Magnetospectroscopy of 2D electron gas: Cusps in emission spectra and Coulomb gaps

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A theory of photoluminescence due to radiative trapping of 2D electrons from a confinement layer at neutral impurity centers is presented. In the dependence of the emission band position on the filling factor ν there are cusps at fractional values $\nu = p/q$; these cusps are closely related to cusps in the ground state energy of interacting 2D electrons; the latter cusps are attributable to the formation of incompressible liquids at these points. The effect of the finite state interaction on the cusp shape is investigated, and the possibility of determining Coulomb gaps from cusp strengths is discussed.

The incompressible two-dimensional (2D) electron liquid,[1] which manifests itself in the fractional quantum Hall effect,[2] is one of the most remarkable objects of the modern solid state physics. Review articles on this subject are found in Refs. 3–5. However, the number of experimental methods that make it possible to measure the parameters of the 2D liquid is highly restricted. Recently spectroscopic observations of the 2D liquid have been reported.[6–9] In particular, many features on the dependence of the position of the photoluminescence band, which corresponds to the radiative trapping of 2D carriers by neutral acceptors, on the filling factor ν have been observed (see Refs. 6 and 7 and the bibliography cited there). These features can be attributed to the electron–electron interaction and can be described in terms of steps. There are now only a few theoretical papers on the photoluminescence in a strong magnetic field, which are not based on the mean-field approximation, and which accordingly can be used to describe the quantum liquid. The effect of spin polarization on optical spectra was studied in Refs. 10 and 11 and the Auger processes were analyzed in Refs. 10 and 12. It was shown in Ref. 12 that under certain conditions the energy Δ_r of the magneto-roton minimum[13] can be found from the shape of the impurity emission spectrum. We will show in this paper that the gap Δ for quasielectron–hole pairs[2] which is created can be found from the dependence of the center-of-gravity $\bar{\omega}$ of the emission band which corresponds to the trapping of electrons from the 2D liquid by neutral impurity centers on ν. According to the theory of fractional valuesΔ_r
where p is an integer, and q is an odd integer, in the dependence $\bar{\omega}(\nu)$ one can expect to see cusps and discontinuities in the derivative, $d\bar{\omega}/d\nu$, at these points which are connected with the gaps, Δ, in them. The fact that the effect of the impurity centers on 2D electrons is weak in the initial state is a distinctive feature of this type of transitions, which apparently can be ignored. This assumption is of crucial importance in the determination of Δ from experimental data on cusps strengths, which will be discussed below.

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The model used below is as follows. In the initial state the electron density is homogeneous in the confinement layer and an impurity center at which an electron is trapped is neutral. The distance h of the impurity center from the confinement layer and the magnetic length $l = \sqrt{c/eH}$ ($\hbar = 1$) exceed considerably the layer width and the center radius $r_{\rm imp}$. These two quantities and the initial state interaction of the impurity center with 2D electrons are ignored. The potential of the impurity center in the final state is a Coulomb potential. At a temperature $T = 0$ the system of electrons in the initial state is at the ground level i. If this level is degenerate, all the states belonging to it are equally populated. The strength of the magnetic field is $\omega_c \gg \epsilon_C = e^2/\kappa l$, where ω_c is the cyclotron frequency, ϵ_C is the Coulomb energy, κ is the dielectric constant. Mixing of different Landau levels is disregarded. The quantum transition occurs at the point $\vec{r}_0$, which is the point of the 2D layer closest to the impurity center. This recombination scheme is especially well suited for describing the trapping of electrons from a deep quantum well by neutral shallow acceptors that reside near it. Numerical calculations are performed in the spherical geometry.[14] Therefore, all equations are written for a homogeneous system with a finite number of particles.

The normalized first moment of the emission spectrum (i.e., divided by its zero moment) determines the position of the center-of-gravity. It is

$$\bar{\omega} = E_i - \langle H_f \rangle_{av} \quad , \tag{1}$$

where E_i is the energy of the i level, i.e., of the ground state of N interacting particles. H_f is the Hamiltonian of the system in the final state, i.e., the Hamiltonian of $(N-1)$ interacting particles $\vec{r}_1,...,\vec{r}_{N-1}$ in the 2D layer subjected to a repulsive Coulomb center at the point $\vec{r}_I$, where the impurity center resides (see Ref. 12). The symbol $\langle ... \rangle_{\rm av}$ stands for averaging over the wave functions $\Psi_\alpha(\vec{r}_1...\vec{r}_N)$ of all the states α that belong to the i level provided that the condition $\vec{r}_N = \vec{r}_0$ is satisfied. For the energy of the "excess" electron in the initial state, the difference of the ground Landau level in the conduction band and the impurity level is chosen as the reference point. Hence $\bar{\omega}$ includes only the Coulomb interaction energy.

It is convenient to introduce the pair correlation function

$$g(|\vec{\rho}_N - \vec{\rho}_{N-1}|) = (N-1)Al^2 \sum_\alpha \int |\Psi_\alpha(\vec{r}_1 \dots \vec{r}_N)|^2 d\vec{r}_1 \dots d\vec{r}_{N-2}/g_i \quad ,$$

$$\int g(\rho) d\vec{\rho} = N - 1 \quad , \tag{2}$$

where g_i is the multiplicity of the i level, A is the area of the 2D layer, and $\vec{\rho} = \vec{r}/l$. In the thermodynamic limit $g(r) \to \nu/2\pi$ as $\rho \to \infty$. The function g depends exclusively on the difference $\rho = |\vec{\rho}_N - \vec{\rho}_{N-1}|$ since it possesses, due to summation over α in (2), the full symmetry of the system before the optical transition if the impurity center is neutral and does not perturb 2D electrons. For the same reason, all integrals contained in $\langle H_f \rangle_{\rm av}$ can be expressed in terms of the function g. The Hamiltonian H_f is

$$H_f = \sum_{jk=1}^{N-1} V(|\vec{\rho}_j - \vec{\rho}_k|) - \sum_{j=1}^{N-1} V(|\vec{\rho}_j - \vec{\rho}_I|) \quad , \; j < k \quad , \tag{3}$$

where $V(\rho)$ is the electron-electron interaction potential. The potential of the positive background, which accounts for the electric neutrality of the system, is omitted since it does not contribute to energy differences which are our final results. Since

$$\frac{N}{2}\int V(\rho)g(\rho)d\vec{\rho}=E_i \quad , \tag{4}$$

Eq. (1) can be rewritten as

$$\begin{aligned}\bar{\omega} &= \int\{V(|\vec{\rho}_0-\vec{\rho}\,|)-V(|\vec{\rho}_I-\vec{\rho}\,|)\}g(|\vec{\rho}_0-\vec{\rho}\,|)d\vec{\rho} \\ &= 2E_i/N-\int V(|\vec{\rho}_I-\vec{\rho}\,|)g(|\vec{\rho}_0-\vec{\rho}\,|)d\vec{\rho} \quad .\end{aligned} \tag{5}$$

The integration in (4) and (5) is performed over the 2D layer.

Let us first consider the short-range potential $V(r)$. In this case the second term in (5) can be omitted when the distance $h=|\vec{r}_I-\vec{r}_0|$ of the point $\vec{r}_I$ from the confinement layer is larger than the potential radius. Taking into account that $\partial E_i/\partial N=\mu$, where μ is the chemical potential, and $\mu(\nu)$ is a discontinuous function of ν at $\nu=\nu_{pq}$[15]

$$\delta\mu\equiv\mu(\nu_{pq}+0)-\mu(\nu_{pq}-0)=q\Delta \quad , \tag{6}$$

we easily find from (5)

$$\Delta=(\nu/2q)\,\delta\{\partial\bar{\omega}/\partial\nu\} \quad . \tag{7}$$

The function $\bar{\omega}$ must therefore show cusps at the points ω_{pq}, and the discontinuity in the derivative makes it possible to determine the gap. It is clear from (5) that these cusps are closely related to cusps of the function[15] $E_i(\nu)$. We define the quantity $\delta\{\partial\bar{\omega}/\partial N\}$ as the cusp strength.

The long-range Coulomb behavior $V(r)$ makes it necessary to take into account the second term in (5); both terms contribute to the cusp strength. The singularities of $\bar{\omega}(\nu)$ appear due to a nonanalytical behavior of g treated as a function of ν. The previous computations imply[16,17] that the behavior $g(\rho)=g_1(\rho)+|\nu-\nu_{pq}|g_2(\rho)$ near the points $\nu=\nu_{pq}$, where g_1 and g_2 are smooth functions of ν. Such a behavior results in cusps in both integrals in Eq. (5). If $h\to 0$, then $\bar{\omega}=0$. This means that the intrinsic and extrinsic terms cancel each other and the interaction does not affect the position of the emission band. For intermediate values of h the extrinsic contribution to the cusp strength may be large, but is less than the intrinsic contribution. The general expression for the cusp strength is

$$\delta\{\partial\bar{\omega}/\partial\nu\}=2q\Delta/\nu-\int V(|\vec{\rho}_I-\vec{\rho}|)\,\delta\{\partial g(|\vec{\rho}_0-\vec{\rho}|)/\partial\nu\}d\vec{\rho} \quad . \tag{8}$$

In the expansion of Eq. (8) at $h\gg 1$ the term $\sim h^{-1}$ is absent due to normalization condition (2), and at $h\ll l$ the expansion begins with the term $\sim h^2$.

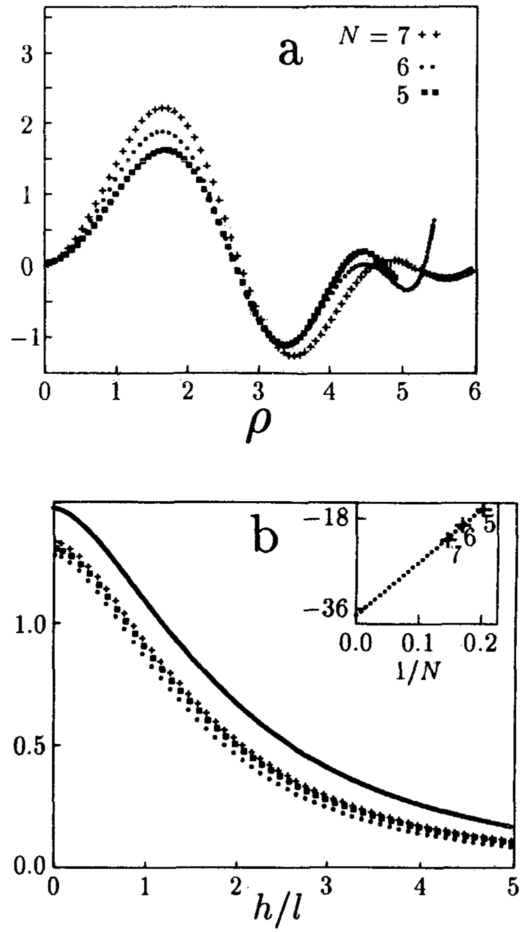

FIG. 1. (a) The function $\delta\{\partial g(\rho)/\partial\nu\}$ found in the spherical geometry for different number N of particles; $\nu = 1/3$. (b) Extrinsic contribution to the cusp strength [the second term in Eq. (8)] versus the distance h, $\nu = 1/3$. The heavy line is obtained by extrapolation $1/N \to 0$. The energy is in ϵ_C units. Inset—the value of the coefficient at $(l/h)^3$ in (9) found for different numbers of particles.

The function $g(\rho)$ was found by us in the spherical geometry[14,18] for three values of ν: $\nu = 1/3$ and at the two neighboring points. The function $\delta\{\partial g(\rho)/\partial\nu\}$ at $\nu = 1/3$ found from these data is shown in Fig. 1a. It exhibits the ρ^2 behavior at $\rho \ll 1$ and one strong oscillation. Oscillations at $\rho > 5$ decrease with increasing N. This function was used to calculate in the plane geometry the last term in (8) as a function of h; it is shown in Fig. 1b. In the limit cases

$$\delta\{\partial\overline{\omega}/\partial\nu\} \approx \begin{cases} 18\Delta - 37\epsilon_c(l/h)^3 & \text{for} \quad h \gg l \ , \\ 0.77(h/l)^2 & \text{for} \quad h \ll l \ , \end{cases} \qquad (9)$$

the coefficients in (9) were found by extrapolation $1/N \to 0$. A rapid decrease of the extrinsic term with increasing h could make it much easier to determine the gaps from experimental data on the cusp strengths.

Figures 2 and 3 show the first moment of the spectrum and the width γ of the

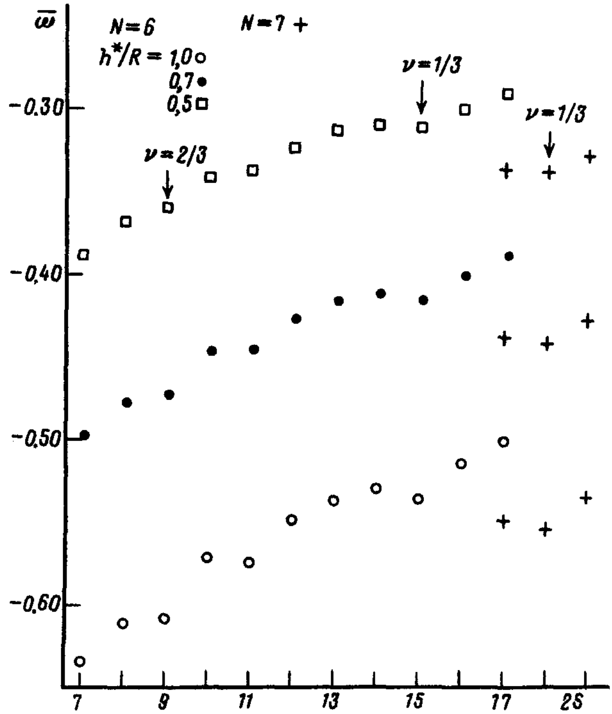

FIG. 2. Dependence of the center-of gravity $\bar{\omega}$ of the emission spectrum on $2S$. Cusps correspond to nondegenerate states. Calculations were performed at $N=6$ for three values of the parameter h^*/R. The $\nu=1/3$ cusp for $N=7$ is also shown. Energy $\bar{\omega}$ is in ϵ_C units.

emission band versus the integer parameter $2S$, $S=(R/l)^2$, R is the radius of a sphere, $N=6$. The width γ is defined as $\gamma^2=\langle\omega^2-\bar{\omega}^2\rangle$. The impurity center is inside the sphere at a distance h^* from its north pole.[12] The dependence $\bar{\omega}(2S)$ is shown in Fig. 2 for three values of h^*/R; it is natural to set $h^*=R$ in correspondence with $h=\infty$ in the plane geometry. Three cusps are seen against a smooth background; all of them correspond to nondegenerate i states. The cusp $2S=15$ corresponds to $\nu=1/3$ [according to the equation $2S=(N-1)/\nu$[18]], the cusp $2S=9$ corresponds to $\nu=2/3$ [according to the charge symmetry relation: $\nu\rightarrow(1-\nu)$, $N\rightarrow(2S+1)-N$], and the cusp $2S=11$ has no definite assignment. The cusps $\nu=1/3$ for $N=7$ are shown for comparison. The cusp strength increases with h^* in accordance with (9). In Fig. 3 $\gamma(2S)$ is shown for three values of h^*/R. It is obvious that γ depends strongly on S. The minima of γ correspond to all the cusps. The depth of these minima increases with increasing distance of an impurity center from the 2D layer.

The basic assumption used above is that an impurity center does not affect 2D electrons in the initial state. This interaction is small as compared to ϵ_C in the parameter $r_{\rm imp}/h$. Nevertheless, it may prove to be important since it reduces the symmetry and lifts the degeneracy of the ground level. This may result in discontinuities of the

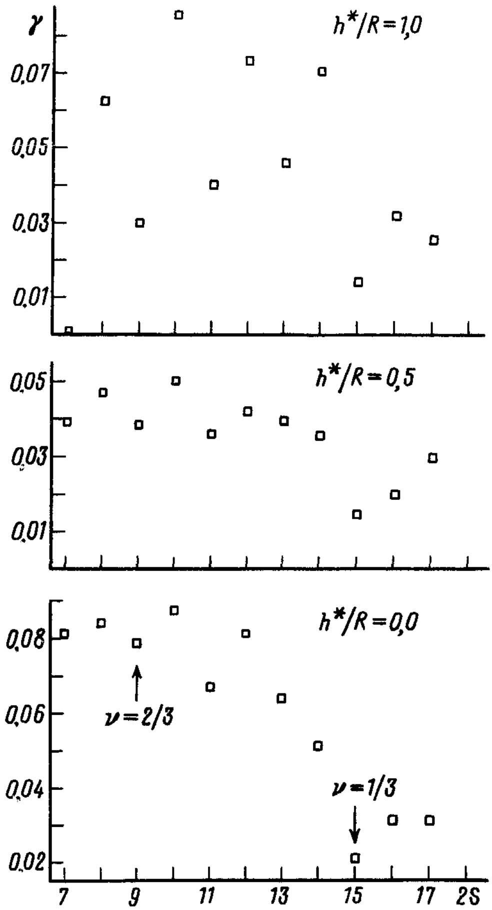

FIG. 3. Width γ of the emission spectrum vs. $2S$ at $N=6$ for three values of h^*/R. γ is in ϵ_C units.

function $\bar{\omega}(\nu)$ (and not only of its derivative) having the scale of Δ_r, (see, for example, Ref. 12). The actual effect of the initial state interaction on the optical spectrum depends on the values of T, h, and $r_{\rm imp}$. The discontinuities must be smeared by disorder and also by the nonequilibrium distribution of photoproduced charge carriers. The recombination scheme used above imposes some restrictions on the value of h. For a shallow acceptor near a deep quantum well, the appropriate condition is $h \ll l^2/r_{\rm imp}$. The dependence of the shape of $\bar{\omega}(\nu)$ curves on the recombination scheme must be also investigated.

For the discussion of experimental data from the standpoint of this paper, see the paper published in this issue of JETP Letters.[19]

In conclusion, gaps in the energy spectrum of the incompressible 2D liquid can be found from cusps of the function $\bar{\omega}(\nu)$ for radiative trapping of 2D electrons at neutral impurities. Due to a rapid decrease of the extrinsic contribution to the cusp strength with h, this method may prove to be useful for actual distances of impurity centers from the confinement layer.

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