Limiting electric field of the Wakefield plasma wave

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(Submitted 26 April 1991)
Pis'ma Zh. Eksp. Teor. Fiz. **53**, No. 11, 540–544 (10 June 1991)

Electric fields larger than the fields in a steady electron plasma wave can be generated in a time-varying Wakefield track after the completion of the laser pulse and beyond the relativistic electron bunch. The results of a computer simulation are presented. The results confirm that these fields can effectively accelerate the electrons captured during the inversion of electron waves in the Wakefield track.

The large size of present-day high-energy particle accelerators (several tens of kilometers)[1] is attributable to the restriction imposed on the electric field strength in the accelerator vacuum chamber because of the breakdown at its walls. The possibility of appreciably increasing the acceleration rate has stimulated interest in collective methods of acceleration in a plasma, among which an accelerator design based on the use of the Wakefield fast plasma wave generated by laser pulses[2] or bunches of relativistic electrons[3] stands out.

The objective of the present study is to determine the maximum permissible field of the Wakefield plasma wave and to numerically simulate the acceleration in such a field.

In the preceding studies (see, for example, Ref. 4) the maximum field of the Wakefield plasma wave was linked with the limiting amplitude of the steady wave of the electron density, in which all the physical quantities depend on the combination of the space and time variables $\zeta = x - v_{ph} t$, where v_{ph} is the phase velocity of the wave.

 0021-3640/91/110565-05$01.00

The limiting field of such a wave in a plasma with cold electrons has been known for some time.

$$E_{max} \simeq \kappa_0 \sqrt{2(\gamma_{ph} - 1)}, \tag{1}$$

where

$$\gamma_{ph} = (1 - v_{ph}^2/c^2)^{-1/2}, \quad \kappa_0 = m_e c \omega_p / e, \quad \omega_p = (4\pi e^2 n_o / m_e)^{1/2},$$

n_0 is the unperturbed electron density. At amplitudes higher than (1) the wave experiences an inversion, which causes its regular structure to be distorted and which leads to the appearance of captured electrons.

The solution of the Vlasov-Poisson system of equations shows that the thermal spread of electrons facilitates their capture by a wave and thus leads to a decrease, at the isotropic temperature of the plasma electrons, of the limiting field of the wave to a value

$$E_{max} \simeq \kappa_0 (T_e/m_e c^2)^{-1/4} \quad \text{for} \quad 1 \gg T_e/mc^2 \gg \gamma_{ph}^{-2}. \tag{2}$$

At the anisotropic temperature, when $1 \gg T_{e\|}/mc^2 \gg \gamma_{ph}^{-2}$, $T_{e\perp}/mc^2$, the limiting field can be determined from the result of Ref. 5:

$$E_{max} \simeq \kappa_0 (T_{e\|}/m_e c^2)^{-1/4} \ln[\gamma_{ph}^{1/2} (T_e/m_e c^2)^{1/4}]^{1/2}. \tag{3}$$

In the case of high transverse temperature $1 \gg T_{e\perp}/m \gg T_{e\|}/m_e c^2$, γ_{ph} we obtain the expression

$$E_{max} \simeq \kappa_0 (T_{e\perp}/m_e c^2)^{-1/2}. \tag{4}$$

Let us now determine whether the Wakefield fields which are higher than the fields in a steady wave can be excited. If the difference between the velocity of a relativistic electron bunch and the speed of light is disregarded, an arbitrarily large field[6] can be excited by this bunch with a density $n_0/2$ and a steep trailing edge (Fig. 1). When the difference between the velocity of the bunch, v_b, and the speed of light is taken into account, the solution is valid only up to the maximum field beyond the bunch (the solid line), i.e., at distances $\Delta\zeta < (cL_b\omega_p)^{1/2}$ from the trailing edge of the bunch.

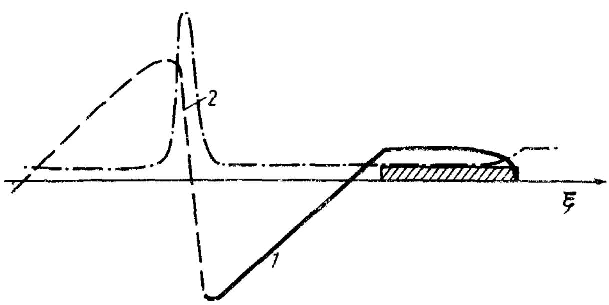

FIG. 1. The diagram illustrates the excitation of the Wakefield electric fields (curve 1) by a relativistic electron bunch ($n_b = n_0/2$—the hatching). The plasma electron density profile is represented by the dot-dashed line. Curve 2 represents the region of possible inversion of the Wakefield plasma wave.

If the maximum values of the field, E_{max}, are smaller than the limiting value in a steady wave, $v_{ph} = v_b$ [the minimum value in (1)–(4)], the solution will hold even at large distances from the bunch (the dashed curve in Fig. 1).

If, on the other hand, E_{max} is larger than the limiting field of the steady wave, the electrons will accelerate to a velocity $v_x > v_{ph}$ in the maximum field, causing the wave to flip.

The structure of the Wakefield fields beyond the wave inversion is transient. The region of multiple flux of electrons, which appears as a result of inversion of the wave, spreads toward the trailing edge of the bunch at a relative velocity $d\zeta/dt \simeq c - v_{ph}$. Because this velocity is low, the superlimiting field can exist long enough. If the retardation of the beam electrons is taken into account, it is possible to obtain fields in a flipped Wakefield track beyond the electron beam, which are stronger than (2)–(4), but not stronger than (1). Shaping of the beam in order to reduce the retarding field cannot produce conditions suitable for generation of fields stronger than $\kappa_0\sqrt{2\gamma_{ph}}$.

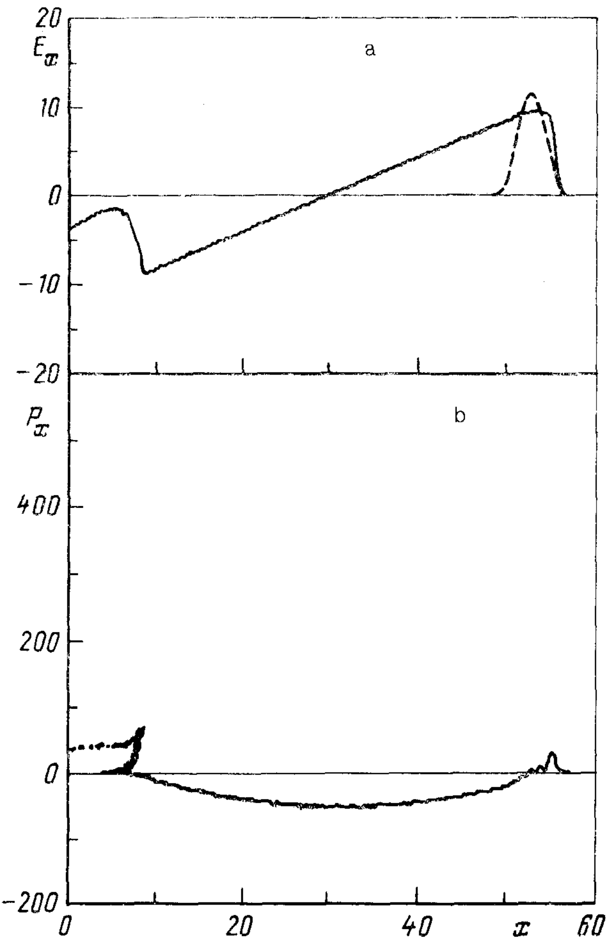

FIG. 2. (a) The Wakefield electric field and (b) the logitudinal electron momentum in the Wakefield track after the completion of the laser pulse at the time immediately after the inversion, $t = 60T_0$. The dashed line shows the profile of the laser pulse intensity.

The limiting field (1) can be exceeded by exciting the Wakefield electric fields by a relativistically strong laser pulse.[7] The phase velocity of the Wakefield field, v_{ph}, in this case is equal to the group velocity of the leading edge of the pulse, $v_g \simeq 1 - \omega_p^2/(2\omega_0^2)$, i.e., $\gamma_{ph} = \omega_0/\omega_p$, where ω_0 is the carrier frequency of the laser field.

Excitation of the Wakefield electric fields by a square laser pulse with a steep leading edge $[\delta\tau < (a_\perp \omega_p)^{-1}]$, where $a_\perp = eE_\perp/m\omega_0 c$ is a dimensionless amplitude of the field of the laser pulse at the leading edge, causes the ponderomotive force to accelerate the plasma electrons in the direction of their motion to a velocity $v_x = c(1 - 2/a_\perp^2)$. If $v_x < v_{ph} = v_g$ (i.e., $a_\perp < \sqrt{2}\gamma_{ph}$), the electrons are not captured near the laser pulse. After completion of the pulse, the electric field in the inverted Wakefield track can reach a value

$$E_{max} = \kappa_0 \gamma_{ph}, \tag{5}$$

which is clearly larger than the limiting field of the steady wave.

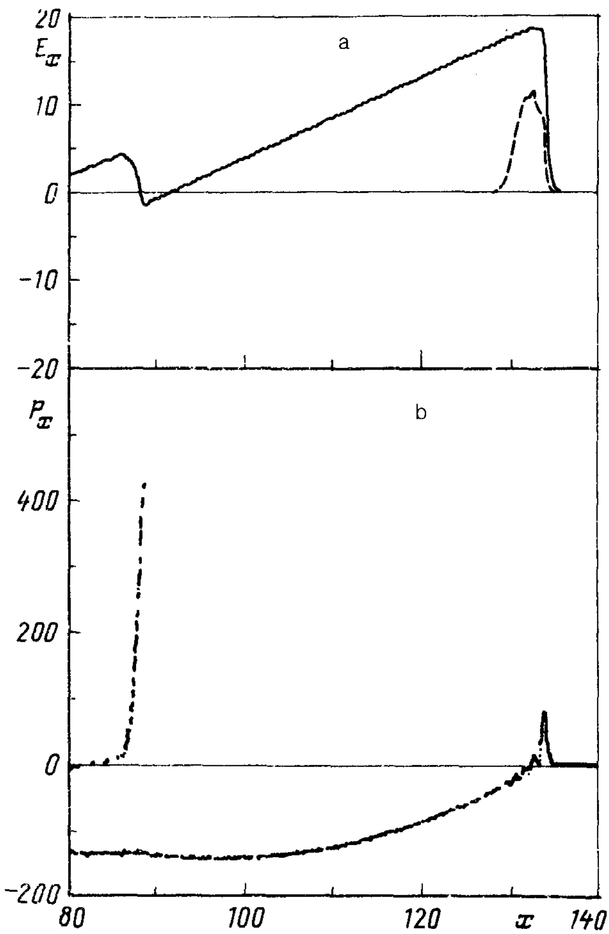

FIG. 3. The same as in Fig. 2, but at $t = 140T_0$.

Analysis of a more realistic laser pulse with a smoother leading edge $[\delta\tau > (\omega_p a_\perp)^{-1}]$ showed that electrons are not captured in the region of the pulse at characteristic field amplitudes $a_\perp < \gamma_{ph}^3 (\omega_p \delta\tau)^2$. The electric field in the Wakefield track in this case cannot be greater than the limiting value

$$E_{max} \simeq \kappa_0 (\omega_p \delta\tau) \gamma_{ph}^2. \tag{6}$$

Estimates of the time scale of the laser pulse field distortion[7,8] confirm that the approximation of the laser pulse used in our analysis is correct.

The results of a numerical simulation of the excitation of a flipping Wakefield plasma wave by the particle method is shown in Figs. 2 and 3. The profiles of the logitudinal electric field of the separation charges (in units of κ_0), and also the intensity of the laser pulse $a_2 \perp$ (in arbitrary units), are shown in the upper part of these figures. The phase planes of electrons ($p_\parallel / m_e c$ versus x) are shown in the lower part of the figures. The coordinate x is measured from the plasma boundary (left boundary), through which the laser pulse enters the plasma; it is normalized to $\lambda_0 = 2\pi c/\omega_0$. In our example $\gamma_{ph} = \omega_0/\omega_p = 8$ the pulse length is $\tau = 5T_0 \simeq \omega_p^{-1}$ ($T_0 = 2\pi/\omega_0$) and its amplitude is $a_\perp = 30$.

The initial stage of inversion of the Wakefield plasma wave is shown in Fig. 2 ($t = 60T_0$). The maximum field which accelerates the electrons captured during the inversion in this case reaches a value of $10\kappa_0$, which exceeds the limiting field of the steady wave (1) by a factor of 2.5.

The effective acceleration of electrons captured by the Wakefield electric field is shown in Fig. 3 ($t = 140T_0$). At this time, the nonlinear deformation of the laser pulse[8] leads to a reduction of the accelerating field which drives the captured electrons. A fraction of the captured electrons, however, acquires during this time a large amount of energy, $E_{\max} \approx \gamma_{ph}^3 m_e c^2$. Since the amplitude of the wave decreases considerably beyond the inversion region, we conclude that the bulk of the energy transferred to the plasma by the laser pulse is used to accelerate the captured electrons.

The progress which has been achieved in the supershort laser pulse compression[9] justifies the belief that superlimiting electric fields which were discussed above can be generated in a plasma. In a plasma with $n_0 = 10^{17}$ cm,$^{-3}$ a typical plasma used in laser-acceleration experiments, we obtain $E_{\max} \approx 4 \times 10^9$ V/cm for the given numerical simulation parameters.

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Translated by S. J. Amoretty