Fermionic Lagrangian realization of cosets G/H

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(Submitted 15 October 1990)
JETP Lett. **52**, No. 10, 1167–1169 (25 November 1990)

A field-theoretic realization of cosets $SO(N)/H$ in terms of Majorana spinor models is proposed.

As is well known, the algebraic G/H constructions of Goddard, Kent, and Olive[1] admit a realization within the framework of two-dimensional Lagrangian quantum theory in terms of Wess-Zumino-Novikov-Witten (WZNW) models, in which the H-degrees of freedom are gauged in a suitable manners.[2-4] In these articles, guided by the general idea of two-dimensional Fermi-Bose equivalence,[5,6] we propose a field-theoretic spinor interpretation of cosets $SO(N)/H$.

We restrict the discussion to the consideration of Majorana spinors in curved two-dimensional spacetime. The action of the N free Majorana fermions Ψ is invariant under the $SO(N)$ group of global rotations. In order to gauge the degrees of freedom associated with a subgroup H of $SO(N)$, we introduce a gauge interaction between the spinors and the vector fields A_μ^a ($a = 1,...,\dim H$) according to the rule

$$S_F(\Psi, A_\mu) = i\int d^2z\sqrt{g}(\eta \not\nabla \eta + \varphi^i \not\nabla \varphi^i + J^a A^a), \tag{1}$$

where (η,φ^i) form N Majorana spinors Ψ. Here the φ^i ($i = \overline{1,k}$) transform like the fundamental representation of the subgroup H for each value of the index i. The parameter k is determined by the manner in which H is embedded in $SO(N)$. The vector fields A_μ^a belong to the adjoint representation of H, and J_μ^a is the covariant current

$$J_\mu^a = i\varphi^i \gamma_\mu t^a \varphi^i. \qquad ((2)$$

Here the t^a are the generators of the Lie algebra h

$$[t^a, t^b] = if^{abc}t^c, \quad \mathrm{Tr}(t^a t^b) = 2\delta^{ab}.$$

The action (1) does not contain the kinetic term for the gauge fields, which therefore play the role of Lagrange multipliers. From the equations of motion $\delta S_F/\delta A_\mu^a = 0$ we find

$$J_\mu^a = 0. \tag{3}$$

Therefore, the currents associated with the subgroup H vanish at the classical level.

The quantum dynamics of this model is characterized by the path integral

$$Z = \int D\Psi DA_\mu \exp[iS_F(\Psi, A_\mu)]. \tag{4}$$

To calculate this integral, we use the well-known variable substitution[2-4]

$$A_+ = (\nabla_+ hh^{-1}), \quad A_- = (\nabla_- \tilde{h}\tilde{h}^{-1}), \tag{5}$$

where $A_\pm$ are the components of the vector A_μ in the light-cone notation, and h and $\tilde{h}$ are independent group elements from H. In the gauge-invariant (vector) regularization scheme the Jacobian of the transformation (5) can be written as[4]

$$J(A_\mu|h,\tilde{h}) = \exp[-ic_2(H)S_{WZNW}(h^{-1}\tilde{h})]$$

$$\times \exp[i\mathrm{Tr}\int d^2x\sqrt{g}(b_+ \nabla_- c + b_- \nabla_+ \bar{c})], \tag{6}$$

where S_{WZNW} is the action of the WZNW model,[6] $c_2(H)$ is the quadratic Casimir operator in the adjoint representation of the group H, and c, $\bar{c}$, $b_\pm$ are Grassmann auxiliary fields taking values in the adjoint representation of H.

As a result of the substitution (5), the gauge part of action (1) takes the form

$$S_g = i\int d^2x\sqrt{g}[\varphi^i \not\nabla \varphi^i + J_+(\nabla_- \tilde{h}\tilde{h}^{-1}) + J_-(\nabla_+ hh^{-1})]. \tag{7}$$

Using the gauge-covariant regulator, it is easy to prove that

$$\int D\Psi \exp[iS_F(\Psi, A_\mu)] = e^{-ikS_{WZNW}(h^{-1}\tilde{h})} \times \int D\Psi \exp[iS_F(\Psi, 0)]. \tag{8}$$

In the final result, the integrand of the path integral (4) becomes the WZNW model with the level $-[k + c_2(H)]$ (cf. Ref. 4) invariant under the gauge transformations

$$h \longrightarrow \lambda h, \quad \tilde{h} \longrightarrow \lambda\tilde{h}, \quad \lambda \in H. \tag{9}$$

Therefore, we can impose the algebraic (ghostless) gauge $\tilde{h} = 1$. For the path integral (4) we then obtain the expression

$$Z = \int DhD\Psi Db_+ Db_- DcD\bar{c}$$

$$\times \exp[-i(k + c_2(H))S'_{WZNW}(h) + iS_F(\Psi, 0) + i\int d^2x\sqrt{g}(b_+ \nabla_- c + b_- \nabla_+ \bar{c})], \tag{10}$$

where S'_{WZNW} differs from S_{WZNW} by the sign in front of the Wess-Zumino term.

The corresponding central charge c of the Virasoro algebra can now be easily found from the Weyl anomaly of the functional (10):

$$\begin{aligned} c &= \frac{N}{2} + \frac{2(-k - c_2(H))\dim H}{2(-k - c_2(H)) + c_2(H)} - 2\dim H \\ &= \frac{\dim SO(N)}{1 + c_2(SO(N))/2k_G} - \frac{\dim H}{1 + c_2(H)/2k_H}, \end{aligned} \tag{11}$$

where $k_G = 1$ is the level of the Kac-Moody algebra $\widehat{SO}](N)$ and $k_H = k$ is the level of the Kac-Moody algebra $\hat{H}$, depending on how H is embedded in $SO(N)$, as in the algebraic approach.[1] It is easy to see that this expression is identical to the value of the central charge of the conformal model on the coset $SO(N)/H$ (Refs. 1–4). We have therefore demonstrated the possibility of a fermionic field realization of cosets.

As far as the field theoretic description of more complicated cosets G/H (for example, $SO(N)/H$ with $k_G \neq 1$) is concerned, for this we need more complicated models of fermions (not only Majorana ones) with a set of auxiliary gauge fields. We plan to study such models in future publications.

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Translated by Patricia Millard