Phase transitions and new types of order in the Ising model with stray field defects

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A theory of phase transitions of a d-dimensional Ising system with stray field defects, which occur as a result of increasing degree of disorder, is derived. New types of ordered phases with $1 < d \leqslant 2$ are predicted.

The properties of disordered systems have been the topic of sustained interest for many years. One of the most important recent achievements in this field is the proof that the lower critical dimension of the Ising model with stray field defects is $d = 2$ (see, for example, the review article in Ref. 1 and the bibliography cited there). This result corresponds to a statement of the problem in which the dimension which is chosen is such that a uniformly magnetized state of the sample is unstable with respect to the case in which the sample is partitioned into domains in the presence of a negligible disorder. In the opposite limiting case of a strong disorder, the system is found in the disordered state, regardless of the dimension. At $d > 2$, therefore, a phase transition occurs as the degree of disorder is increased. Our purpose was to study this transition. We will show that for $1 < d \leqslant 2$ there are new types of order with zero spontaneous magnetization, and that the corresponding phases also undergo a phase transition to a disordered state. The analysis carried out below is conceptually similar to the scaling theory of the Anderson localization (see, for example, Ref. 2), with which we will compare it to the extent that it is possible.

We recall first the solution of the problem of the lower critical dimension. A possible method, which is useful in the analysis below, involves the determination of the surface tension of a domain wall as a function of the dimension L. This dependence stems from the fact that the defects, which cause the wall to be irregular, decrease its surface energy. The corresponding relative contribution is given by[1]

$$\frac{\Delta\sigma}{\sigma_0}=\frac{3}{2}\left(\frac{\Delta a^{2-d}}{\Gamma^2}\right)^{2/3}\frac{1-(L/a)^{(2/3)(2-d)}}{2-d}. \quad (1)$$

Here σ_0 is the bare (starting) surface tension, Γ is the effective rigidity of the domain wall, Δ is the mean-square fluctuation of the stray field, which we define for convenience in such a way that the common factor on the right side of Eq. (1) would be 3/2 and a is the cutoff scale in the plane of the wall. The slight disorder limit corresponds to the inequality $k=\Delta a^{2-d}/\Gamma^2 \ll 1$. For $d>2$ the first term in Eq. (1) is a small negative contribution, compared with unity, to the surface tension, and the second term is a positive dimensional correction which vanishes as $L\to\infty$. For $d\leqslant 2$, however, the sign of this correction changes, the correction begins to increase, and at a certain critical value of L_c it reduces the total surface tension to zero, causing the states with a uniform magnetization to be unstable. Curiously, a similar circumstance occurs in the localization theory, where the role of σ_0 is played by the conductivity, L_c corresponds to the localization length, and Eq. (1) corresponds to the interference increment in the conductance.[2] To describe the phase transition (the analog of the Anderson transition), Eq. (1) must be reformulated in terms of the renormalization group. A similarity to the localization theory suggests that the key parameter of the renormalization group is a quantity such as the conductance. Such is the difference in the free energies of the systems in which the spins at the walls are set in the opposite direction and in the same direction, respectively. The resultant excess energy is appropriate to call the total rigidity G. If $G(L\to\infty)\to 0$, then this is an indication that the phase is disordered. In any other case ($G\to\text{const}$ or $G\to\infty$) the phase is an ordered phase. If $G(L\to\infty)=\sigma L^{d-1}$, the order is characterized by the spontaneous Ising magnetization, while the last relation determines the surface tension σ. Analysis of the $G(L)$ dependence is very useful, since it identifies new type of order different from the Ising order at $1<d\leqslant 2$.

The Gell-Mann–Low function $\beta(G)=\partial \ln G/\partial \ln L$ can be reconstructed exactly as in the case of the localization theory. If we have an ordered phase with a spontaneous magnetization, it corresponds to a scale-independent σ in the limit $L\to\infty$ or $G=\sigma L^{d-1}$. We thus have in zeroth order

$$\beta(G)=d-1.$$

Using expression (1) to determine the next approximation of $\beta(G)$ in the limit $G\to\infty$, we find that $\beta(G)=d-1-k^{2/3}(L/a)^{2/3(2-d)}$. Substituting into this expression the zeroth-order result $G=\sigma L^{d-1}$, we find

$$\beta(G) = d - 1 - k^{2/3}(G/\sigma a^{d-1})^{\frac{2}{3}\frac{2-d}{d-1}} . \tag{2}$$

In the strong-disorder limit ($G \to 0$) we expect that the change in the boundary conditions affects only a narrow region near the edge of the sample. Accordingly, $G(L) \sim \exp(-L/L_c)$ and, by analogy with the localization theory, we have

$$\beta(G) = \ln G + \text{const.}$$

Using the asymptotic formulas for the large and small values of G, we can postulate the dependence $\beta(G)$, which is shown in Fig. 1 for the case $k \ll 1$. It should be noted that the last curve can, strictly speaking, be reconstructed only when $d \to 2$ from below. There is no reason, however, that the $\beta(G)$ curve should change qualitatively even under less severe restrictions, $1 < d < 2$.

The resultant renormalization group equation

$$\frac{\partial \ln G}{\partial \ln L} = \beta(G)$$

has an unstable fixed point G_c in the case $d \geqslant 2$, and two fixed points: G_{c1} (the unstable point) and G_{c2} (the stable point) in the case $1 < d < 2$. The first case resembles the localization theory. If the initial value is $G_0 \simeq \sigma_0 a^{d-1} < G_c$, then $\beta < 0$, $G(L \to \infty) \to 0$, and we therefore have a disordered phase. If $G_0 > G_c$, then $\beta > 0$ and $G \to \infty$, which suggests that we have an ordered state. In the case $d > 2$ the total rigidity asymptotically approaches the law $G = \sigma L^{d-1}$, indicating that we have an Ising disorder. With $d = 2$, the total rigidity approaches the dependence $G = gL^{1-k^{2/3}}$ where g is the corresponding specific rigidity. It can be asserted that the ordered state is characterized in this case by a spontaneous zero-valued magnetization, since the exponent in the last relation is less than unity. The ordered phase is even less rigid in the case $1 < d < 2$. If $G_0 > G_{cl}$, the total rigidity in this case approaches G_{c2}. In contrast with the localization theory, passage through the transition point with changing k is achieved as a result of the corresponding displacement of the $\beta(G)$ curve, while holding the initial condition G_0 nearly constant.

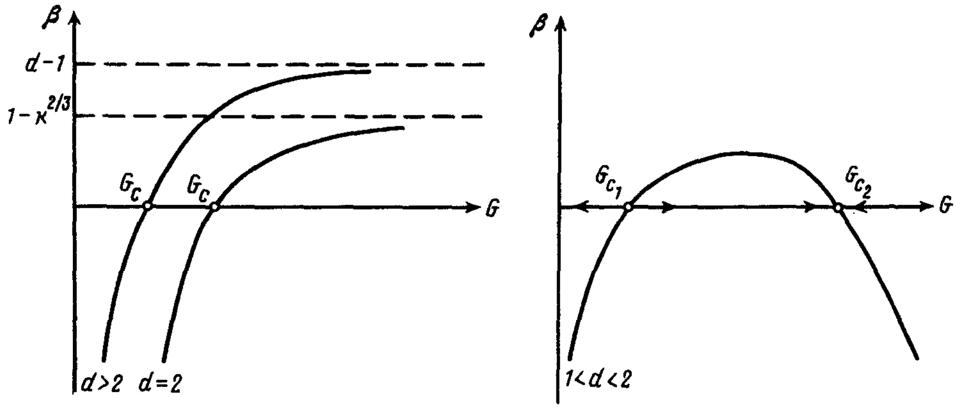

FIG. 1.

Since the analysis of the phase transition for $G = G_c\,(d \geqslant 2)$ exactly repeats the analysis of the Anderson transition, we will immediately write the final results. Near the transition point the correlation length diverges in accordance with the relation

$$L_c \simeq a\left|\frac{G_0 - G_c}{G_c}\right|^{-\frac{1}{y(d)}},$$

where $y = \partial\beta/\partial \ln G\,|G = G_c$, and the surface tension and specific rigidity $g(d = 2)$ vanish as

$$\sigma \sim (G_0 - G_c)^{\frac{d-1}{y(d)}}, \qquad g \sim (G_0 - G_c)^{\frac{1-k_c^{2/3}}{y(2)}}.$$

Since the initial value of G_0 is a finite value, we can assert that $k_c < 1$ at the transition point $(d = 2)$. A crude estimate of the index y is possible only at $d > 2$ if it is assumed that G_c is determined primarily by expression (2). We thus have

$$y(d) = \frac{2}{3}(d - 2).$$

In contrast with the localization theory, however, such an estimate for $d \to 2$ is clearly incorrect, since for $d = 2$ the right side of (2), in general, does not depend on G.

We note in conclusion that the single-parameter nature of the theory discussed above seems to be justifiable because of the absence of renormalization of the wall rigidity Γ, and of the mean-square fluctuation of the stray field,[1] Δ.

There is also another basic fact which states that the $\beta(G^{-1})$ dependence is nonanalytic in nature in the limit $G \to \infty$. This fact distinguishes the arguments given above from those of the scaling theory of the Anderson localization. This condition is a corollary of the expression for the correction [(1)] to the surface tension, which is of an essentially nonperturbative origin.[1] Although the analytic properties of the correction are not used (just as in the localization theory) in constructing the Gell-Mann–Low function, this circumstance by itself deserves attention.

I wish to thank the referee for this comment.

[1] T. Natterman and J. Villain, Phase Transitions **11**, 5 (1988).
[2] A. A. Abrikosov, *Principles of the Theory of Metals*, Nauka, Moscow (1987).

Translated by S. J. Amoretty