Mixed Thirring (1,0) supersymmetry model

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A Thirring (1,0) supersymmetry model has been analyzed. The left-hand sector of this model is realized by non-Abelian bosons on the $SO(n)$ group manifold and the right-hand sector is realized by the heterotic fermions. A quantum displacement of the level in the Kac–Moody $SO(n)$ algebra is shown to be possible. A relationship between this effect and the behavior of the scalar potential in the four-dimensional heterotic string model is analyzed.

Two-dimensional models on group manifolds play an important role in the (super) string theory.[1] Among them, the theories in which the left-hand and the right-hand 2D modes interact with each other, have recently attracted special interest. The Thirring fermion model preceded the data on the dynamic systems.[2] Various versions of the Thirring model have been obtained by using either the fermions or bosons or fermion and boson 2D fields simultaneously for the realization of the chiral modes. In general, the question of equivalence between the indicated formulations remains open.

In the present letter we considered the (1,0) supersymmetry Thirring model with a mixed Fermi–Bose realization of the non-Abelian chiral degrees of freedom which interact with the (1,0) supersymmetry.[3] The dynamics of such a system is described by a (1,0) superfield action

$$S_{FBTM} = kS_{WZW}(g;\Lambda_+^{=},\Gamma_=) + S_F(\eta_-^I), \qquad (1)$$

where

$$S_{WZW}(g;\Lambda_+^{=},\Gamma_=) = S_{WZW}(gh;\Lambda_+^{=}) - S_{WZW}(h;\Lambda_+^{=}),$$

$$S_{WZW}(g;\Lambda_+^{=}) = -\frac{i}{8\pi}\int d^3z E^- \mathrm{Tr}\{(g^{-1}\nabla_+ g)(g^{-1}\nabla_= g) + \Lambda_+^{=}(g^{-1}\nabla_= g)^2$$

$$+\int_0^1 dy(\tilde g^{-1}\frac{d}{dy}\tilde g)[\nabla_+(\tilde g^{-1}\nabla_=\tilde g) - \nabla_=(\tilde g^{-1}\nabla_+\tilde g)]\},$$

$$S_F(\eta_-^I) = -\int d^3z E^- \eta_-^I \nabla_+ \eta_-^I,$$

$$\nabla_= h h^{-1} \equiv \Gamma_= = 4\pi i S^{IJ}\eta_-^I\eta_-^J,$$

$$\tilde g(z;y) = \begin{cases} 1, & y=0, \\ g(z), & y=1. \end{cases}$$

Here $\Lambda_+^{=}$ is a Lagrangian multiplier which is used to take into account in action (1) the constraint on the (left) chirality of the non-Abelian scalar superfields $g(z)$, which "live" on the group manifold G, and k is the order of the Kac–Moody algebra $\hat{G}$. The matrix S^{IJ}, which takes on the values of G in the Lie algebra, mediates the interaction of the left non-Abelian scalars $g(z)$ with the right heterotic spinnors $\eta^I_-(z)$ ($I = 1,...,N$), with which the group $SO(N)$ is associated. The remaining notation corresponds to (1,0) covariant formalism of the (1,0) supergravitation.[3] As we can see, the non-Abelian groups connect with the right-hand and left-hand sectors can be described independently in the theory under consideration [action (1)]. The mixed supersymmetry Thirring model (1) in this regard is therefore similar to the (1,0) lefton–righton Thirring model,[4] in which the left-hand and the right-hand (non-Abelian) bosons interact.

We will now restrict the analysis to the case $G = SO(n)$ and assume that $n \leqslant N$, $N = \alpha n + N'$, where α is an integer between 1 and $[N/n]$. Let g (the $n \times n$ orthogonal matrices, $\det g = 1$) transform as a fundamental representation of the group $SO(n)$. In general, analysis of model (1) is difficult for arbitrary values of the matrix $(S^{IJ})^{ab}$ [$a,b = \overline{1,n}$ correspond to the fundamental representation of $SO(n)$]. A nonperturbative analysis of theory (1) can, however, be carried out for some special values of the coupling constants. In particular, let us assume that nonzero components of the matrix $(S^{IJ})^{ab}$ are those components whose indices I and J vary between 1 and αn. We choose

$$(S^{AB})^{cd} = \frac{1}{k}\delta^{ij}\delta^{a[c}\delta^{d]b}, \quad A \equiv a,i, \quad B \equiv b,j, \quad i,j = \overline{1,\alpha}. \tag{2}$$

Depending on the specific value of α, there can be different actions S_{FTBM}, in which the spinor superfields η^I_- appear in the form

$$\tilde{S}_F(\eta^I_-) = -\int d^3z E^-[\eta^{I'}_- \nabla_+ \eta^{I'}_- + \eta^{i,a}_-(\delta^{ab}\nabla_+ + (g^{-1}\nabla_+ g)^{ab})\eta^{i,b}_-], \tag{3}$$

where I' varies from $\alpha n + 1$ to N'.

We write the partition function

$$Z[S] = \int \mathcal{D}g\mathcal{D}\eta^I_- \exp[iS_{FBTM}]. \tag{4}$$

The Lagrangian multiplier $\Lambda^{=}_{+}$ in action (1) can be omitted since it is, by virtue of the Siegel symmetry, a purely gauge degree of freedom (see, for example, Ref. 5).

The following equation can be used for a fixed value of the coupling constants (2):

$$\int \mathcal{D}\eta^I_- \exp[iS_{FBTM}]|_{S=\frac{1}{k}\mathbb{1}} = e^{i(k-\alpha)S_{WZW}(g)} \int \mathcal{D}\eta^I_- \exp[iS_F(\eta^I_-)]. \tag{5}$$

This equation can easily be proved by using (3) and also the (1,0) superfield intrinsic time technique.[6] As a result, the (1,0) Wess-Zumino-Witten supersymmetry model with a new effective level $\tilde{k} = k - \alpha$ appears within the path integral in Eq. (4). This means that the proposed Thirring model contains a level-shift mechanism. As a result, the theory becomes degenerate if $\alpha = k$.

In the case of a mixed (1,0) supersymmetry Thirring model (1) the right-hand central charge (C_-) and the left-hand central charge (C_+) of the corresponding Virasoro operators generally are not the same since the left-hand and right-hand chiral models are taken into account in the theory independently of each other. The difference $c_+ - c_-$ is a supergravity anomaly. Since the supergravity anomaly is an essentially single-loop anomaly,[7] the indicated difference does not depend on the coupling constants S^{IJ}. In general, the central charges of model (1) therefore have the form

$$c_+ = \frac{\dim G}{1 + c_2(G)/2k} + \frac{1}{2}\dim G + F(S), \quad c_- = \frac{1}{2}N + F(S), \tag{6}$$

where $F(S) - S$ is a dependent contribution which is the same for each central charge, $F(S=0) = 0$.

Using the mixed (1,0) supersymmetry Thirring model (1), we can realize an internal symmetry $G \times SO(N)$ of a certain four-dimensional heterotic string. The coupling constants S^{IJ} in this case can naturally be identified with the massless scalar modes, while the function $F(S)$ in (6) is the scalar potential in the effective string action. In Refs. 8 and 9 the function $F(S)$ was calculated in the form of a series in powers of the matrix S for the fermion model[8] and the bosonized Thirring model.[9] The function $F(S)$ was calculated in the form of an expansion in $1/k$ and the corresponding expression was found in the leading order for the same two above-mentioned Thirring models with a special choice of the coupling constants.

An exact value of the function $F(S)$ can be obtained for theory (1) with the group $SO(n) \times SO(N)$, in which the coupling constants are given by the matrix (2). Using Eqs. (5) and (6), we find

$$F(S = \frac{1}{k}1) = -\frac{\alpha \dim SO(n) c_2(SO(n))}{(1 + \frac{c_2(SO(n))}{2(k-\alpha)})(1 + \frac{c_2(SO(n))}{2k})}. \tag{7}$$

As we can see, the theory has a singularity at $\alpha = k + c_2/2$.

Since the indicated value of the matrix S^{IJ} corresponds to a free theory (at the quantum-mechanical level), all the renormalization β functions vanish. As a result, we have

$$F'(S)|_{S=\frac{1}{k}1} = 0. \tag{8}$$

An extremum of the effective scalar string potential is thus associated with the point $S = (1/k)1$. Such points exist in the bosonized Thirring model.[9]

In conclusion, let us consider a four-dimensional $SO(n) \times SO(N)$ heterotic string. The conditions for the cancellation of supergravity anomalies[11] determine the values of the constants n and N at the level $k(k>0)$: $n = 5$, $N = 44$, $k = 2$. There are thus eight permissible values of the parameter $\alpha (= 1,2,...,8)$. The pathology of the model is linked with two values: $\alpha = 2$—the integral may be degenerate with respect to g, $\alpha = 5$—the singularity in the scalar potential [$F(\alpha = 5) \to \infty$].

From the standpoint of strings, the following points remain unclear: 1) the nature of extremum (8) (is it a minimum, a maximum, or an inflection point?); 2) how is the pathological behavior of the theory in the case of certain configurations of the massless modes to be understood?

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