Continuous topological defects on the ^{3}He A–B interface

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The microscopic structure of topological defects on the ^{3}He A–B interface is considered. An explicit description of a certain class of such defects is presented. Nonequivalence of positive and negative topological charges is demonstrated.

Recently a topological classification of the defects on the ^{3}He A–B interface was proposed.[1–3] Here we consider possible microscopic structure of some of the defects. Characteristic for our solution is a nonvanishing and everywhere-continuous distribution of the order parameter. These properties seem to contradict the topological nature

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of the defects because the defects with nonzero topological charges have as it is well known, a singular "hard" core. Inside the hard core region of the order parameter no longer belongs to the vacuum manifold of a given phase and may vanish. We will show, however, that in some cases this singularity can be eliminated by changing the shape of the interface involving creation of handles.[1)]

The bulks of the A and B phases are described by distributions of the order parameter which has a form $A^{A}_{\alpha i} = \Delta_A d_\alpha (e_{1i} + ie_{2i})$ in the A phase and $A^{B}_{\alpha i} = \Delta_B \exp(i\Phi) R_{\alpha i}$ in the B phase. As a boundary condition we require that the vector $\vec{l} = \vec{e}_1 \times \vec{e}_2$ in the A phase near the interface should be parallel to it.[4,5] Other constraints should be added in order to make the boundary condition complete. They specify for each value of the order parameter $A^{A}_{\alpha i}$ in the A phase a set of permissible values of the order parameter $A^{B}_{\alpha i}$ in the B phase on the opposite side of the interface, and vice versa. In other words, a pair $(A^{A}_{\alpha i}, A^{B}_{\alpha i})$ satisfies the boundary condition if it can be obtained from the pair $(A^{0A}_{\alpha i}, A^{0B}_{\alpha i})$. where $A^{0A}_{\alpha i} = \Delta_A \hat{x}_\alpha (\hat{x}_i - i\hat{z}_i)$, $A^{0B}_{\alpha i} = \Delta_B \delta_{\alpha i}$, by the action of an element of the symmetry group $G = U(1) \times SO(2)^L SO(3)^S$. Here x is normal to the interface, $U(1)$ is the gauge group, $SO(2)^L$ denotes the group of space rotations around x, and $SO(3)^S$ is the group of all spin rotations.

The result of the topological analysis can be summarized as follows:[2,3] a pointlike singularity of the interface is characterized by a triplet (m_ϕ, m_l, m_R), where $m_\Phi, m_l \in Z$ are winding numbers for the phase Φ of the order parameter (both in the A and B phases) and for the vector $\vec{l}$ (in the A phase); the index $m_R \in Z_2$ stands for disclinations in the field of R matrix in the B phase. Here we study two types of defects (Figs. 1a and 1b):

a) pointlike singularities localized at the interface (boojums), for which m_l is even; $m_\Phi = m_r = 0$.

b) singular lines (vortices and disclinations) of the B phase which terminate in the pointlike defect of the interface; in this case $m_\Phi + m_l$ is even

A possible microscopic picture of the defects **a**), **b**) is shown in Figs. 1c and 1d.

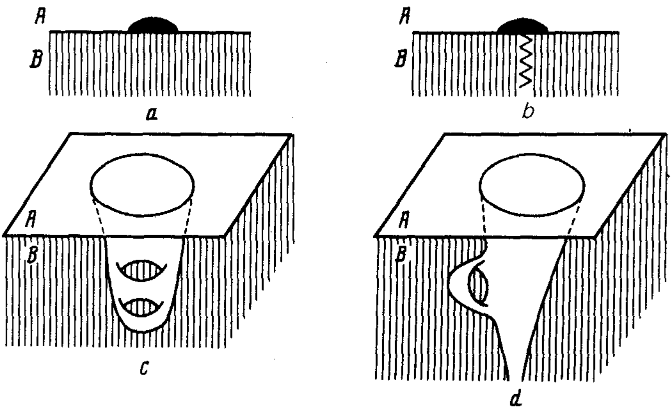

FIG. 1. Schematic diagram of (a) boojums and (b) singular lines in the B phase which terminates at the interface; (c) microscopic structure in the case with $g = 2$, (d) microscopic structure in the case with $g = 1$.

The A–B interface is bent to form a connected surface C which separates the bulks of the A and B phases. This changing of the shape of the interface is energetically preferable if there exists a continuous distribution of the order parameter in the bulk compatible with the boundary conditions on C. One then has the structure which macroscopically looks like the appropriate boojum or vortex but has no singularities in the microscopic order-parameter distribution.

We consider first the case **a**) (boojums). The boundary surface can be compactified at infinity, where the A phase is the interior of the compactified surface. We will then have a compact, two-dimensional, orientable manifold $\tilde{C}$ homeomorphic to a Riemannian surface of some genus g, i.e., to the two-dimensional sphere S^2 with g handles. According to the boundary condition, the vectors $\vec{l}$ form a tangent field on $\tilde{C}$, which is continuous everywhere except at "the infinitely distance" point N which is added to the surface C to make it compact: $\tilde{C} = C \cup \{N\}$. In view of this circumstance, it is necessary to find the obstructions for the existence of such a field. The answer is known as the Euler theorem: the sum of the indices of all singular points of a tangent vector field is $2-2g$ (Euler's characteristic of the Riemannian surface of genus g).

Since the index of the $\vec{l}$ field in N is equal to $2-m_l$, we obtain $2-m_l = 2-2g$ or $m_l = 2g$. We conclude that for $m_l < 0$ such a structure cannot exist. We found that for $m_l = 2$ (in this case $g = 1$, and the appropriate surface $\tilde{C}$ is a torus) there exists a continuous distribution of all other components of the order parameter which includes $\vec{e}_1$, $\vec{e}_2$, $\vec{d}$, $R_{\alpha i}$ and Φ and which satisfies all the boundary conditions. It is shown schematically in Fig. 2.

For larger m_l one can construct similar distributions. They contain pointlike

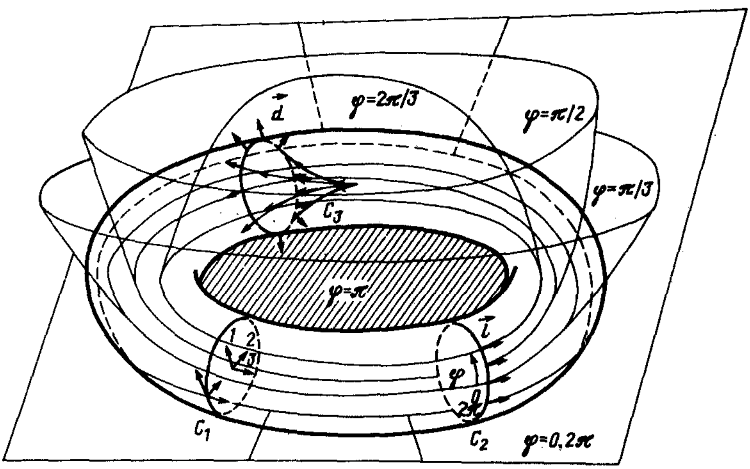

FIG. 2. Schematic diagram of the order parameter distribution for the case $m_l = 2$, $g = 1$. The A phase fills the interior of the torus. Lines of the $\vec{l}$ vector coincide with the parallels of the torus (see Section C_2). Triads $(\vec{e}_1, \vec{e}_2, \vec{l})$ are uniform throughout a given cross section (see, e.g., triad (1,2,3) in the section C_1); $\vec{d}$ vectors on the surface of the torus are perpendicular to it and form a continuous funnel-like structure in the interior (section C_3). The matrix $R_{\alpha i}$ in the B phase in $\delta_{\alpha i}$. Surfaces of a constant phase Φ of the B phase look like closed domes leaning on the parallels of the torus. The disk bounded by the shortest parallel corresponds to $\Phi = \pi$. The horizontal plane surface corresponds to $\Phi = 0,2\pi$.

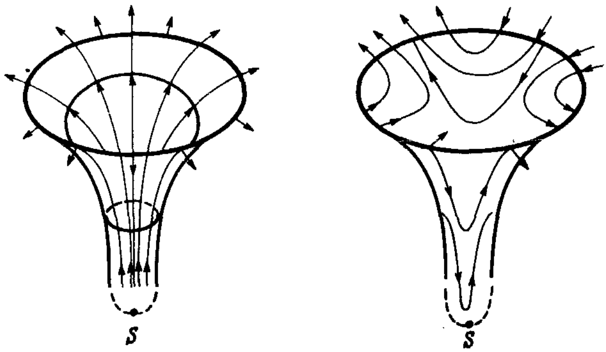

FIG. 3. Lines of the $\vec{l}$ vector on the surface of the funnel in the case $m_l = +1$ (left) and $m_l = -1$ (right). For the former we have $I_S = 1$, which allows for a uniform distribution of $\vec{l}$ near S, while for the latter $I_S = -1$ and thus a texture of $\vec{l}$ near S arises unavoidably.

singularities in the A phase (hedgehogs of the $\vec{d}$-vector distribution). Their total topological charge is $1-g$.

Let us consider now case **b**) of the vortex lines which terminate at the interface. In order to compactify the surface C, one has to add the point N and also to glue "the neck of the funnel" by a point S. The previous considerations of the $\vec{l}$-vector distribution will then apply and we find the index of the $\vec{l}$ field in S to be $I_S = m_l - 2g$. We see that only $I_S = 1$ allows for a space-uniform distribution of $\vec{l}$ near S. Any other I_S involves a texture with large $(\nabla \vec{l})^2$ in the vortex core. (Because for $m_l = +1$ one can set $g=0$ and obtain $I_S = 1$, which is impossible for $m_l = -1$; see Fig. 3.) This observation implies that the vortices with $m_l = +1$ and $m_l = -1$ are not equivalent with respect to their ability to penetrate the B phase.

In conclusion, I should mention that, as suggested by G. Volovik, it is possible that this nonequivalence between different ends of the B-phase quantized vortices was manifested in the Helsinki NMR experiments on the phase boundary under rotation.

I am grateful to G. Volovik for stimulating discussions and all communications.

[1)] As I was informed by G. Volovik, the development of singularities into the shape of the A–B interface was initially suggested by E. Thuneberg.

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