K^0–$\bar{K}^0$ mixing in nonleading order in $1/m_c$

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Corrections in the inverse mass of a charmed quark to the effective local Lagrangian with $\Delta S = 2$ are calculated. The contributions from long and short range are determined in a consistent way. The convergence boundary is estimated for an operator expansion of the nonlocal part of the effective Lagrangian.

The system of neutral kaons provides important information about the overall structure of the standard model,[1] and it is a convenient test model for studying extremely subtle theoretical effects, e.g., the breaking of CP invariance.[2] The significant improvements in the experimental data recently[3] require corresponding improvement in the accuracy of theoretical predictions, so we can see whether theory and experimental are in agreement. The K^0–$\bar{K}^0$ transitions are extremely sensitive to both the mass of the t quark and the numerical values of the mixing angles for the mixing of quarks of different generations.[4] For this reason, theoretical work on this process is attracting much interest. The results are not very reliable because it is necessary to calculate kaon–antikaon matrix elements of an effective electroweak low-energy Lagrangian with $\Delta S = 2$ which has not only a local part[5] (in the limit $m_c \to \infty$) but also a nonlocal part.[6] The matrix element of the K^0–$\bar{K}^0$ transition, determined by the local part of the effective low-energy Lagrangian, is given in the leading approximation in $1/m_c$ by

$$_{out} < \bar{K}^0(k')|K^0(k) >_{in}^{local} = C(M_W, m_t, V, \alpha_s) < \bar{K}^0(k')|16\pi^2 L_H|K^0(k) >$$

$$= C(M_W, m_t, V, \alpha_s) < \bar{K}^0(k')| - m_c^2(\bar{s}\gamma_\alpha d)^2|K^0(k) >, \tag{1}$$

where $C(M_W, m_t, V, \alpha_s)$ is a coefficient function which depends on the masses of the W boson and the t quark, on the mixing-angle matrix for mixing between generations (V), and on the strong-coupling constant α_s. A matrix element of the local operator $(\bar{s}\gamma_\alpha d)^2 = \bar{s}_L \gamma_\alpha d_L \bar{s}_L \gamma_\alpha d_L$ is parameterized by the quantity B_K, which is in turn normalized to the value found through the use of the vacuum-saturation hypothesis:

$$< \bar{K}^0(k')|(\bar{s}\gamma_\alpha d)^2|K^0(k) > = B_K < \bar{K}^0(k')|(\bar{s}\gamma_\alpha d)^2|K^0(k) >^{VS}. \tag{2}$$

Below we assume that all the quarks are left-handed, unless we specify otherwise.

The nonlocal part of the matrix element of the K^0–$\bar{K}^0$ transition is related to the contribution of virtual light u quarks, whose loops on the corresponding Feynman diagrams cannot be collapsed to a point, because there is no long-range cutoff of the integration. The nonlocal part of the matrix element is parameterized by the quantity D, normalized to simply the experimental difference between the masses of the K_L and K_S mesons:

$$_{out} < K^0(k')|K^0(k) >_{in}^{nonlocal} = 2m_K D\Delta m^{\exp} \sim i\int dx T L_u(x) L_u(0), \qquad (3)$$

where $L_u = s_L\gamma_\alpha u_L \bar{u}_L \gamma_\alpha d_L$, and $\Delta m^{\exp} = m_{K_L} - m_{K_S}$. Numerous attempts have recently been undertaken to calculate the parameters B_K (Ref. 7) and D (Ref. 8), but the accuracy of the results is not yet satisfactory (particularly in the case of D). This situation seriously complicates efforts to compare theoretical predictions of the K^0–$\bar{K}^0$ mixing parameters with experimental data.

In this letter we are reporting calculations of corrections to the local part L_H of matrix element (1) in a nonleading order in m_c^{-1}. Since the mass of the c quark is not extremely large in comparison with the typical mass values in the world of light hadrons constructed of (u,d) quarks, e.g., the mass m_ρ of the ρ meson, such corrections may be important. Corresponding corrections for the decays of charmed mesons and baryons were recently studied by several investigators.[9]

We wish to stress that at the order m_c^{-2} the dependence on the boundary between long and short range (the local and nonlocal contributions) appears explicitly, in contrast with the situation in (1), because of operators of dimensionality 8 (e.g., $s\gamma_\mu d s\gamma_\nu \tilde{F}_{\nu\mu} d$, where $\tilde{F}_{\nu\mu}$ is the dual tensor of the gluon field). It is therefore necessary to determine all contributions clearly and consistently. We suggest a way to do this in the present paper.

After the W boson and the t quark are split off, we need to calculate an effective $\Delta S = 2$ Lagrangian in order to determine the matrix element. Since our purpose in this study is to calculate the corrections to L_H in expression (1) [not the corrections to the coefficient function $C(M_W, m_t, V, \alpha_a)$; such calculations, although important, would generally constitute an independent problem], we proceed immediately to the stage in which the c quark splits off.

The effective Lagrangian consists of a heavy (local) part L'_H and a light (nonlocal) part L'_L. By virtue of the GI mechanism, their sum

$$L_{eff} = L_H + L'_L$$

is finite in the uv limit. Each of the amplitudes $L_{H,L}$ must be subjected to a uv regularization separately. We choose this regularization in the form

$$L_H^R = i\int d^4x T L_c(x) L_c(0)(-\mu^2 x^2)^\epsilon,$$

$$L_L^R = i\int d^4x T L_u(x) L_u(0)(-\mu^2 x^2)^\epsilon, \qquad (4)$$

where L_c is the part of the $\Delta S = 1$ Lagrangian which contains the heavy c quark. A dimensional regularization is inconvenient because of complications stemming from the algebra of γ matrices, while a Pauli–Willars regularization would not allow us to evaluate explicitly all the integrals that arise.

For the heavy part we find the expansion (we are omitting the superscript R)

$$16\pi^2 L_H = -m_c^2(\bar{s}\gamma_\alpha d)^2 - \frac{4}{3}O\left(\frac{1}{\epsilon} + \ln\left(\frac{4\mu^2}{m_c^2}\right) - 2C + \frac{4}{3}\right) - \frac{2}{3}(O_2 + O_3), \quad (5)$$

where $C = 0.577...$ is Euler's constant, $O = O_1 + O_2 + O_3$, and

$$O_1 = \bar{s}\gamma_\mu d\bar{s}\gamma_\nu \tilde{F}_{\nu\mu} d, \quad O_2 = \bar{s}\gamma_{(\mu}D_{\nu)} d\bar{s}\gamma_{(\mu}D_{\nu)} d, \quad \gamma_{(\mu}D_{\nu)} = (\gamma_\mu D_\nu + \gamma_\nu D_\mu)/2, \quad (6)$$

$$O_3 = \bar{s}\gamma_\alpha d\bar{s}(\hat{D}D_\alpha + D_\alpha\hat{D})d - \frac{m_s^2 + m_d^2}{4}(\bar{s}\gamma_\mu d)^2 + \frac{m_s m_d}{2}\bar{s}_R d_L \bar{s}_L d_R.$$

Expression (5) constitutes the basic result of this study: an expansion of the local part of matrix element (1) up to order $1/m_c^2$.

The divergent part of expression (5)—the $1/\epsilon$ pole—is canceled by the short-range contribution of the light part. For $T_L(x) = TL_u(x)K_u(0)$ we find an operator expansion at small values of x^2:

$$T_L(x)|_{x^2\to 0} = T_L^{as}(x) = (\bar{s}\gamma_\mu d)^2\frac{1}{4\pi^4 x^6} + O\frac{1}{12\pi^4 x^4}\ . \quad (7)$$

After an integration over small $x(x^2 < \bar{x}^2$ in the Euclidean region) we find

$$L_L = i\int dx T_L(x)(-\mu^2 x^2)^\epsilon = L_L^{short} + L_L^{Long}, \quad (8)$$

where

$$16\pi^2 L_L^{short} = i\int dx T_L^{as}(x)(-\mu^2 x^2)^\epsilon = (\bar{s}\gamma_\mu d)^2\frac{4}{\bar{x}^2} + \frac{4}{3}O\left(\frac{1}{\epsilon} + \ln(\bar{x}^2\mu^2)\right), \quad (9)$$

$$16\pi^2 L_L^{long} = i\int dx T_L(x)(-\mu^2 x^2)^\epsilon, \quad (10)$$

$$x^2 > \bar{x}^2\ .$$

Summing, we finally find

$$16\pi^2 L = 16\pi^2 L_L^{long} + (\bar{s}\gamma_\mu d)^2\left(-m_c^2 + \frac{4}{\bar{x}^2}\right)$$
$$+\frac{4}{3}O\left(\ln\left(\frac{m_c^2\bar{x}^2}{4}\right) + 2C - \frac{4}{3}\right) - \frac{2}{3}(O_2 + O_3), \quad (11)$$

We see from (5) and (9)–(11) that the separation of the contributions depends explicitly on the boundary between long and short range, $\bar{x}$. This boundary value can be estimated numerically from Eq. (7), as the value at which expansion (7) is violated. After we have determined $\bar{x}^2$, we still need a model for the long-range contribution (10), in order to find the matrix element of the complete Lagrangian of the transition.

For numerical estimates we use the vacuum-dominance approximation for the matrix elements of the local operators in the chiral limit (i.e., in the case in which the masses of the light quarks and/or the square masses of the kaon and pions are parametrically small):

$$< \bar{K}^0(k)|O_1|K^0(k) >^{VS} = -\delta^2\left(1+\frac{1}{N_c}\right)\frac{f_K^2 m_K^2}{2}$$

$$< \bar{K}^0(k)|O_2|K^0(k) >^{VS} = -\delta^2\frac{1}{N_c}\frac{f_K^2 m_K^2}{2},$$

$$< \bar{K}^0(k)|O_3|K^0(k) >^{VS} = 0,$$

where the parameter δ^2 is defined by[10] $\langle 0|g_s\bar{d}\gamma_\nu\tilde{F}_{\nu\mu}s|\ K^0(k)\rangle = -if_K\delta^2 k_\mu$ and has the value $\delta^2 = 0.2\ \mathrm{GeV}^2$ (Refs. 10 and 11).

From expression (7), we find

$$T_L^{as}(x) = (\bar{s}\gamma_\mu d)^2\frac{1}{4\pi^4 x^6}\left(1-\frac{\delta^2 x^2}{3}+o(x^2)\right). \qquad (12)$$

The boundary value $\bar{x}^2$ is found from the requirement $x^2\delta^2 = z$, $1 < z < 3$. Substituting this value into (11), we find

$$< \bar{K}^0|16\pi^2 L^{sh}|K^0 >^{VS} = < \bar{K}^0|16\pi^2(L_H + L_L^{short})|K^0 >^{VS}$$

$$= -m_c^2 < \bar{K}^0|(\bar{s}\gamma_\mu d)^2|K^0 >^{VS}\left(1-\frac{4\delta^2}{zm_c^2}+\frac{4\delta^2}{3m_c^2}\left(\ln\left(\frac{zm_c^2}{4\delta^2}\right)+2C-\frac{35}{24}\right)\right)$$

$$= -m_c^2 < \bar{K}^0|(\bar{s}\gamma_\mu d)^2|K^0 >^{VS}\begin{cases}1-0.4 & \text{for } z=1\\ 1+0.1 & \text{for } z=3.\end{cases} \qquad (14)$$

It can be seen from (14) that in the chiral limit and in the vacuum-dominance approximation only the short-range component L^{sh} depends strongly on the separation boundary $\bar{x}^2$, at $\bar{x}^2 \sim \delta^{-2}$, where expansion (10) is violated. Since the result for L_{eff} is generally independent of $\bar{x}^2$, the strong x^2 dependence of L^{sh} suggests that the component L_L^{long} is important at small values of $x^2 \sim 1/\delta^2$ but quite small (?) at large values $\bar{x}^2 \sim 3/\delta^2$. That question, however, requires a special study on the basis of some low-energy model, e.g., calculations on a lattice or with the help of a theory of effective chiral Lagrangians.

In conclusion, we wish to repeat that calculations of the corrections to the effective local $\Delta S = 2$ Lagrangian are important. These corrections must be taken into consideration for an unambiguous determination of the long- and short-range contributions. The numerical values of the corrections, which are parametrically small and proportional to $\delta^2/m_c^2 = 0.12$, depend on the use of the chiral limit and the vacuum-saturation approximation for an estimate of the kaon–antikaon matrix elements. For certain parameter values (e.g., at small values of B_K), this correction may prove to be large. This result means again that a refinement of the numerical value of the parameter B_K, which has been the goal of a substantial effort,[7] will not by itself lead to a better understanding of the entire short-range contribution L^{sh}. The rather strong $\bar{x}^2$

dependence of the result [expression (14)] suggests that the L_L^{long} component, which cancels this dependence, should also vary rapidly with $\bar{x}^2$ near the convergence boundary of expansion (7). This component must therefore be taken into account if we wish to generate a stable prediction. Finding the absolute value of this component, on the other hand, will require the use of a low-energy model.

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