Do quantum chiral bosons need Siegel symmetry?

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Chiral bosons with a Siegel symmetry broken at the quantum level are quantized.

Siegel's pioneering paper[1] was to be the first of a large number of papers devoted to a Lagrangian description of two-dimensional scalar chiral fields and various methods for quantizing them (see, for example, Ref. 2 and the bibliography there). Along approaches which start from a Siegel formulation,[1)] a leading role is assigned to an additional gauge symmetry which is similar to diffeomorphisms in a conformal gauge.[1] Because of this symmetry, the Lagrange multiplier with which the chirality condition is incorporated in the Siegel action is a purely gauge degree of freedom and has no effect on the classical spectrum of the theory. At the quantum level, in contrast, the Lagrange multiplier becomes a dynamic variable because of the breaking of Siegel symmetry.[2] This fact can be interpreted in different ways, depending on one's point of view of Siegel symmetry.

If we look at the symmetry as a gauge symmetry and quantize the chiral bosons with allowance for the gauge invariance by the Faddeev–Popov method, the breaking of the symmetry signifies a contradiction with the standard Siegel model.[3] Some modified actions proposed in recent papers[2–4] describe, along with chiral bosons, fields which are classically nonpropagating fields (scalar[3] and spinor[2,4] fields). As the latter become quantum-dynamic fields, they make it possible to cancel the anomaly in Siegel symmetry.

On the other hand, we could depart from the Faddeev–Popov procedure and refrain from fixing the Siegel symmetry. The interaction of chiral bosons with Lagrange fields would then become important in the quantum theory, because these bosons would acquire a quantum dynamics. Significantly, the Lagrange multipliers could be interpreted in this case as components of the metrics of certain two-dimensional gravitations in light-cone gauges.[1] This circumstance means that we can apply to the research on the quantum Siegel model some methods used in research on an induced 2D gravitation in the light-cone gauge.[5,6]

We denote by Y^i left-moving scalar fields and by Z^m right-moving ones ($i = 1,\ldots, N_L$; $m = 1,\ldots, N_R$). Along the Siegel approach these fields are described by the action

$$S = -\int d^2\sigma (D_+ Y^i \partial_- Y^i + \partial_+ Z^m D_- Z^m), \tag{1}$$

where the derivatives $D_\pm$ and $\partial_\pm$ are defined by[2,4]

$$\partial_\pm = \partial/\partial\sigma^\pm, \quad D_\pm = \partial_\pm - \Lambda_{\pm\pm}\partial_\mp + (\partial_\mp\Lambda_{\pm\pm})\hat{M},$$
$$[\partial_-,\ D_+] = \frac{1}{2}R^L\hat{M}, \qquad R^L = 2\partial_-^2\Lambda_{++},$$
$$[D_-,\ \partial_+] = \frac{1}{2}R^R\hat{M}, \qquad R^R = 2\partial_+^2\Lambda_{--}. \tag{2}$$

Here $\Lambda_{\pm\pm}$ are Lagrange multipliers. Action (1) is invariant under Siegel symmetry:

$$\delta Y^i = \epsilon_+\partial_- Y^i, \qquad \delta\Lambda_{++} = D_+\epsilon_+,$$
$$\delta Z^m = \epsilon_-\partial_+ Z^m, \qquad \delta\Lambda_{--} = D_-\epsilon_-. \tag{3}$$

The breaking of symmetry (3) at the quantum level is seen in the circumstance that the functional

$$\exp[i\Gamma(\Lambda_{++},\quad \Lambda_{--}] = \int DY^i DZ^m \exp(iS) \tag{4}$$

is noninvariant under transformations (3) (Ref. 2):

$$\delta\Gamma = \frac{N_L}{48\pi}\int d^2\sigma\partial_-\epsilon_+ R^L + \frac{N_R}{48\pi}\int d^2\sigma\partial_+\epsilon_- R^R. \tag{5}$$

We thus find an expression for Γ:

$$\Gamma = \Gamma_L + \Gamma_R, \quad \Gamma_{L,R} = -\frac{N_{L,R}}{96\pi}\int d^2\sigma R^{L,R}\frac{1}{\Box^{L,R}}R^{L,R},$$
$$\Box^L = \{\partial_-, D_+\}, \qquad \Box^R = \{D_-, \partial_+\}. \tag{6}$$

From the form of the "covariant" derivatives in (2) we easily see a similarity between the fields $\Lambda_{\pm\pm}$, Λ_{--}, on the one hand, and the metric components h_{++1}, h_{--2} of the two different 2D gravitations in light-cone gauges, on the other:

$$ds_1^2 = d\sigma^+ d\sigma^- + h_{++1}(d\sigma^+)^2,$$
$$ds_2^2 = d\sigma^+ d\sigma^- + h_{--2}(d\sigma^-)^2. \tag{7}$$

Within the framework of this analogy, the functionals Γ_L and Γ_R in (6) are none other than the effective actions of 2D gravitations with metrics (7). The components of the central charges of the right-hand and left-hand Virasoro algebras which come from the fields Λ_{++} and Λ_{--} should thus be the same as the components which come from the corresponding 2D gravitations, which were calculated in Refs. 5 and 6 (to which we refer the reader interested in the details). We write expressions for the central Virasoro charges, taking account of the contributions from the fields Y^i and Z^m:

$$c_R = N_R + N_L + c(h_1) + c(h_2) - 26,$$
$$c_L = N_L + N_R + c(h_1) + c(h_2) - 26, \tag{8}$$

$$c(h_{1,2}) = \frac{3k_{1,2}}{k_{1,2}+2} - 6k_{1,2}. \tag{9}$$

The parameters $k_{1,2}$ in (9) arise from renormalizations of the numerical factors in (6) and are the same as the central charges of $SL_{1,2}(2, R)$ current algebras.[5,6]

If the theory in (1) is to describe quantum chiral bosons, the corresponding central charges $c_{L,R}$ must be of the form

$$c_R = N_R, \qquad c_L = N_L. \tag{10}$$

Equations (8) become the same as (10) under the conditions

$$N_L + c(h_1) + c(h_2) = 26,$$
$$N_R + c(h_1) + c(h_2) = 26. \tag{11}$$

It follows that we have $N_L = N_R = N$ and $k_1 = k_2 = k$. As a result, we find the equation

$$N + \frac{6k}{k+2} - 16k = 26, \tag{12}$$

which differs from the corresponding equation in the theory of 2D gravitation of Ref. 6 in that the $c(h)$ contribution is doubled.

From (12) we find

$$k_{\pm} = \frac{N - 44 \pm \sqrt{(N-20)(N+28)}}{24}. \tag{13}$$

We see that the values of k are real if $N \geqslant 20$ (or $N \leqslant -28$). In the theory of an ordinary quantum 2D gravitation which is interacting with n scalar fields, the limitation $n \geqslant 26$ or $n \leqslant 1$ arose. This is one of the problems of a continuous formulation of a quantum induced gravitation, since the "physical" values of n must lie on the interval $1 < n < 26$.

In the case of a theory of chiral boson strings,[2-4] the "physical" values of N are determined from the condition

$$d + N = 26, \tag{14}$$

where d is the dimensionality of the physical space-time. Taking the limitation $N \geqslant 20$ into account, we thus reach the conclusion that chiral strings in which the chiral sector is described by a Siegel action can be formulated in a noncontradictory way in the dimensionalities $d = 0, 1, 2, 3, 4, 5, 6$.

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[1)] We will not be discussing approaches other than the Siegel approach to a Lagrangian representation of chiral bosons.

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