Anomalous thermodynamic characteristics of an S/N superlattice

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The density of states has been found for superconducting and normal monoatomic layers of a superlattice. The energy corresponding to the maximum of the density of states, $\tilde{\Delta}$, may be considerably higher than the standard value according to the BCS theory, $1.76T_c$. This circumstance might explain the large value of the ratio $2\tilde{\Delta}/T_c \approx 5$–$6$ for high-T_c superconductors.

The high-T_c superconductors have a pronounced, layered-type electron anisotropy, which reflects the particular crystal structure of these materials (Ref. 1, for example). At present, most researchers lean toward the interpretation that CuO_2 layers are responsible for the superconductivity in these materials. In addition to these layers, the high-T_c superconductors contain layers of other types, in which one might expect to see evidence of metallic properties (in $YBa_2Cu_3O_7$, these would be layers of CuO, Y, and Ba chains; in the Bi and Tl compounds, they would be BiO, TlO, Sr, and Ca layers; etc.). In addition, recent technological progress[2] in the fabrication of heterostructures raises the hope that it would someday become possible to create artificial S/N systems with monoatomic layers.

In this connection let us consider the properties of a system consisting of an alternation of superconducting (S) and normal monolayers (N) monolayers. We assume that a Cooper pairing occurs only in the S layers and that the hopping integral for the hopping between layers satisfies $t \ll E_F$. It is possible that this model, which was proposed by Bulaevskii and Zyskin,[3] correctly conveys the basic features of S/N superstructures. The Hamiltonian of the system is

$$H = \sum_{p,n,i,\sigma} \{\xi(\vec{p}) a^{+}_{ni\sigma}(\vec{p}) a_{ni\sigma}(\vec{p}) + t(a^{+}_{ni\sigma}(\vec{p}) a_{n,-i,\sigma}(\vec{p}) + a^{+}_{n,-i,\sigma}(\vec{p}) a_{ni\sigma}(\vec{p})$$

$$+\text{c.c.})\} + \frac{\lambda}{2} \sum_{p_1,p_2,n,\sigma} a^{+}_{n1\sigma}(\vec{p}_1) a^{+}_{n,1,-\sigma}(-\vec{p}_1) a_{n,1,-\sigma}(-\vec{p}_2) a_{n1\sigma}(\vec{p}_2), \qquad (1)$$

where the operator $a^{+}_{ni\sigma}(\vec{p})$ creates an electron with a momentum $\vec{p}$ and a spin σ in layer i of unit cell n (which consists of S and N layers). The index i takes on the values $i = 1$, for an S layer, and $i = -1$, for an N layer. Here $\xi(\vec{p}) = E(\vec{p}) - E_F$. We assume for simplicity that the electron dispersion $E(\vec{p})$ is the same in the N and S layers.

It is possible to derive an exact solution of the equations for the normal (G_{ij}) and anomalous (F^+_{ij}) Green's functions ($i,j = \mp 1$ are the indices of the layers in the unit cell). In particular, we have

$$G_{ii}(\omega,\vec{p},q) = [\omega_-(\omega_+^2 - \tilde{T}^2) + (i-1)\omega_+\Delta^2/2]/(\omega^2+E_1^2)(\omega^2+E_2^2),$$

$$F^+_{ii}(\omega,\vec{p},q) = -\Delta\omega_-\omega_+/(\omega^2+E_2^2)(\omega^2+E_2^2),$$

$$\omega_\mp = i\omega \mp \xi, \tag{2}$$

$$E^2_{1,2} = \xi^2 + \tilde{T}^2 + \Delta^2/2 \pm (4\xi^2\tilde{T}^2 + \Delta^4/4 + \tilde{T}^2\Delta^2)^{1/2},$$

$$\tilde{T} = t(1+e^{iq}).$$

As t increases, the transition temperature of the system of course decreases (because the neighborhood of the N layers becomes more influential), but there is also a decrease in the parameter Δ. It is a simple matter to derive an equation for the ratio $\Delta(0)/T_c$:

$$0 = \int d\xi dq \sum_{k=1}^{2}\left(\frac{\mathrm{th}(E_k(\xi,q,0)/2T_c)}{E_k(\xi,q,0)}\left(1-(-1)^k\frac{T}{2\xi}\right)\right.$$
$$\left.-E_k^{-1}(\xi,q,\Delta(0))\left(1-(-1)^k\frac{T^2+\Delta^2(0)/2}{(4\xi^2\tilde{T}^2+\Delta^4/4+\tilde{T}^2\Delta^2)^{1/2}}\right)\right). \tag{3}$$

The dashed line in Fig. 1 shows the ratio $\Delta(0)/T_c$ as a function of the hopping integral t (again, for hopping between layers). It is easy to see that this quantity differs from the standard value according to the BSC theory, $\Delta(0)/T_c = 1.76$ at $t=0$. It doubles at large values of t.

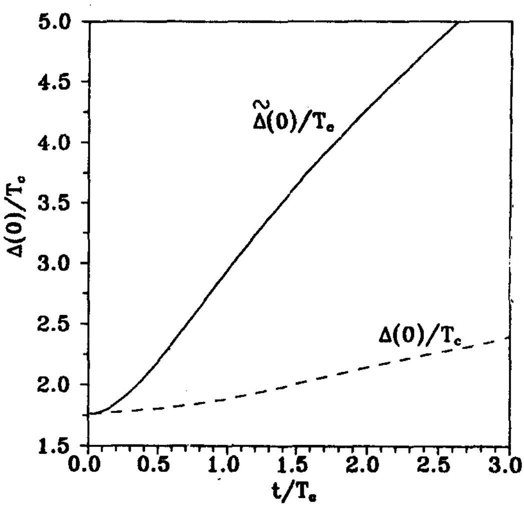

FIG. 1. The ratio Δ/T_c versus the hopping integral t.

However, we would like to know how the parameter $\Delta(0)$ is related to those characteristics of a system which are usually measured in actual experiments. Let us look at a calculation of the density of states $\rho_i(E) \sim \mathrm{Sp}[\mathrm{Im} G_{ii}(i\omega \to E + i\delta)]$, which can be found from tunneling measurements (among other ways). Using expression (2) for G_{ii}, we find

$$\rho_i(E) = \frac{\rho(0)|E|}{2\pi} \int_0^\pi dq \left\{ \left(1 + \frac{\tilde{T}^2 - i\Delta^2/2}{\sqrt{4E^2\tilde{T}^2 + \Delta^4/4 - \tilde{T}^2\Delta^2}}\right) |\xi_1|^{-1} \right.$$

$$\left. + \mathrm{sign}\left(\sqrt{4E^2\tilde{T}^2 + \Delta^4/4 - \tilde{T}^2\Delta^2} - 2\tilde{T}^2\right)\left(1 - \frac{\tilde{T}^2 - i\Delta^2/2}{\sqrt{4E^2\tilde{T}^2 + \Delta^4/4 - \tilde{T}^2\Delta^2}}\right)|\xi_2|^{-1} \right\}, \tag{4}$$

where $\xi_{1,2}^2 = E^2 + \tilde{T}^2 - \Delta^2/2 \pm \sqrt{4E^2\tilde{T}^2 + \Delta^4/4 - \tilde{T}^2\Delta^2}$. Figure 2 shows the results of numerical calculations of the density of states in the S layers (the solid line) and in the N layers (the dashed lines) for various values of t.

As was pointed out in Ref. 3, the superconductivity in this case is a gapless superconductivity. Analysis of expression (4) shows that the functional dependence $\rho(E)$ is considerably more complex than as described in Ref. 3. There are two logarithmic singularities on the plot of $\rho(E)$; the singularity at low energies has a greater effect in the N layer. With increasing t, the singularity shifts up the energy scale, tending toward the value $E = \Delta/2$ in the large-t limit. The singularities in the S and N layers become essentially identical here. The singularity at $E > \Delta$ is observed at energies

$$E = \tilde{\Delta} = (4t^2 + \Delta^2/2 + \sqrt{4t^2\Delta^2 + \Delta^4/4})^{1/2} \tag{5}$$

(see the solid line in Fig. 1). The singularity is more obvious in the S layer. It is easy to see that $\tilde{\Delta}$ incrases rapidly with increasing t. Golubov and Kupriyanov[4] have derived a functional dependence $\rho(E)$ with two singularities for a model of a "thick" S/N bilayer.

Since the correlation length along the direction perpendicular to the layers is on the order of the interplanar spacing in the high-T_c superconductors,[1] we can use the estimate $t \sim T_c$. It is obvious from Fig. 2 that at these values of the hopping integral the quantity $\tilde{\Delta}$ is quite far from its "actual" value $\Delta(0)$ and corresponds to $2\tilde{\Delta}/T_c \approx 5$. This peak on the density of states can be interpreted as the value of Δ in tunneling experiments and can explain the large value of the ratio $\Delta(0)/T_c$ in the high-T_c superconductors.

At $E = \Delta$, a jump appears in the density of states in the S layer; with increasing value of the hopping integral t, this jump shrinks. At large values of t ($t > 0.25\Delta$), an additional jump appears at

$$E = (4t^2 + \Delta^2/2 - \sqrt{4t^2\Delta^2 + \Delta^4/4})^{1/2}.$$

This second jump is seen in the density of state for both the N and S layers. A fine

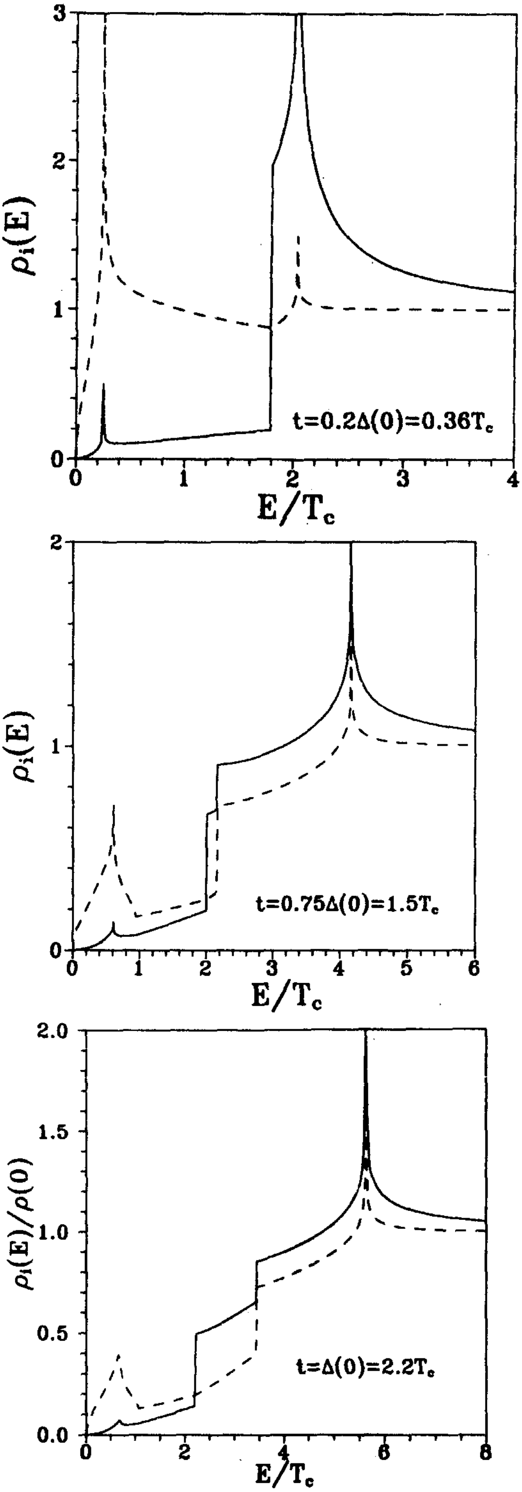

FIG. 2. Density of states at superconducting layer (solid line) and at a normal layer (dashed line) for various values of the hopping integral t.

structure of this sort inside the "gap" has indeed been observed in several experiments.[5,6]

The value found for the ratio $\Delta(0)/T_c$ from tunneling measurements may thus be considerably higher than the standard BCS values. In real high-T_c systems the presence of a large number of layers, with various values of the hopping integrals for hopping between these layers, disrupts the gapless nature of the superconductivity,[3] but it does not qualitatively change the results derived here. In a numerical analysis of a model with five different layers in the unit cell, Tachiki *et al.*[7] found indications of a fine structure. Because of the complexity of their model,[7] found indications of a fine structure. Because of the complexity of their model,[7] however, it is not possible to draw any conclusions about the changes in the characteristics of this system with an increase in the coupling between layers.

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