Non-Abelian *SU* (2) neck with a seven-dimensional insertion

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An entity consisting of two parts has been constructed: 1) A four-dimensional region outside the horizon of events, which is a non-Abelian $SU(2)$ black hole; 2) a region inside the horizon, which is a solution of seven-dimensional Einstein-Kaluza-Klein-Yang-Mills equations.

1. INTRODUCTION

We will construct a nonsingular stationary entity which is globally arranged in the following way: 1) Beyond the horizon of events it is a non-Abelian black hole with a gauge group $SU(2)$ which was constructed in Ref. 1; 2) inside the horizon of events it is a seven-dimensional space N^7 (insertion) with a Euclidean time. In this metric the first four dimensions correspond to the standard coordinates of Einstein's four-dimensional space-time and the remaining dimensions are the coordinates on the gauge group $SU(2)$. To construct this entity, it must be assumed that 1) the gravitation acts on the entire N^7 space, i.e., even on the gauge group $SU(2)$; 2) on the horizon of events the seven-dimensional geometric quantities of $SU(2)$ insertion are spliced with the corresponding quantities of the 4D world, and also with the Yang-Mills $SU(2)$ field.

2. SEVEN-DIMENSIONAL EINSTEIN-KALUZA-KLEIN-YANG-MILLS EQUATIONS

The Lagrangian for the insertion N^7 can be written in the form

$$L=\sqrt{-G}\left(-\frac{R}{16\pi\gamma}-\frac{1}{4}F^{a}_{AB}F_{a}^{AB}\right), \tag{1}$$

where $A, B = 0, 1, 2, 3, 4, 5, 6$; $a = 4, 5, 6$; R is a scalar 7D curvature, and $(F_{aAB} = \partial_A W_{aB} - \partial_B W_{aA} + \epsilon_{abc} W_{bA} W_{c_B})$ is the curvature of the connection W_{aA}, which is governed by the fact that the additional measurements in the insertion form a group. $F^{a}_{AB} = h^{ab}F_{bAB}$,$h_{ab}$ is the metric on the gauge group $SU(2)$, i.e., the metric in the additional measurements.

The 7D metric is chosen in the following spherically symmetric form:

$$ds^2=-e^{\lambda(t)}dt^2-e^{-\nu(t)}dR^2-r^2(t)(d\theta^2+\sin^2\theta d\phi^2)-e^{\nu(t)}(dy_1^2+dy_2^2+dy_3^2), \tag{2}$$

where t is the Euclidean time, t, θ, and ϕ are the standard polar coordinates, and y_1, y_2, and y_3 are the coordinates on the gauge group $SU(2)$. The nonmetric connection in the additional measurements of the gauge group $SU(2)$ in the polar coordinate system (t,θ,ϕ) are sought in the form[2]

$$\vec{W}_t = \vec{W}_R = 0,$$
$$\vec{W}_\theta = (-\sin\phi, \cos\phi, 0)\frac{(V(t)-1)}{e} \quad (3)$$
$$\vec{W}_\phi = (-\cos\phi\cos\theta, -\sin\phi\cos\theta, \sin\theta)\sin\theta\frac{(V(t)-1)}{e},$$

where e is the gauge coupling constant.

Since the coordinate t can be subjected to an arbitrary gauge transformation, the variation can be done only with respect to λ, ν, and V:

$$e^{-\lambda}\left(\frac{r'^2}{r^2}+\frac{2r'\nu'}{r}\right)-\frac{1}{r^2}=\frac{\kappa}{2}e^{-\nu}\left(\frac{(V^2-1)^2}{2r^4}-e^{-\lambda}\frac{V'^2}{r^2}\right), \quad (4)$$

$$\frac{r''}{r}+\frac{r'^2}{2r^2}-\frac{r'\lambda'}{2r}-\frac{e^\lambda}{2r^2}=0, \quad (5)$$

$$(e^{-\frac{\lambda}{2}}V')'=e^{\frac{\lambda}{2}}\frac{(y^2-1)V}{2}. \quad (6)$$

Here the prime denotes the derivative of t, and $\kappa = 16\pi\gamma/e^2$.

Integration of the first two equations in (4) and (5) after some simplifications gives the following result:

$$e^\lambda = r'^2, \quad (7)$$

$$e^\nu = \frac{\alpha}{4}\int_x^{x_H}\left[\left(\frac{dV}{dx}\right)^2-\frac{(V^2-1)^2}{2x^2}\right]\frac{dx}{x}. \quad (8)$$

Here we have introduced a dimensionless variable $x = r/r_0$, $x_H = r_H/r_0$, $\alpha = \chi/r_0^2$, and r_0 and r_H are constants. The Yang-Mills equations (6) can be rewritten in the following form, after substituting (7) in them:

$$x^2\frac{d^2V}{dx^2} = V(V^2-1). \quad (9)$$

This equation has two solutions. the first one is regular at $x = 0$ and it can be expanded in a series in x for $0 \leqslant x \leqslant 1$

$$-V_1(x) = 1+\sum_{1=1}^{\infty} a_i\left(\frac{r}{r_0}\right)^{2i} \quad (10)$$

$a_1 = -1$, $a_2 = 3/10$; for $i > 2$

$$a_i = \frac{3\sum_{j<i} a_{i-j}a_j + \sum_{j+k<i} a_{i-j-k}a_j a_k}{4i^2-2i-2}.$$

The second solution is regular at $x = \infty$ and it can be expanded in a series

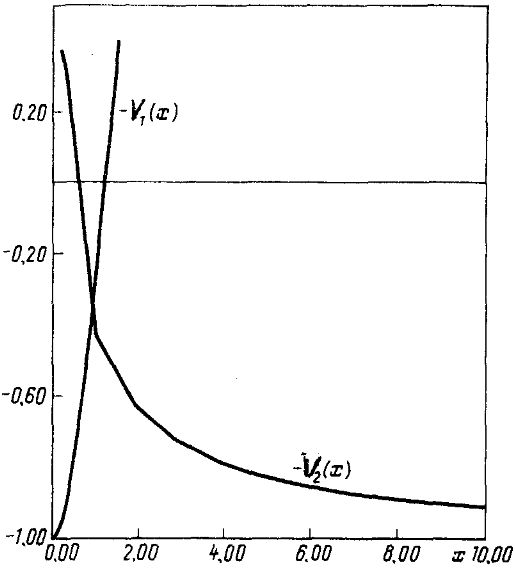

FIG. 1. The solution of $V_1(x)$ is similar at $x=0$. The solution of $V_2(x)$ is similar at $x=\infty$.

$$-V_2(1/x) = 1 + \sum_{i=1}^{\infty} b_i \left(\frac{r_0}{r}\right)^i \tag{11}$$

$1/x < 1$, $a_1 = -1$, and $a_2 = 3/4$; for $i > 2$:

$$b_i = \frac{3\sum\limits_{j<i} b_{i-j} b_j + \sum\limits_{j+k<i} b_{i-j-k} b_j b_k}{i^2 + i - 2} .$$

These two solutions can be extended beyond the range of determination of the series by numerically integrating Eq. (9). The result is shown in Fig. 1.

3. SPLICING A NON-ABELIAN BLACK HOLE

Let us splice the corresponding 4D quantities of the outer region, which were found in Ref. 1, to the 7D quantities of the inner region, which were found above, in the following way:

$$g_{\theta\theta}({}_4r = r_H) = r_H = G_{\theta\theta}({}_7r = r_H), \tag{12}$$

$$f({}_4r = r_H) = f_n^*(\alpha) = V({}_7r = r_H) . \tag{13}$$

The quantities on the left side of the equations are 4D quantities, which were determined in Ref. 1, and the quantities on the right side are 7D quantities corresponding to them [which were determined on the $SU(2)$ insertion]; ${}_4r$ is a 4D spacelike coordinate in the non-Abelian black hole,[1] $g_{\mu\nu}$ is a 4D metric tensor $(\mu,\nu = t,r,\theta,\phi)$, $f(r)$ determines the 4D Yang-Mills field in the region ${}_4r \geqslant r_H$ by analogy with (3), $f({}_4r)$ can have, on the horizon of events, only discrete values of $f_n^*(\alpha)$ which are specified by natural numbers n, and r_H is the radius of the horizon of events. It is easy to see that

(12) and (13) are satisfied. It should be noted that the equalities $g_{rr} = G_{RR}$ and $g_{tt} = G_{tt}$ need not necessarily be satisfied, since one quantity in each pair is an arbitrary quantity because of the gauge degree of freedom of the coordinate ${}_4r$ or the Euclidean time ${}_5t$. In addition, it is very important that

$$G_{y_1y_1} = G_{y_2y_2} = G_{y_3y_3} = 0. \tag{14}$$

This condition must be satisfied in order that the auxiliary coordinates [$SU(2)$ gauge group] would split off beyond the horizon of events (with $r > r_H$).

4. DISCUSSION

On the basis of the discussion above it can be asserted that two stationary nonsingular entities have been constructed. These entities are globally arranged in the following way: 1) There is a 4D "tail" which is a 4D non-Abelian black hole and which is situated beyond the horizon of events ($r > r_H$); 2) inside the horizon of events there is a 7D insertion, in which the additional measurements are the $SU(2)$ gauge group; 3) the additional measurements split off at the horizon of events, the dimensionality changes abruptly, and the Euclidean time becomes the Einstein time. A similar particle-like entity; a neck with an outer region, which is the stationary part of the Reissner-Nordström solution, and with a $U(1)$ insertion, was constructed in Ref. 3. In contrast with the entity obtained here, the signature of the metric does not change at the surface of the horizon of events of the neck with a $U(1)$ insertion and only the dimensionality changes abruptly.

[1]M. S. Volkov and D. V. Gal'tsov, Yad. Fiz. **51**, 1171 (1990) [Sov. J. Nucl. Phys. **51**, 747 (1991)].
[2]P. Van Nienwenhuizen, D. Wilkinson, and M. J. Perry, Phys. Rev. **D12**, 778 (1975).
[3]V. D. Dzhunushadiev, *Additional Coordinates in Multidimensional Space as Coordinates on a Gauge Group*, VINITI, 1990, No. 2005-B90.

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