Phase transition to the superconducting state in a strong magnetic field

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(Submitted 16 October 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **52**, No. 10, 1141–1145 (25 November 1990)

The phase transition from the Abrikosov vortex lattice state to the normal state is shown to be able to split into two transitions. The intermediate vortex liquid phase differs from the normal phase by the presence of screening for current j which flows parallel to the external magnetic field.

1. It was assumed an established fact until now that a phase transition to a superconducting state in an external magnetic field is a consequence (in the case of type-II superconductors) of a spatially periodic solution (the Abrikosov vortex lattice) of the Ginzburg–Landau equations. This conclusion was drawn, however, without regard for the effect of thermal fluctuations near the phase transition point, which is quite justifiable for ordinary superconductors, where the region of strong critical fluctuations is unobservably narrow. The situation is quite different in high-T_c superconductors: the fluctuation region in YBaCuO is noticeable even in the absence of an external magnetic field. Its relative width $\Delta T/T_c$ is given by the Ginzburg number, $\mathrm{Gi} \sim 10^{-2}$ (Ref. 1); with an increase in the external field B, the width of the fluctuation region, $\Delta T(B)$, increases rapidly, as follows from the resistive[2] and thermodynamic[3] measurements. It is accordingly of interest to theoretically analyze the phase transition to the superconducting state at $B \neq 0$ outside the framework of the Ginzburg-Landau-Abrikosov (GLA) mean-field theory. First of all, it is important to determine the order parameter that describes this state. A simple answer would be to say that below $T_c(B)$ we have an Abrikosov lattice state with strongly developed fluctuations of the vortex lines. A study of vortex lattice (VL) stability with respect to melting by using the Lindemann criterion[4] (which was confirmed by Monte Carlo modeling[5]) has shown, however, that melting of the VL occurs at $B = B_M(T) \ll H_{c2}^{(0)}(T)$, where $H_{c2}^{(0)}(T)$ is the upper critical field in the GLA theory. On the other hand, study of the stability of the normal state with respect to the superconducting fluctuations in a strong magnetic field ($B \gg B_{fl} = \mathrm{Gi} H_{c2}^{(0)}(0)$, shows[6] that this state loses stability even at $B \simeq H_{c2}^{(0)}(T)$ (the relative width of the fluctuation region is $\Delta T(B)/T_c \simeq (B/B_{fl})^{2/3} \mathrm{Gi} \ll 1$; Ref. 7). The question therefore arises as to whether an intermediate phase, which is distinct from the high-T_c (normal) phase and the Abrikosov phase, can exist. Such a state would be a vortex line fluid without a translation order but one which conserves the superconducting correlations in the direction of the background field B_0. We will show that this assumption is not contradictory; specifically, the vortex line fluid can screen the current j that flows parallel to B_0. This property sets it apart from the normal (high-T_c) phase.

2. Our task is to derive an effective functional of the free energy $\mathscr{F}_v$, which will describe the fluid of the interacting vortex lines with a specified average density,

$n_0 = B_0/\Phi_0$. Such a problem has been discussed extensively.[8-10] It was shown in Refs. 8–10 that $\mathcal{F}_v$ is equivalent to a Euclidean action of a two-dimensional quantum nonrelativistic Bose liquid, where the vortex lines are the given world particle lines of this fluid. The corresponding Planck's constant is equal to the actual temperature of the system T, while the effective temperature of the fluid is $T_* = T/L_z$, where L_z is the size of the system in the direction of the field $B_0 \| \mathbf{z}$ (we have in mind the periodic boundary conditions along z, which physically correspond to the toroidal geometry of the sample; we will assume below that $L_z^{-1} = 0$, i.e., we will consider the ground state of the Bose liquid). In Refs. 8–10, however, the interaction of vortices was assumed (for simplicity) to be a local interaction. In fact, high-T_c superconductors are characterized by the Ginzburg-Landau parameter, $\kappa = \lambda/\xi \sim 10^2$; i.e., in the main region of the fields, $H_{c1}^{(0)}(T) \ll B_0 \ll H_{c2}^{(0)}(T)$, the London decay length of the interaction between the vortices is large: $\lambda \gg n_0^{-1/2}$. A systematic method of derivation of the effective action in this case is the dual transformation of the original Ginzburg–Landau functional with a fluctuating electromagnetic field. If the "vortex size" is $\xi \ll n_0^{-1/2}$ [i.e., $B_0 \ll H_{c2}^{(0)}(T)$], such a dual transformation is similar to the one described for the lattice model of a superconductor in Refs. 11 and 12 (where the finite background field, $B_0 \neq 0$, was not considered). This transformation leads to the functional $\mathcal{F}_v$ (see also Ref. 13) of the type

$$\mathcal{F}_v = m_B \sum_i \int ds_i + \int d^2r dz \left[ig\left(\mathbf{J} - \frac{1}{\Phi_0} \operatorname{curl} \mathbf{A} \right) \vec{\chi} + \frac{(\operatorname{curl} \vec{\chi})^2}{8\pi} + \frac{(\operatorname{curl} \mathbf{A})^2}{8\pi} \right], \tag{1}$$

where ds_i is a segment of the path of the ith vortex "particle," $m_B = (\Phi_0/4\pi\lambda)^2$ is the mass of these particles, $\mathbf{J}$ is the density of the remaining 3D current of the particles: $\mathbf{J} = (J_x, J_y, n)$, $\nabla\mathbf{J} = \partial_\alpha J_\alpha + \partial_z n = 0$; $\vec{\chi} = (a_\alpha, \vec{\chi})$ is a three-component vector field which accounts for the long-range interaction of the vortices, $g = \Phi_0/4\pi\lambda$, and $\mathbf{A}$ is the electromagnetic vector potential. The functional integration $\exp[-\mathcal{F}_V/T]$ is carried out along the particle paths and the fields $\vec{\chi}$ and $\mathbf{A}$ on condition that $\langle \operatorname{curl} \mathbf{A} \rangle = (0,0,B_0)$.

Further analysis can be performed by switching from a formally 3D invariant ("relativistic") action $\mathcal{F}_v$ for the particles to a nonrelativistic action $\mathcal{F}_B$ for a Bose field ϕ ($|\phi|^2 = n$) which interacts with the "scalar potential" χ and the vector potential a:

$$\begin{aligned} \mathcal{F}_B = \int d^2r\, dz \{ \phi^* [T\frac{\partial}{\partial z} - \frac{1}{2m_B}(T\nabla_\alpha - iga_\alpha)^2 - ig\chi]\phi \\ + \frac{i}{4\pi\lambda}[\epsilon_{\alpha\beta}(\partial_z A_\alpha - \partial_\alpha A_z)a_\beta + \epsilon_{\alpha\beta}\partial_\alpha A_\beta \chi] \\ + \frac{1}{8\pi}[(\epsilon_{\alpha\beta}\partial_\alpha a_\beta)^2 + (\partial_z a_\alpha)^2 + (\partial_\alpha \chi)^2] + \frac{1}{8\pi}(\operatorname{curl} \mathbf{A})^2 \} \end{aligned} \tag{2}$$

[expression (2) is based on the gauge $\partial_\alpha a_\alpha = 0$, $\nabla \mathbf{A} = 0$]. In the functional $\mathcal{F}_B$ the coordinate z is the virtual time; the field χ and the transverse component of the field A_α are responsible for the instantaneous interaction of the vortices, while the fields a_α and A_0 and the longitudinal component of A_α are responsible for the delayed interaction.

In the small-density limit, $n_0\lambda^2 \ll 1$, action (2) describes 2D Bose gas with a local interaction and a superfluid ground state ("entangled" flux liquid[9,11]). The arguments used in Ref. 6 show that in the inverse limit $n_0\lambda^2 \gg 1$ a Bose liquid with a nonsuperfluid ground state can exist in a certain density interval n_0. As will be shown below, such a state corresponds to a vortex-liquid phase with a longitudinal current screening.

3. Let us consider the effect of a Bose field ϕ on the effective action of a gauge field a_α. This (polarization) component of the quadratic approximation has the form $(1/8\pi)\Pi(c)a_\alpha(c)a_\alpha(-c)$. Diagonalizing the quadratic form with respect to a_α and A derived from (2), with allowance for the polarization term, we obtain a correlation function for the fluctuation of the component A_z of the vector potential

$$D_{zz}(c) = \langle A_z(k_z,k_\perp)A_z(-k_z,-k_\perp)\rangle$$
$$= \left(1 + \frac{\Pi(c)}{k^2}\right)\frac{T}{k^2+\lambda^{-2}+\Pi(c)}\left(1-\frac{k_z^2}{k^2}\right) \quad (3)$$

In the superfluid phase of the Bose liquid the quantity $\Pi(0)$ has a nonzero value and is given by the expression (in the zeroth approximation, i.e., ignoring the quantum fluctuations of the ground state) $\Pi(0) = 4\pi g^2 n_0/m_B = 4\pi n_0$. As a result, expression (3) in the long-wavelength limit becomes the standard Coulomb correlation function (in the gauge $\nabla A = 0$) for a medium with a permeability $\mu = \Pi(0)/(\Pi(0)+\lambda^{-2}) < 1$. In other words, the superfluid phase of the vortex Bose liquid corresponds to the high-T_c phase of the superconductor (normal metal).[9] The diamagnetic susceptibility $\chi_{DM} = -\Phi_0/4\pi\lambda^2 B_0$ with respect to the field $\delta\mathbf{H}\perp\mathbf{B}_0$ is related to the presence of the pretransition superconducting fluctuations. Note that χ_{DM} is equal in magnitude and opposite in sign to the differential susceptibility of the Abrikosov lattice with respect to the variation of the background field B_0.

In the normal (nonsuperfluid) phase of the Bose liquid we have $\Pi(c) \simeq \eta k^2$ as $k \to 0$. As a result, we find from (3) a massive-field correlation function for A_z

$$D_{zz}(c) = \frac{T}{k^2+\lambda_{eff}^{-2}}\left(1-\frac{k_z^2}{k^2}\right), \quad (4)$$

where $\lambda_{\rm eff}^2 = \lambda^2(1+\eta)$. This suggests that there is a screening of the z components of the vector potential A and of the current j.

The correlation function $D_{\alpha\beta}(c)$ of the transverse components of the vector potential has a Coulomb form in any case, since the screening of the longitudinal field $\delta\mathbf{B}\|\mathbf{B}_0$ cannot be changed in the absence of pinning.

4. The method described above can be used only for studying the equilibrium thermodynamics. It cannot be used to directly calculate the resistivity. We will present below qualitative arguments in support of the fact that the phase of the "normal" vortex liquid has a zero-valued linear resistivity $\rho_{\rm lin}$ for the current $j\|\mathbf{B}_0$. The energy dissipation due to the flow of current is caused by the creation and growth of the vortex rings lying in the plane $x,y\perp\mathbf{B}_0$. In the presence of a background field B_0 such a vortex ring is a deformation of the initial distribution of the vortex lines. The configuration of vortex lines corresponding to the section of such a ring is shown in Fig. 1a. The projection of such a configuration onto the x, y plane is the vortex ring, shown in

FIG. 1.

Fig. 1b. In a system with periodic boundary conditions along z (toroidal geometry) the formation and motion of such rings is caused by the breaking and reconnection of the vortex lines [in contrast with the dislocation loops, whose endless growth (which leads to the melting of the vortex lattice[14]) is not linked with the reconnection of the vortex lines]. In terms of the 2D Bose liquid, the configuration shown in Fig. 1 corresponds to a many-particle ring exchange of Bose particles. The superfluid state is characterized by the presence of arbitrarily remote ring exchanges,[15] i.e., by the presence of vortex rings of arbitrary size in a vortex system and at $j = 0$, which accounts for the finite value of ρ_{lin}. On the other hand, the normal phase of the Bose liquid has a small component of the long-range ring exchange, since the action associated with it increases linearly with the number of particles that participate in the exchange. In other words, the energy of the transverse vortex ring in the intermediate phase of the vortex fluid is proportional to its length. The free energy of the vortex ring of radius r in the presence of a current j_z thus behaves as $f(r) = \text{const}*r - (\Phi_0/c)j_z \pi r^2$. The dissipation rate is given by the probability $\sim\exp(-f(r_c)/T)$ for the appearance of rings of critical size $r_c \sim j_z^{-1}$, which correspond to the maximum of $f(r)$. As a result, we obtain an electric field $\mathscr{E}(j_z) \sim \exp(-\text{const}/j_z)$. This means that $\rho_{\text{lin}} = 0$ in the normal phase of the vortex fluid, consistent with the above conclusion that the current $\mathbf{j}\|\mathbf{B}_0$ is screened. To measure the screening of the longitudinal current, we must use samples of toroidal geometry (only in this case will the condition $\mathbf{j}\|\mathbf{B}_0$ be satisfied in the entire sample); this restriction does not apply to the measurement of ρ_{lin}.

5. We have shown that a phase transition from the normal (N) state to the superconducting state in a strong magnetic field can split into two phase transitions with an intermediate phase of the vortex fluid (VF). This phase is characterized by the absence of linear resistance and by a screening of the current (for $\mathbf{j}\|\mathbf{B}_0$). Many questions such as the critical behavior near a N$\rightarrow$VF$\rightarrow$VL transition and the role of defects in the vortex fluid phase remain unanswered. Finally, although it is very likely that the vortex fluid phase exists, its existence has not been proved conclusively.

We wish to thank L. B. Ioffe, A. I. Larkin, and A. V. Chubukov for many useful discussions. We also thank D. Nelson for sending us a preprint[10] of the article before publication.

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Translated by S. J. Amoretty