Localization of atoms in a nonuniformly polarized resonant field as the result of a coherent trapping of population

A. V. Taĭchenachev, A. M. Tumaĭkin, M. A. Ol'shanyĭ, and V. I. Yudin
Novosibirsk State University, 630090, Novosibirsk

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A new physical effect is predicted: a spatial localization of atoms over distances much shorter than the wavelength of the light at the caustics of nonuniformly polarized wavefronts. The transition $\{j_g = 1,\ j_e = 1\}$ is used as an example to discuss the spatial distribution of atoms under conditions corresponding to a coherent trapping of population in resonant fields with a nonuniform polarization and a nonuniform intensity. The density of atoms is shown to be proportional to the local energy density of the field.

1. As was shown in Ref. 1, when atoms with an angular momentum $j_g = 1$ in the ground state and $j_e = 1$ in an excited state are in a plane wave with an arbitrary elliptical polarization $\vec{e}$, there exists a stationary solution $|\phi\rangle$ of the Schrödinger equation which does not interact with the field, i.e., one for which $(\vec{d}\cdot\vec{e})|\phi\rangle = 0$ ($\vec{d}$ is the dipole-moment operator). This state has a vanishing natural width, since it constitutes a coherent superposition of the wave functions of Zeeman sublevels of the ground state. In the coordinate representation, in the c.m. frame of the atom, this superposition is

$$|\phi> = F(\vec{r}) \sum_{m=1,0} b_m|j_g, m>; \quad \frac{\hat{p}^2}{2M}F(\vec{r}) = \epsilon F(\vec{r}), \tag{1}$$

where the coefficients b_m are determined by the field polarization and are independent of the coordinates, and $F(\vec{r})$ is an arbitrary eigenfunction of the kinetic-energy operator. Because of radiative relaxation processes, atoms accumulate in the state $|\phi\rangle$; i.e., a coherent trapping of population occurs.

It turns out that in fields which have a spatially nonuniform polarization and a spatially nonuniform intensity there is again a stationary state (referred to below as the "nonuniform stationary state") which does not interact with the field and which has several unexpected properties.

Let us analyze the spatial distribution of atoms in a nonuniform stationary state of this sort. We will show that the density of atoms $n(\vec{r})$ is proportional to the local energy density of the field, $w(\vec{r})$. Thinking of fields which have singularities of the $w \propto r^2$ type as $r \to 0$, we can speak in terms of a new physical effect: a localization of atoms over distances much shorter than the wavelength (λ) of the light at the caustics of the nonuniformly polarized wavefronts: $n \propto r^2$ as $r \to 0$ (in the geometric-optics limit).

2. Let us briefly describe our method for finding the specific form of the nonuniform stationary states $|\phi(\vec{r})\rangle$. We consider an ensemble of atoms with angular momenta $\{j_g = 1, j_e = 1\}$ which are interacting resonantly with a nonuniform monochromatic field

$$\vec{E}(\vec{r},t) = e^{i\omega t}\vec{e}(\vec{r}) + \text{c.c.} \tag{2}$$

The complex vector amplitude $\vec{e}(\vec{r})$ satisfies the Helmholtz equation

$$(\Delta + k^2)\vec{e}(\vec{r}) = 0; \quad k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \tag{3}$$

and the transversality condition $\vec{\nabla}\cdot\vec{e}(\vec{r}) = 0$. We seek the state $|\phi(\vec{r})\rangle$ as a superposition of the type in (1), but we assume that the coefficients $b_m(\vec{r})$ depend on the coordinates (i.e., that there is a correlation between the internal and translation degrees of freedom). The conditions under which the operator representing the interaction with field (2) vanishes $[(\vec{d}\cdot\vec{e}(\vec{r})|\phi(\vec{r})\rangle = 0)]$ and the condition for a stationary state $(\vec{p}^{\,2}/2M|\phi(\vec{r})\rangle = \epsilon|\phi(\vec{r})\rangle)$ are written as follows:[1,2]

$$\sum_{m,q=\pm 1,0} (-1)^{1-\mu} \begin{pmatrix} 1 & 1 & 1 \\ -\mu & q & m \end{pmatrix} e_q(\vec{r})F(\vec{r})b_m(\vec{r}) = 0, \tag{4}$$

$$\left(\frac{\hat{\vec{p}}^2}{2M} - \epsilon\right) F(\vec{r})b_m(\vec{r}) = 0; \quad \hat{\vec{p}} = -i\hbar\vec{\nabla}. \tag{5}$$

Here $\begin{smallmatrix} a & b & c \\ d & e & f \end{smallmatrix}$ is the $3jm$-symbol, and $e_q(\vec{r})$ are the circular components of the vector $\vec{e}(\vec{r})$. If the field polarization vector $\vec{\mu}(\vec{r}) = \vec{e}(\vec{r})(|\vec{e}(\vec{r})|^2)^{-1/2}$ varies along all three coordinates (x, y, z), then a solution of system (4), (5) exists only if $\epsilon = (\hbar k)^2/(2M)$, and this solution is unique: $F = \text{const}$, $b_m(\vec{r}) = e_m(\vec{r})$ (this assertion is true except in several particular cases which we will not discuss here). Using the normalization $\int\langle\phi(\vec{r})|\phi(\vec{r})\rangle d^3\vec{r} = 1$, we write this solution in the form

$$|\phi(\vec{r})\rangle = [V \langle|\vec{e}(\vec{r})|^2\rangle_V]^{-1/2}\{e_{+1}(\vec{r})|j_g + 1\rangle + e_0(\vec{r})|j_g, 0\rangle + e_{-1}(\vec{r})|j_g, -1\rangle\}, \tag{6}$$

where the angle brackets mean the expectation value over the normalization volume V: $\langle\ldots\rangle_V = \int\ldots d^3\vec{r}V^{-1}$. We wish to stress that (6) is an exact stationary solution of the problem, and that it incorporates all radiative processes, including recoil effects. The problem of the three-dimensional cooling of atoms below the recoil energy on the transition $\{j_g = 1, j_e = 1\}$ has been treated in a corresponding formulation.[2]

3. Let us examine the spatial distribution of N atoms: $n(\vec{r}) = \langle\phi(\vec{r})|\phi(\vec{r})\rangle$. It is easy to see that $n(\vec{r})$ is proportional to the local energy density of the field, $w(\vec{r})$:

$$n(\vec{r}) = |e(\vec{r})|^2/\langle|\vec{e}|^2\rangle_V V \equiv w(\vec{r})/\langle w\rangle_V V. \tag{7}$$

In momentum space, the atoms are localized on the sphere $\vec{p}^2 = (\hbar\vec{k})^2$; i.e., the momentum distribution has a vanishing effective width. Expression (6) thus describes ultracold atoms which are concentrated at the maxima (antinodes) of the field. This circumstance can be utilized to develop spatial lattices of the density of atoms in a resonant gas. For fields which have singularities, expression (7) demonstrates a new physical effect: a localization of atoms at the caustics of nonuniformly polarized wavefronts. As an example we could discuss the localization at the focus of a parabolic mirror. The incident light beam is chosen to be linearly polarized. When we take the change in polarization upon reflection into account, we find that the polarization vector of the total field, $\vec{\mu}(\vec{r})$, varies over distances on the order of λ.

4. Let us summarize.

I. The localization described by (7) is the result of an interference of states with different momentum directions. It has no interpretation in terms of forces, in sharp contrast with the semiclassical limit ($\hbar k \ll p$).

II. Aspect *et al.*[3] have described the experimental observation and numerical analysis of a one-dimensional ultradeep cooling associated with a coherent trapping of population in a nonuniform field. Their work suggests that it might be possible to observe the effect predicted in this letter.

We have observed corresponding phenomena on transitions of other types: $\{j_g = 1,\ j_e = 0\}$, $\{j_g = 2,\ j_e = 1\}$, and $\{j_g = 3/2,\ j_e = 1/2\}$.

[1] V. S. Smirnov, A. M. Tumaĭkin, and V. I. Yudin, Zh. Eksp. Teor. Fiz. **96**, 1613 (1990) [Sov. Phys. JETP **69**, 913 (1989)].

[2] M. A. Ol'shanyĭ and V. G. Minogin, *Report to the Tenth Vavilov Conference on Coherent and Nonlinear Optics*, Novosibirsk, 1990.

[3] A. Aspect, E. Arimondo, R. Kaiser *et al.*, Phys. Rev. Lett. **61**, 826 (1988); A. Aspect, E. Arimondo, and R. Kaiser *et al.*, J. Opt. Soc. Am. B **6**, 2112 (1989).

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