One-electron depressions on a thin film of helium

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The nonmonotonic behavior of the characteristics of one-electron depressions on a thin film of helium as a function of the thickness of this film is discussed. In the limit of very thin films, the formation of one-electron depressions becomes unfavorable from the energy standpoint.

The problem of the properties of one-electron depressions on a thin film of helium has been studied quite thoroughly on the theoretical side. We have in mind the first variational calculations on this formation, which were carried out by Monarkha[1] in the limit of absolute zero, through the successful application of Feynman's technique[2] to the description of polaron states, which made it possible to introduce a temperature in the problem of depressions on a thin film of helium,[3,4] to study the dynamic properties of such depressions,[5–7] etc. The various approaches which have been taken to derive a theory of one-electron depressions merge quite well and lead to the general qualitative conclusion that with decreasing thickness d of the helium film, the binding energy W_ξ of the one-electron depression increases monotonically, and fairly rapidly [asymptotically, $W_\xi(d) \propto d^{-4}$]. The onset of this behavior in the case of depressions formed on a thin helium film itself on a metal substrate results in breakdown and an escape of the electron into the helium.[8]

However, the usual conclusion that the binding energy of the depression increases monotonically with decreasing d actually turns out to be wrong. Our purpose in the present letter is to formulate and prove the assertion that the formation of one-electron depressions on a thin helium film becomes unfavorable from the energy standpoint in the limit $d \to 0$.

1. In order to formulate the problem of the existence of a depression, we introduce several necessary definitions. The variational calculation of Ref. 1 leads to the following expressions for the energy of the depression, W_ξ, for its localization length l_ξ, and for the depth of the pit at its center, $\xi(0)$:

$$W_\xi = -\frac{F^2}{4\pi\alpha}\left(\ln\frac{1,3}{\tilde{\kappa} l_\xi} - \frac{1}{2}\right), \quad \tilde{\kappa}^2 = \frac{\rho\tilde{g}}{\alpha}, \quad \tilde{g} = \frac{3\Delta}{\rho d^4}, \tag{1}$$

$$l_\xi^2 = 4\pi\alpha\hbar^2/(mF^2), \quad \tilde{\kappa} l_\xi \ll 1, \tag{2}$$

$$\xi(0) \cong -\frac{F}{2\pi\alpha}\ln\frac{1}{\tilde{\kappa} l_\xi}. \tag{3}$$

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Here α is the surface tension of the liquid helium, ρ is its density, $\tilde{g}$ is the effective acceleration of van der Waals origin, Δ is the van der Waals constant, and F is the force which tends to press the electron toward the free surface of the film. In the case $d < \gamma_\infty^{-1}$, which is the case of interest below, the quantity F is usually defined by[1-8]

$$F = \Lambda_s/d^2 \qquad \Lambda_s = \frac{e^2(\epsilon_s - 1)}{4(\epsilon_s + 1)} \tag{4}$$

where ϵ_s is the dielectric constant of the substrate, d is the thickness of the helium film under the depression, γ_∞^{-1} ($\gamma_\infty = [me^2(\epsilon - 1)]/[4(\epsilon + 1)\hbar^2]$) is the distance over which the electron is localized by the interaction with the liquid substrate, and ϵ is the dielectric constant of the liquid helium.

Using definitions (1)–(4), we can easily understand the assertion that W_ξ is a monotonic function of d. The combination $\tilde{\kappa} l_\xi$ in the argument of the logarithm in (1) turns out to be independent of d when we take F from (4):

$$\tilde{\kappa}^2 l_\xi^2 = \frac{12\pi\Delta\hbar^2}{m\Lambda_s^2} < 1. \tag{5}$$

The energy W_ξ is thus negative and increases monotonically with decreasing d, in accordance with $W_\xi \propto d^{-4}$.

The definition of F in (4) is inaccurate. That definition is valid only in the region

$$\gamma_\infty^{-1} > d \gg \gamma_d^{-1}, \tag{6}$$

where γ_d^{-1} is the localization length of an electron above a helium film for the case of a localization caused by the image potential in the substrate. In the case $d < \gamma_d^{-1}$, the law $F \propto d^{-2}$ breaks down, and the combination $\tilde{\kappa} l_\xi$ begins to depend on d. This effect can be modeled schematically by[1)]

$$F = \frac{\Lambda_s}{(d + \gamma_d^{-1})^2}, \quad \tilde{\kappa}^2 l_\xi^2 \propto \frac{(d + \gamma_d^{-1})^4}{d^4} = \begin{cases} 1, & d\gamma_d > 1 \\ \gg 1, & d\gamma_d < 1 \end{cases} \tag{7}$$

According to (7), the parameter $\tilde{\kappa} l_\xi$ begins to increase in the region $d_{\gamma d} < 1$, in accordance with $\tilde{\kappa}^2 l_\xi^2 \propto d^{-4}$. In the case $\tilde{\kappa} l_\xi > 1$, the localization energy W_ξ in (1) becomes positive; a positive value corresponds to a destruction of the depression.

2. A definition of the functional dependence $F(d)$, which is more systematic than that in (7), is

$$F(d) = \frac{\partial < V >}{\partial d}, \quad < V > = -\Lambda_s \int_0^\infty \frac{f^2(z)dz}{z + d}, \tag{8}$$

where $f(z)$ is the normalized solution of the wave equation

$$\frac{\hbar^2}{2m} f'' + \left[E + \frac{\Lambda_s}{d + z} \right] f = 0, \quad f(z)|_0 = 0, \tag{9}$$

which corresponds to the minimum energy E_1. The variational problem in (9) with the trial function

$$f_1(z) = 2\gamma_d^{3/2} z \exp(-\gamma_d z) \tag{9a}$$

leads to an equation for γ_d:

$$\frac{\partial < \hat{H} >}{\partial \gamma_d} = 0, \quad < \hat{H} > = -\frac{\hbar^2}{2m}\gamma_d^2 + 4\Lambda_* d^2 e^{2d\gamma_d}\Gamma(-2, 2d\gamma_d)\,, \tag{10}$$

where $\Gamma(-\nu,\mu)$ is the incomplete gamma function. In the limit $d \to 0$, a solution of Eq. (10) yields

$$\gamma_d|_{d\to 0} = \gamma_0 = m\Lambda_*/\hbar^2. \tag{10a}$$

Analysis of definitions (8)–(10) shows that the saturation of the force $F(d)$ which is modeled by (7) persists in a more systematic formalism. The assertion

$$\tilde{\kappa} l_\xi|_{d\to 0} \longrightarrow \infty \tag{11}$$

thus remains in force, so depressions cannot exist in the region $d\gamma_0 < 1$.

[1)] The saturation in the dependence $F(d)|_{d\to 0}$ was first noted by Paalanen and Iye.[9]

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