# Determination of the Coulomb gaps under the conditions of fractional quantum Hall effect by the magnetoluminescence method

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The results of an analysis of the spectral position curves of the luminescence line, which were measured experimentally and calculated theoretically, are presented. The luminescence line corresponds to a radiative recombination of 2D electrons with the holes at a neutral impurity center in a $\delta$-doped AlGaAs/GaAs heterostructure under the conditions of fractional quantum Hall effect. The Coulomb gaps are determined.

The ground-state energy of the interacting 2D electrons, $E(N)$, which are situated in a strong transverse magnetic field, plotted as a function of the total number of particles, $N$, exhibits cusps as a result of filling, which correspond to noninteger factors $\nu \equiv n_s/N_0 = p/q (N = n_s \cdot S)$, where $n_s$ is the concentration, $S$ is the area, $N_0 = eH/hc$ is the capacity of the Landau level, $H$ is the magnetic field, $p$ is an integer, and $q$ is an odd integer).[1] Such a filling gives rise to incompressible Fermi-liquid states (IFL states or Laughlin's states) which arise from the fractional quantization of the Hall resistance.[2,3] An important quantity in the fractional quantum Hall effect is the Coulomb gap between the ground state of the IFL and the continuum of the quasiparticle excitations with the fractional charges $e^* = \pm e/q$ ($e$ is the electron charge). The Coulomb gap $\Delta_q$ and the discontinuity in the chemical potential $\xi$ at the cusp of the function $E(N)$ are related by a simple relation[4]

$$\delta\xi = \frac{\delta E}{\delta N}\bigg|_{+} - \frac{\delta E}{\delta N}\bigg|_{-} = q\Delta. \qquad (1)$$

Under the conditions of the fractional quantum Hall effect the Coulomb gaps usually are measured by means of an activated magnetic transport.[5] More precisely, this method is used to determine the "mobility gaps" which are highly sensitive to any disorder in the system. It should be borne in mind that the magnetic-transport method runs into virtually insurmountable difficulties in the ultraquantum limit ($\nu \ll 1$) because of the increasing effects of strong localization as one advances into the region of progressively decreasing values of $\nu$.

A spectroscopic method of determining the size of Coulomb gaps in the fractional quantum Hall effect was proposed in Refs. 6–8. This method was initially used in the case of a 2D electron channel in an Si field transistor[6,7] and later for a $\delta$-doped AlGaAs/GaAs heterostructure with 2D electrons.[10] The magneto-optic method makes use of the radiative recombination of 2D electrons with the holes situated at the neutral acceptor center placed a fixed distance from the electron layer.[8] The dependence of the spectral position of the luminescence line in this case is studied as the filling factor is varied (the magnetic field is varied while the concentration $n_s$ is kept constant). It was found that for fractional values of $\nu$ such a dependence shows that abrupt changes occur on the energy scale. On the basis of strictly qualitative considerations, which have no rigorous theoretical foundation, such structural features were attributed to abrupt changes in the chemical potential in the interacting electron system as a result of the condensation in the IFL. It was assumed that the energy of the emitted photon during the recombination, i.e., when the electron leaves the 2D layer, is related to the difference in the ground-state energies of the 2D system before and after the recombination: specifically, $\hbar\bar{\omega} \sim \xi \equiv E(N) - E(N-1)$ [$E(N)$ and $E(N-1)$ are the ground-state energies of a system with the number of electrons $N$ and $N-1$]. This assumption means that the interacting electron system adiabatically follows the tunneling recombination of a 2D electron with an acceptor hole. If these assumptions are correct, then the size of the Coulomb gap, which corresponds to a specified value of $\nu$, can be determined by using Eq. (1) from the magnitude of the experimentally measured abrupt change in the spectral position of the luminescence line in the quantum Hall effect.

So far there have been no theoretical calculations of the magnetic luminescence spectra which were measured under the conditions of fractional quantum Hall effect. The article by Apal'kov and Rashba[9] in this issue of JETP Letters is actually the first study that addresses this topic. Using numerical methods for a homogeneous 2D system with a finite number of particles, they have calculated the spectral position of the luminescence line (the first-order moment), which corresponds to the recombination of a 2D electron with a neutral acceptor, as a function of the filling factor. The model-based approximations used by them closely approximate the conditions of the experimental study of AlGaAs/GaAs heterostructures with $\delta$ doping carried out previously.[10] We recall that the magnetic field was fairly large in that experiment: $\hbar\omega_c > e^2/\kappa l_H$ ($\omega_c$ is the cyclotron frequency, $l_H = [ch/eH]^{1/2}$ is the magnetic length, and $\kappa$ is the dielectric constant), so that the mixing of various Landau levels could be disregarded. The distance of the electron channel from the layer of the neutral acceptors was greater than the width of the channel and greater than the radius of the acceptor centers. In the initial state the impurity center was neutral and in the final state it was charged.

The theory[9] was formulated under the assumption that in the initial state the neutral centers perturb the 2D electron system only slightly. In the analysis the effect produced by disorder was omitted. On the basis of such an approach, the most important conclusion of the theory, in our view, is the fact that in the limit of large distances of the $\delta$ layer from the electron channel, the spectral position of the luminescence line follows the behavior of the energy of the 2D system in the ground state; specifically, $\hbar\bar{\omega}\sim 2E(N)/N$. Since this is a conceptually new conclusion, we thought that it would be very useful to compare the results of the calculations of Ref. 9 with experiment.

In Fig. 1a we compare the variation of the spectral position of the luminescence

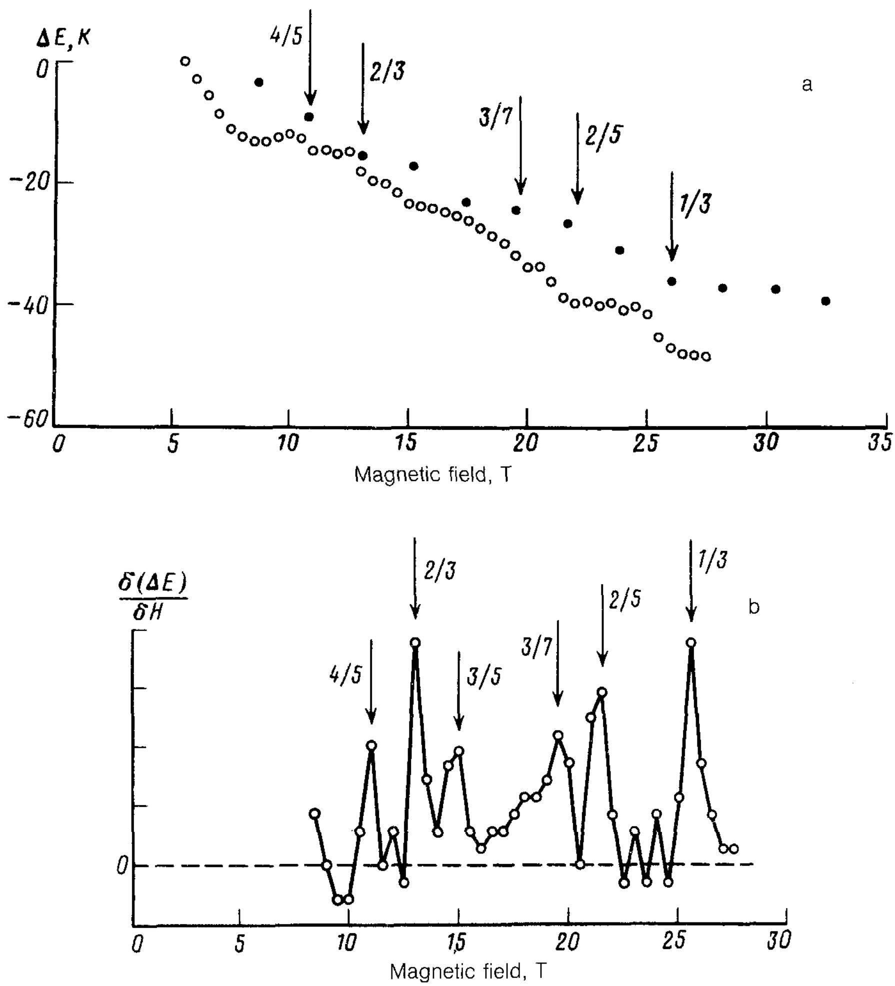


FIG. 1. (a) Change in the spectral position of the luminescence line after subtracting the term $1/2\hbar\omega_c$, produced as a result of variation of the magnetic field: open circles—experimental measurements[10]; filled circles—theoretical calculations.[9] In the experiment, $n_s=2\times10^{11}$ cm$^{-2}$. (b) The derivative $\delta(\Delta E)/\delta H$ of the experimentally measured curve for the variation of the spectral position of the luminescence line in a magnetic field (the ordinate is in arbitrary units).

line, $\Delta E(H)$, as a result of variation of the magnetic field, measured experimentally (open circles) and calculated theoretically, for six particles (filled circles). The component $1/2h\omega_e$ was subtracted from these curves. The arrows in Fig. 1a show the fractional values of $\nu$, near which some structural features (cusps) are observed on the $\Delta E(H)$ curves. The comparison is made for the case in which the $\delta$ layer of the acceptors is situated approximately 250 Å from the 2D channel. Since the adjustable parameters were not used in the comparison, we believe that the agreement between experiment and calculation is satisfactory.

Let us carry out a more detailed comparison. According to calculations, the structural features associated with the fractional values of $\nu$ always have "downward-pointing cusps." This feature is attributable to the behavior of the $E(H)$ function which characterizes the ground state of the 2D system. In addition to such downward-pointing cusps, we have experimentally observed some upward-pointing cusps near the fractional values of $\nu$ in low magnetic fields $H$ (large $\nu$). The result of a numerical differentiation of an experimentally measured curve of $\Delta E(H)$ is shown in Fig. 1b. The derivatives $\delta(\Delta E)/\delta H$ in the $H$ function show the presence of narrow peaks in the immediate vicinity of the fractional $\nu$. The height of these peaks clearly shows that the derivative changes abruptly near the cusps. The symmetric nature of the peaks shows that the upward- and downward-pointing cusps are approximately the same for fractional $\nu$. According to the theory, however, the derivative $\delta(\Delta E)/\delta H$ is a discontinuous function when $H$ is varied, and must have a series of steps for fractional $\nu$. These differences stem exclusively from the presence of upward cusps on the experimental curves.

We see from the theory that the Coulomb gap can be determined from the discontinuity of the derivative $\delta(\Delta E)/\delta\nu$ near the downward-pointing cusp. For the case in which the $\delta$ layer of the acceptors is situated far from the 2D channel we obtained an analytic expression[9] in which the Coulomb gap is related to the amplitude of the downward-pointing cusp $\delta(\delta(\Delta E)/\delta H)$:

$$\Delta = \frac{\nu}{2q}\delta\left(\frac{\delta(\Delta E)}{\delta\nu}\right) = \frac{H}{2q}\delta\left(\frac{\delta(\Delta E)}{\delta H}\right). \qquad (2)$$

Using this expression, we can easily determine the Coulomb gaps near all the experimentally measured downward-pointing cusps. The corresponding results are shown in Fig. 2. The results of an analysis of the experimental data using expression (2) for $\nu = P/3$ ($P = 1,2$) is shown in Fig. 2a. The solid line represents the limiting value of the Coulomb gap for $\nu = 1/3$, plotted as a function of $H$, calculated for an infinitesimally thin, ideal layer.[11] Also shown in this figure are the Coulomb gaps (crosses) determined after introducing the correction for the finite distance of the $\delta$ layer from the 2D channel, using the procedure outlined in Ref. 12. Figure 2b shows the Coulomb gaps for fractional values of $\nu$, $\nu = p/q$ ($p = 1,2,3,4$; $q = 5,7,9$), which were determined from the experimental curves in a similar manner. Note that the Coulomb gaps determined on the basis of the procedure used in Ref. 9 closely approach (but do not exceed!) the limiting theoretical values. Note also that the Coulomb gaps determined in this manner are approximately three times larger than those found previously from the same experimental curves for the abrupt changes in the spectral positions of the lines.[10]

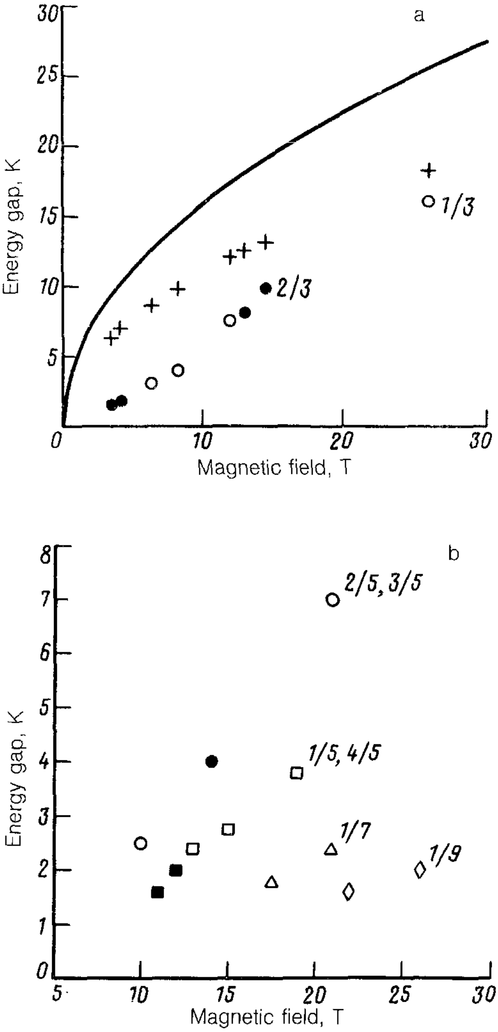


FIG. 2. (a) Coulomb gaps for fractional filling factors $\nu = p/3$ (where $p = 1,2$) determined from the downward-pointing cusps on the measured $\Delta E(H)$ curves, using Eq. (2) (open and filled circles) and those determined after introducing a correction factor for a finite separation of the $\delta$ layer from the electron channel (crosses). The open and filled circles correspond to the filling factors of 1/3 and 2/3. (b) Coulomb gaps for $\nu = p/q$ (where $q = 5, 7, 9$), determined from the experimental $\Delta E(H)$ curves using the procedure described in the paper. The filled squares and the filled circles correspond to the filling factors 4/5 and 3/5.

We can summarize the results discussed here as follows. The theory developed by Apal'kov and Rashba[9] accurately describes the general picture of the magnetoluminescence, under the conditions of fractional quantum Hall effect, on the basis of the model-based representations outlined above. It also formulates a fundamentally new procedure for determining the Coulomb gaps in magneto-optics. There are, however, some differences in the conclusions obtained experimentally and theoretically. First of all, they concern the upward-pointing cusps in the experimentally observed behavior of the luminescence line near the fractional values of $\nu$. The calculated curves have no such cusps. The theory also ignores disorder, which is always present in actual systems. As a result, the linewidths in the calculations have a similar origin and do not exceed the scale of the Coulomb gaps. In the experiments, however, the luminescence lines are broadened nonuniformly, and their widths are markedly larger than the calculated widths. In the case of fractional values of $\nu$, a slight line broadening (by $\sim 10\%$) has been observed experimentally, while the uniformly broadened lines in the fractional quantum Hall effect always become narrower, according to theory. The reasons for these discrepancies are not yet understood. Finally, the role of the interaction between the neutral center and the 2D electrons in the initial state, and also the extent of the deviation from equilibrium of the electron distribution in the initial and final states in the optical experiments remain unresolved problems.

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