (1979).
[2] B. Rosenstein, B. J. Warr, and S. H. Park, Phys. Rev. Lett. **62**, 1433 (1989).
[3] N. V. Krasnikov, A. B. Kyatkin, and N. A. Tavkhelidze, Preprint UWThPh-1989-66, Vienna University, Vienna (1989).
[4] J. Wess, Lectures given at Princeton University, Princeton University Press, 1981.
[5] A. Chatterjee, Phys. Rep. **186**, 249 (1990).

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