Condensate of monopoles and confinement in an SU(2) lattice gauge theory

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The existence of a condensate of Abelian monopoles at a temperature below the critical value is demonstrated through a numerical simulation of lattice SU(2) gluodynamics at a finite temperature. This result is evidence in favor of a confinement mechanism based on a model of a dual superconductor.

Let us examine the possibility of describing confinement in lattice theories on the basis of a mechanism proposed by 't Hooft and Mandelstam.[1] Greatly simplified, the idea is this: By fixing the gauge, one projects the non-Abelian theory onto a U(1) theory, whose Lagrangian is assumed to be

$$\mathcal{L} = \frac{1}{g^2} G_{\mu\nu}^2 + |D_\mu \phi|^2 + \lambda(|\phi|^2 - 1)^2, \qquad (1)$$

where ϕ is the field of monopoles, $D_\mu = \partial_\mu + iB_\mu$, and $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$, where

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B_μ is a field which is the dual of the electromagnetic field: $G_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha$. At the level of the classical equations of motion, the existence of a condensate of monopoles gives rise to a string between the + and − charges (the quark and the antiquark). This string leads to a linear potential at long range, while at short range the quark and antiquark interact through a Coulomb potential. A Lagrangian of this sort might play the role of an effective IR Lagrangian if it could be derived directly from the quantum-chromodynamics Lagrangian. In the absence of such a derivation, it is worthwhile to attempt a numerical test of its existence. By virtue of the compact nature of the gauge group, monopoles exist in the vacuum of lattice theories. Their interaction with the gauge field naturally occurs via a long derivative, so the most nontrivial term in the effective Lagrangian is the last one, which leads to the appearance of a condensate of monopoles. Polikarpov *et al.*[3] have proved the existence of such a condensate for lattice electrodynamics, through a numerical calculation of the effective potential of the monopoles. The monopoles were singled out in an SU(2) lattice gauge theory in Ref. 2, and it was shown that their density is high in the confinement region. It was shown in Ref. 4 that in SU(2) lattice gluodynamics the tension in the string is proportional to the fractal dimension of the monopole currents. In the present letter we propose a method for calculating the magnitude of the monopole condensate, C, in this theory. We also study the temperature dependence of C. The temperature serves as a parameter which we can vary to vary the tension in the string. The quantity simplest to calculate is the propagator of a monopole-antimonopole pair:

$$\mathcal{G}(x) = < \bar{\phi}(x)\phi(x)\bar{\phi}(0)\phi(0) > . \tag{2}$$

If a monopole condensate exists, and $\langle\phi(x)\rangle = C$, then in the limit $|x| \to \infty$ we have $\mathcal{G}(x) \to A[\exp(-m|x|)]/|x|^\alpha + C$. We have carried out a fit with this formula to find the numerical values for propagator (2). The resulting temperature dependence of C^2 is shown in Fig. 1 (the results are expressed in units of the critical temperature T_c). Shown for comparison in the same figure is the temperature dependence found

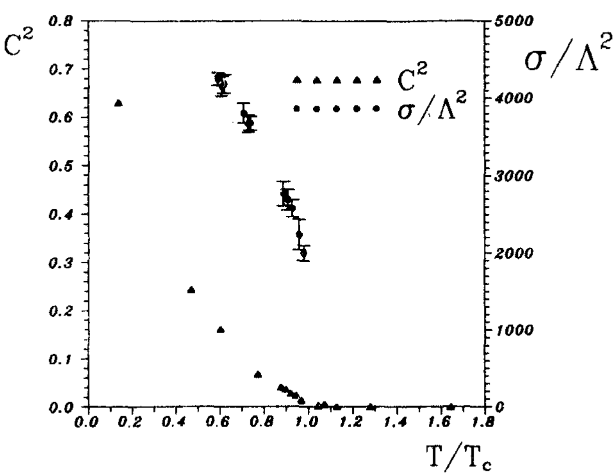

FIG. 1. Temperature dependence of C^2 and σ/Λ^2. The data on σ are from Ref. 5.

for the string tension σ in Ref. 5. In the confinement region, the condensate is nonzero; it vanishes at the confinement-deconfinement transition point. We wish to stress that determining the condensate is not a trivial matter. Formally, this process is totally independent of the string tension. Thus the fact that we have $C \neq 0$ in the region with $\sigma \neq 0$ can be taken as a serious argument in favor of the 't Hooft-Mandelstam confinement mechanism.

We turn now to some technical details. The calculations were carried out on an $8^3 \times 4$ lattice. The dimension of the lattice along the "temporal" direction was smaller than the spatial dimensions. This situation corresponds to the introduction of a finite temperature. A variation of the seed charge in the theory corresponds to a variation of the temperature. To distinguish the monopoles from the configuration of gluon fields, generated by the Monte Carlo method, we proceed as in Refs. 2: We switch to the maximal Abelian gauge, and we use Gauss's theorem to calculate the magnetic charge in each three-dimensional unit cell of the lattice. We do not know the explicit expression for the monopole creation operator ϕ, so we cannot calculate the condensate C directly. We calculate Green's function (2). For this purpose we use the method proposed in Ref. 6: We single out coupled clusters of monopole currents in the given configuration of fields, and we sum over all pairs of points on the lattice which are at a distance $|x|$. If a given pair of points belongs to a coupled cluster, we add 1 to sum N. The Green's function is calculated from $\mathcal{G} = N/D$, where D is the total number of points on the lattice which are at a distance $|x|$.

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[1]G. 't Hooft, in *High Energy Physics* (ed. Zichichi), Editrice Compositori, Bolognia, 1976; S. Mandelstam, Phys. Rev. C **23**, 245 (1976).
[2]A. S. Kronfeld, G. Schierholz, and U.-J. Wiese, Nucl. Phys. B **293**, 461 (1987); A. S. Kronfeld, M. L. Laursen, G. Schierholz, and U.-J. Wiese, Phys. Lett. B **198**, 516 (1987).
[3]M. I. Polikarpov, L. Polley, and U.-J. Wiese, Phys. Lett. **253**, 212 (1990).
[4]T. L. Ivanenko, M. I. Polikarpov, and A. V. Pochisky, Phys. Lett. **252**, 631 (1991).
[5]K. Kanaya and H. Satz, Phys. Rev. D **34**, 3193 (1986).
[6]M. I. Polikarpov, A. V. Pochinsky, and B. N. Yurchenko, "Properties of Vortex Loops in 3D *XY* model," Preprint of Pisa University IFUP-TH 42/90, 1990.

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