Gauge model of gravitation

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A "nonminimal" gauge mechanism for a gravitational interaction is proposed on the basis of the group properties of the operation of parallel translation in Minkowski space. An analog of the Reissner–Norström solution is derived.

There is a fairly broad spectrum of ideas regarding the gauge nature of the gravitational interaction.[1–3] We are offering here yet another opinion, which is based on a broader understanding of symmetry, as transformations in the course of which the change in an action functional is proportional to the flux of some vector across the boundary of an integration region.

In M_4, a parallel translation of the fields $\psi(x)$ by a small vector ϵ^k can be treated as a transformation of the "internal symmetry" which does not involve the coordinates:

$$\psi(x) \to G\psi(x) \approx (1+\epsilon^k \nabla_k)\psi(x), \tag{1}$$

where $G \approx +\epsilon^k \nabla_k$ is the global Abelian Lie group, whose generators ∇_k are the operators of covariant differentiation in M_4.

A noninvariance of the action under transformation (1) is a fundamental feature of G. For the Lagrangian $L(\psi, \nabla_k \psi)$, for example, the following fundamental equality, which we will call "G invariance," holds:

$$\delta L = \frac{\partial L}{\partial \psi}\delta\psi + \frac{\partial L}{\partial(\nabla_k \psi)} \nabla_k (\delta\psi) = \nabla_n (L\epsilon^n). \tag{2}$$

From this equality at the extrema of the fields $\psi(x)$ follows a continuity equation for the canonical energy-momentum tensor.

Upon a localization of G, with $\nabla_k \varepsilon^n \neq 0$, Eq. (2) is violated. Introducing the vector field $\Gamma_k = \Gamma^n_k \nabla_n$ in the Lagrangian (since the generators of G are ∇_n), with (1) transformed in accordance with

$$\Gamma_k \to G\Gamma_k G^{-1} - \nabla_k G G^{-1} = (\Gamma^n_k + \epsilon^i \nabla_i \Gamma^n_k - \Gamma^i_k \nabla_i \epsilon^n - \nabla_k \epsilon^n)\nabla_n, \tag{3}$$

we restore G invariance if we make the substitution

$$L(\psi, \nabla_k \psi) \to \Lambda(\psi, D_k \psi) = \det(G^s_p)^{-1} L(\psi, D_k \psi). \tag{4}$$

Here $D_k = \nabla_k + \Gamma_k = (\delta^n_k + \Gamma^n_k)\nabla_n$ is a G-covariant derivative, and the matrix $(G^s_p)^{-1}$ is the inverse of $G^s_p = \delta^n_k + \Gamma^s_p$.

It follows from the commutator $[D_i, D_k]\psi = (G^n_{ik}\nabla_n)\psi = G^n_{ik} D_n \psi$ that the "operator-valued curvature tensor"

$$F^n_{ik}\nabla_n = (D_i G^n_k - D_k G^n_i)\nabla_n = (\nabla_i \Gamma^n_k - \nabla_k \Gamma^n_i + \Gamma^m_i \nabla_m \Gamma^n_k - \Gamma^m_k \nabla_m \Gamma^n_i)\nabla_n$$

is nonlinear in the potentials Γ^n_k, although G is Abelian. We treat the tensor $G^n_{ik} = F^m_{ik}(G^n_m)^{-1}$ as the gravitational field strength, because of the transformation law, which is analogous to (1).

The general expression for $GL(4,R)$ and for the G-invariant Lagrangian of the gravitational field, bilinear in G^n_{ik}, is

$$\Lambda = \frac{1}{16\pi g^2}\det(G^s_p)^{-1}\{\alpha G^k_{mn} G^k_{mn} + \beta G^m_{kn} G^n_{km} + \gamma G^n_{kn} G^m_{km}\}. \tag{5}$$

A repeated superscript or subscript means a summation over the metric in M_4; α, β, and γ are numbers, $g = \sqrt{k}/c^2$ is a coupling constant; k is the Newtonian gravitational constant; and c is the velocity of light. The existence of a classical Newtonian limit in the form $G^0_0 = 1 - (k/c^2)(m/r)$ limits (5) to three versions: (1) $\alpha = 0$, $\beta = -1$, $\gamma = 1$; (2) $\alpha = -1$, $\beta = 1$, $\gamma = 1$; (3) $\alpha = \frac{1}{2}$, $\beta = 0$, $\gamma = 1$. We prefer the first.

We consider G-invariant Lagrangian (4), which is invariant under the transformation $\psi \to U\psi$, where U is a global matrix-valued Lie group. In the course of the localization, the invariance is obviously disrupted. Making the substitution

$$\Lambda(\psi, D_k\psi) \to \Lambda(\psi, D_k\psi + A_k\psi), \tag{6}$$

where the vector field A_k takes on a value in the Lie algebra of the group U and transforms in accordance with

$$A_k \to UA_kU^{-1} - D_kUU^{-1}, \tag{7}$$

we restore the local invariance of the Lagrangian.

We will apply this mechanism for restoring local U–G invariance to the field A_k, since the corresponding field strength $F_{ik} = D_iA_k - D_kA_i + [A_i, A_k]$ does not, according to (4), transform as $F_{ik} \to UF_{ik}U^{-1}$. The reason is that noncommuting operators D_k are used. Introducing the new definition of the field strength,

$$A_{ik} = D_iA_k - D_kA_i + [A_i, A_k] - G^n_{ik}A_n, \tag{8}$$

we verify by direct substitution of (7) into (8) that A_{ik} has the necessary transformation properties and that the Lagrangian

$$\Lambda = -\frac{1}{16\pi q^2}\det(G^s_p)^{-1}\mathrm{Tr}(A_{ik}A_{ik}), \tag{9}$$

where q is a coupling constant, is locally U–G invariant.

Examining the energy-momentum conservation laws by this approach, we see that these laws are unambiguous and are satisfied identically. For the closed system from (2), for example, we find the following equation for the total canonical energy-momentum tensor by using the definition of Lagrangian (4), (5), (9) and transformation laws (1), (3) at the extrema:

$$\frac{\partial\Lambda}{\partial(\nabla_i\psi)}\nabla_k\psi + \frac{\partial\Lambda}{\partial(\nabla_iG^s_p)}\nabla_kG^s_p - \delta^i_k\Lambda = \nabla_n\left\{\frac{\partial\Lambda}{\partial G^m_{sp}}(G^m_k)^{-1}G^n_sG^i_p\right\}, \tag{10}$$

where Λ is the Lagrangian of the system of fields.

The distinction from the general theory of relatively is seen clearly in an analog of the Reissner–Nordström solution. For Lagrangian (5), (9) in a spherical coordinate system, for example, we find the following solution (this can be done more simply by direction variation):

$$G^0_0 = \frac{(r-r_0)^2 - r_1^2}{(r-r_0)^2 + r_0(r-r_0) - r_1^2}, \quad G^1_1 = G^2_2 = G^3_3 = \frac{r^2}{(r-r_0)^2 + r_1^2},$$

$$A_{10} = \frac{q}{r^2}\left\{\frac{r^2}{(r-r_0)^2 + r_1^2}\right\}^2, \quad A_0 = \frac{q}{r}\frac{r^2 - \frac{1}{2}r_0r}{(r-r_0)^2 + r_0(r-r_0) - r_1^2}, \tag{11}$$

where $r_0 = g^2mc^2$, $r_1 1/gq$, m is the "seed" mass of the object, and q is its charge. For the total energy of this system we have, according to (10),

$$E = mc^2 - \frac{1}{4} q^2 \frac{r_0}{r_0^2 + r_1^2}. \tag{12}$$

A distinctive feature of solution (11) is the regular behavior of the gravitational and electromagnetic fields at the point $r = 0$.

We would like to close with two points. (1) In October of 1990, at the school and seminar on "Topological field theories and exact solutions in the general theory of relativity" in Krasnoyarsk, A. Yu. Morozov showed that the transformation $A_k \to B_k = (G^n_k)^{-1} A_n$ pus Lagrangian (9) in the standard form if B_k is analyzed in a Riemannian space with a metric $g^{ik} = G^i_n G^k_n$. We can thus draw some close parallels with the general theory of relativity. (2) For non-Abelian fields, we might expect to find a nonzero vacuum expectation value of the field A_k, since (9) also contains terms quadratic in A_k. This possibility might lead to some other than the Higgs mechanism for the formation of masses.

[1]N. P. Konopleva and V. N. Popov, *Gauge Fields*, Atomizdat, Moscow, 1980.
[2]D. D. Ivanenko, P. I. Pronin, and G. A. Sardanashvili, *Gauge Theory of Gravitation*, Izd, Mosk. Univ., 1985.
[3]A. A. Logunov, Teor. Mat. Fiz. **85**, 3 (1990).

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