Giant resonances at the threshold of the continuum: New soft mode in nuclei

S. A. Fayans
I. V. Kurchatov Institute of Atomic Energy, 123182, Moscow

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It is shown with the help of simple arguments that nuclei which have an isolated nucleon shell $nlj\tau$ with low binding energy should have very strong transitions with the symmetry 0^+, 2^+, ... from this shell into the state $\epsilon lj\tau$ near the threshold of the continuum. This prediction is confirmed by self-consistent calculations of the transition strength functions of ^{11}Li nuclei.

In the last few years interest in the investigation of the properties of nuclei far from the stability band has increased substantially. Attention is now being focused on nuclei at the limit of the nuclear chart. Such nuclei have a number of exotic properties. In particular, anomalously extended distributions of nuclear matter, which indicate the existence of a neutron "envelope" in them,[1] have been found in comparatively light nuclei near the drip-line, and experiments[2] have shown that the probabilities of their dissociation with the emission of neutrons are surprisingly large.[2] Because of this circumstance, the possibility that such nuclei have a soft dipole mode, which could increase substantially the contribution of the electromagnetic (Coulomb) mechanism in the total dissociation cross section, is being widely discussed.[3]

In this paper it is pointed out that nuclei near the drip-line with an isolated weakly bound nucleon shell should have very strong low-energy transitions $nlj\tau \rightarrow \epsilon lj\tau$ from this shell into the continuous spectrum ($nlj\tau$ is the standard set of single-particle quantum numbers: the principal quantum number n, and orbital angular momentum l and total angular momentum j, the isospin τ; ϵ is the energy in the continuous spectrum). This can easily be shown by using the example of monopole excitations in the field $V_0 = r^2 Y_{00}$ and making use of the well-known sum rule for the reduced probabilities $B(E0;\omega)$:

$$\int \frac{dB(E0;\omega)}{d\omega}\omega d\omega = \frac{\hbar^2}{2m}\frac{1}{\pi}(< r^2 >_n N + < r^2 >_p Z), \tag{1}$$

where $\langle r^2 \rangle_{n(p)}$ is the mean-square radius of the neutron (proton) distribution. Since the density is an additive quantity, we can single out in Eq. (1) the contribution of the least strongly bound shell, say, the neutron shell, which is customarily called the valence shell (we shall refer to everything else as the contribution of the core), can be singled out:

$$\int \frac{dB(E0;\omega)}{d\omega}\omega d\omega = \frac{\hbar^2}{2m}\frac{1}{\pi}(< r^2 >_n^{core} N_{core} + < r^2 >_p^{core} Z_{core} + < r^2 >_n^{val} N_{val}), \tag{2}$$

where N_{val} is the number of neutrons in the valence shell, and

$$< r^2 >_n^{val} = \int R^2_{nlj(\tau=n)}(r) r^4 dr. \tag{3}$$

Here $R_{nlj(\tau=n)}(r)$ is the radial wave function of the valence neutrons. In the limit $\epsilon_{nlj\tau} \to 0$, i.e., as the energy of the valence level approaches zero, $\langle r^2 \rangle_n^{\text{val}} \to \infty$, and the valence shell represents the main contribution to the monopole sum rule. This means that in this limit virtually all of the strength of the 0^+ transitions is determined by the excitation of nucleons from this shell, and the main question is how this strength is distributed. It is not difficult to see that the strength is actually concentrated at very low energies. The probability of the monopole transition $nlj\tau \to \epsilon lj\tau$ into the continuous spectrum in the pure shell model is given by the expression

$$\frac{dB(E0;\omega)}{d\omega} = \frac{N_{val}}{4\pi}\left(\int R_{nlj\tau}(r) R_{\epsilon lj\tau}(r) r^4 dr\right)^2, \tag{4}$$

where $R_{\epsilon lj\tau}(r)$ is the radial wave function of the neutrons in the continuum, which is normalized to $\delta(\epsilon - \epsilon')$, where $\epsilon = \epsilon_{nlj\tau} + \omega$ and ω is the transition energy. As $\omega \to 0$, the matrix element in Eq. (4) formally reduces to the diagonal matrix element (3), which, as noted above, becomes infinite as the valence level approaches the start of the continuum. Thus all of the strength of the 0^+ transitions of the nucleons from the valence shell in the limit $\epsilon_{nlj\tau} + 0$ is concentrated at low values of ω. For finite values of $\epsilon_{nlj\tau}$, as follows from dimensional considerations, the position and width of the corresponding peak should be of the order of magnitude $|\epsilon_{nlj\tau}|$. Taking into account the "residual" interaction is not likely to change these conclusions significantly.

An analogous analysis for multipole excitations in the fields $V_L = r^L Y_{LM}$ shows that nuclei with a weakly bound shell should have strong low-energy transitions with all even L from 0 to $L_{\max} = 2j - 1$. As regards transitions with odd L, for which the valence level also contributes significantly to the corresponding sum rule (except for transitions with $L = 1$, which, as suggested in a number of works,[3,4] form the "soft dipole mode"), such transitions can also be stronger at low values of ω. But there are no grounds for expecting that the enhancement will be as sharp as in the case of even L. We also note that the multipole transitions from core nucleons should be distributed over a wide range of values of ω, as in normal stable nuclei.

Figure 1 shows the strength functions for monopole, dipole, quadrupole, and octupole transitions in ^{11}Li, calculated in the self-consistent theory of finite Fermi systems, taking into account the NN interaction and ignoring it. The mean-field potential and the nucleon distributions were calculated using the energy density functional method.[5] The mean-square radius of the distribution of matter in ^{11}Li was found to be 3.01 fm and the binding energy of the neutron valence shell $0p_{1/2}$ was found to be 0.884 MeV, which agrees reasonably well with experiment. The mean-square radius of the wave function of the valence electrons in this case is 19.35 fm^2. The contribution of these two neutrons to the monopole sum rule (1) therefore is $\simeq 40\%$.

As one can see from in Fig. 1, for low energies the strength function of the 0^+ transitions has a sharp peak at 1.5 MeV with a width $\simeq 1$ MeV, picking up $\simeq 30\%$ of

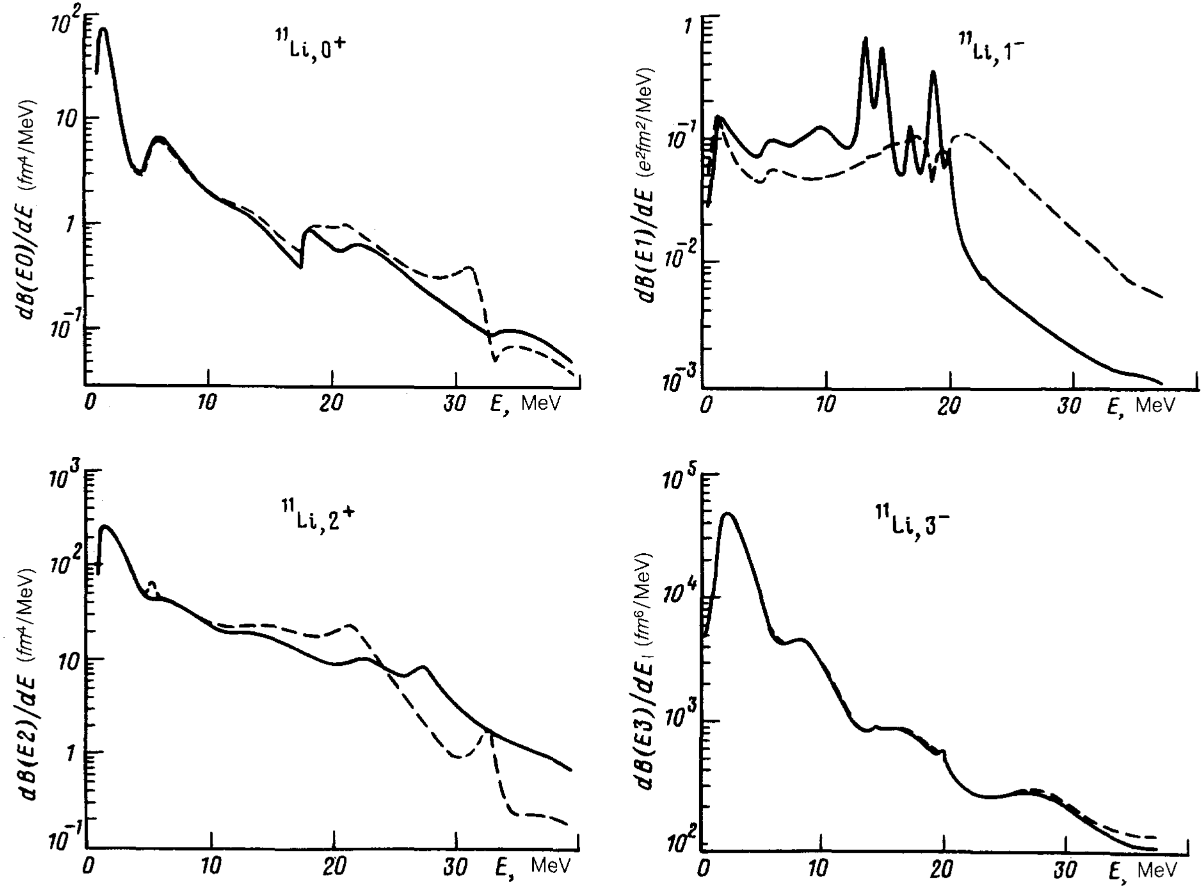

FIG. 1. Distribution of the strength of isoscalar monopolar (0^+), quadrupolar (2^+), octupolar (3^+), and isovector dipolar (1^-) transitions in ^{11}Li. The solid curves represent calculations performed without regard for the *NN* interaction and the dashed curves represent calculations which take the *NN* interaction into account.

the sum rule (in the energy interval from 1 to 4.7 MeV; 40% of the sum rule is picked up in the interval up to $\simeq 7$ MeV). The response of the valence neutrons to a quadrupole field $r^2 Y_{2M}$ makes the same contribution ($\simeq 40\%$) to the sum rule as in the monopole case, but the peak, at $\simeq 1.6$ MeV, due to the soft 2^+ transitions is broader: its width is $\simeq 2.1$ MeV and only $\simeq 17\%$ of the sum rule is picked up in the energy interval up to 5 MeV, while 40% of the sum rule is picked up in the interval up to $\simeq 11$ MeV. This difference is attributed to the fact that in the quadrupole case there are no transitions from the valence shell which are diagonal in lj.

The distribution of the strengths of the 1^- and 3^- transitions for the external fields $(V_0^n, V_0^p) = (-ZerY_{1M}/A, NerY_{1M}/A)$ and $r^3 Y_{3M}$, respectively, are shown on the right side of Fig. 1. Two valence neutrons contribute 71% to the energy-weighted sum rule for the octupole transitions and only $ZN_{\text{val}}/AN \simeq 7\%$ to the model-independent dipole sum rule

$$\int \frac{dB(E1;\omega)}{d\omega}\omega d\omega = \frac{9}{4\pi}\frac{\hbar^2 e^2}{2m}\frac{NZ}{A}, \tag{5}$$

which is independent of both the density distribution and the energy of the valence shell. This factor accounts for the qualitative difference in the behavior of the strength functions of 1^- and 3^- transitions. The peak at $\simeq 2.1$ MeV due to soft octupole transitions has a width of $\simeq 2.5$ MeV, the interval up to the first minimum at $\simeq 7$ MeV picks up $\simeq 52\%$ of the sum rule and the interval up to 11.7 MeV picks up 71%. Disregarding the NN interaction, which in the dipole case plays a more important role, the peak due to the soft 1^- transitions has a maximum at $\simeq 1.3$ MeV and a width $\Gamma \simeq 2.9$ MeV; the interval up to the first minimum at $\simeq 4.25$ MeV picks up only 2.6% of the sum rule (5) and the interval up to $\simeq 7$ MeV picks up 7% of the "valence contribution." When the interaction is included, this peak shifts slightly down the energy scale (the peak crests at $\simeq 1$ MeV) and it becomes sharper ($\Gamma \simeq 1.5$ MeV); the region up to the first minimum at $\simeq 4.5$ MeV picks up $\simeq 2.1\%$ of the sum rule (5) and the interval up to 9 MeV picks up 7%. The calculations for 1^- and 3^- transitions were performed by introducing a damping of $\gamma = 125$ keV for the quasiparticles in order to represent visually the contribution of the transitions between discrete single-particle proton states; this is reflected, in particular, in the appearance of a palisade of peaks from 12 to 20 MeV in the unperturbed dipole strength function (a method for performing such calculations was developed in Ref. 6). For this reason, the real escape-widths of the peaks of the soft 1^- and 3^- transitions are 0.5 MeV smaller than the values indicated. The dipole transition strength distributions calculated in Ref. 4 differ appreciably from our distributions, evidently because of the different choice of the effective NN interaction and the dimension of the coordinate grid (to obtain the correct numerical results, the far "tail" of the wave function of the valence neutrons, up to $R_{\max} \simeq 25$ fm, must be taken into account).

The foregoing analysis is based on the picture of elementary particle-hole excitations. In light nuclei clusterization effects are important. For the ^{11}Li nucleus the two-particle continuum starts earlier than the single-particle continuum, since only $\simeq 200$ keV is required to remove two neutrons. In a model in which this nucleus is represented as the system "dineutron + ^{9}Li core" (no pair of the three particles—two neutrons and ^9Li—forms a bound state, and the existence of ^{11}Li with a large neutron halo can be explained by Efimov's effect[7] and by Migdal's prediction[8]), an intense (in any case, monopolar) soft mode should also arise.

Analogous peaks due to soft transitions of positive parity should also exist in other neutron-excess nuclei near the drip line (^6He, ^8He, ^{11}Be). Since they pick up a significant fraction of the sum rule, they can be giant resonances at the threshold of the continuum—a new soft mode. These peaks can be observed in any imaginable nuclear inelastic scattering reactions (in the reverse kinematics), because they are excited with equal intensity in both isovector and isoscalar external fields. In particular, in the case of forward scattering at small angles the probability for the excitation of a monopolar soft mode is large, and it is important to take this fact into account when separating from the total dissociation cross section the contribution of the electromagnetic (Coulomb) mechanism.

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