Proximity effect with arbitrary transmission of the *NS* boundary

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A decrease in the transmission of the *NS* boundary or an increase in the thickness of the *N* and *S* layers was found to greatly increase the effective electron–electron attraction, which accounts for the experimentally observed increase of the T_c of the *NS* contacts and superlattices.

The existing theories of the proximity effect in *NS* systems (*N* is the normal metal and *S* is the superconductor) have been developed for a low transmission level[1] and high transmission level[2] of the interface between metals. The increase in the critical temperature T_c with increasing thickness of the layers, d_N and d_S, which was observed in *N/S* superlattices, unfortunately cannot be adequately described either in terms of the high[2] or low[1] transmission level of the interface (see the review article in Ref. 3). We have investigated the dependence of the critical temperature of *NS* contacts and superlattices on the transmission σ of the potential barrier between metals, on the thickness of the layers d_N and d_S, and on the strength and sig of the electron–electron interation of the *N* region. We found that a decrease in the rate of electron transfer between the *N* and *S* region (i.e., a decrease in σ or increase in d_N and d_S) leads to an appreciable increase in the electron–electron attraction in the *S* region and, in contrast, to a decrease in the interaction in the *N* region. Accordingly, the experimentally observed increase in T_c in the *NS* systems can be naturally explained from the standpoint of physics.

Near T_c the order parameter $\Delta(z)$ of a plane *NS* contact, in which the region $-d_N<z<0$ is occupied by a normal metal (*N*) and the region $0<z<d_S$ is occupied by a superconductor metal (*S*) is described by the equation

$$\Delta(z)=V(z)T\sum_{\omega}{}'\int_{-d_N}^{d_S} dz' H_\omega(z,z')\Delta(z'), \tag{1}$$

where $V(z>0)=V_S$ and $V(z<0)=V_N$ are the energies of the electron–electron interaction, the prime on the summation denotes the cutoff at the Debye frequency ω_D, $\omega=\pi T(2n+1)$, T is the temperature, and $n=0, \pm 1, \pm 2, \ldots$.

In the "dirty" limit, where the shortest of all the length scales in each metal is the mean free path l_i $(i=N,S)$, the kernel $H_\omega(z,z')$ in Eq. (1), which has a range $\xi_i=(D_i/2|\omega|)^{1/2}\gg l_i$, satisfies the differential equation[2]

$$[2|\omega|-D(z)\partial^2/\partial z^2]H_\omega(z,z')=2\pi N(z)\delta(z-z'). \tag{2}$$

Here $D(z)$ and $N(z)$ are respectively the diffusion coefficient and the density of states

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of the electrons at the Fermi level, and $D_i = v_i l_i/3$, where v_i is the Fermi velocity.

The boundary conditions at $z=0$ for Eq. (2) can be reproduced in the limit $T \to T_c$ from the conditions determined in Ref. 4 for linearized Usadel equations or derived directly from an exact integral equation for $H_\omega(z,z')$. For a plane NS boundary we have

$$D_S(\partial H_\omega/\partial z)_{(z=+0)} = D_N(\partial H_\omega/\partial z)_{(z=-0)}$$

$$= (\sigma_S v_S/4)H_\omega(+0,z') - (\sigma_N v_N/4)H_\omega(-0,z'), \tag{3}$$

$$\sigma_i = <\sigma v_{ix}/(1-\sigma)v_i>; \quad \sigma_s v_s N_s = \sigma_N v_N N_N, \tag{4}$$

where the contact transmissions σ_N and σ_S averaging over the angles (between the velocity vector and the normal to the boundary) are related by the detailed balance relation (4). The de Gennes boundary conditions,[2] which are characterized by the continuity of the quantity $H_\omega(z,z')/N(z)$ on going through the $z=0$ plane, are a special case of conditions (3) and correspond to the high transmission limit $\sigma_i \gg l_i/\xi_i$.

The effect of N and S metals on each other across the boundary is particularly strong in the Cooper limit, where their thicknesses d_i are small compared with the shortest length of the characteristic lengths ξ_i, i.e., $\xi_{i\omega_D} = (D_i/2\omega_D)^{1/2}$. Solving Eqs. (1)–(3) jointly with the condition $(\partial H_\omega/\partial z)(z = -d_N, d_S) = 0$, we find for the T_c of an NS contact the equation

$$\Lambda_N(T_c)\Lambda_S(T_c) - \chi(\Gamma/T_c)[c_N\Lambda_N(T_c) + c_S\Lambda_S(T_c)] = 0. \tag{5}$$

We will use here the following notation:

$$\Lambda_i(T_c) = \lambda_i^{-1} - \ln(2\gamma\omega_D/\pi T_c); \quad \lambda_i = N_i V_i;$$

$$\chi(\Gamma/T_c) = \Psi(1/2) - \Psi(1/2 + \Gamma/2\pi T_c) + \ln(1 + \Gamma/\omega_D) \tag{6}$$

$$c_i = N_i d_i/(N_S d_S + N_N d_N); \quad \Gamma = \Gamma_N + \Gamma_S; \quad \Gamma_i = \sigma_i v_i/8d_i ,$$

where $\Psi(x)$ is the digamma function, $\gamma = 1.781$ is Euler's constant, and we assumed here for simplicity that the Debye frequencies ω_D in N and S are identical. In addition, c_N and c_S, the relative densities of states of electrons on the N and S sides of the contact and Γ_N and Γ_S, the velocities at which they move from N to S and back again, are related by the relation $c_N\Gamma_N = c_S\Gamma_S$. The value of T_c is determined by the largest root in Eq. (5).

The results of a numerical analysis of the plot of $T_c(\Gamma)$ [Eq. (5)] for $c_N = c_S = 0.5$ and $2\pi T_S/\omega_D = 0.2$ (T_S is the critical temperature of the insulating film S) and various values of the parameter $A = \lambda_N^{-1} - \lambda_S^{-1}$ are shown in Fig. 1. Here the solid curves 1–3 correspond to the electron–electron attraction in N ($\lambda_N > 0$) and curves 5 and 6 correspond to the region of electron–electron repulsion ($\lambda_N < 0$). The dashed curve 4 corresponds to $\lambda_N = 0$ (for the region $\Gamma \ll \omega_D$ it reproduces the

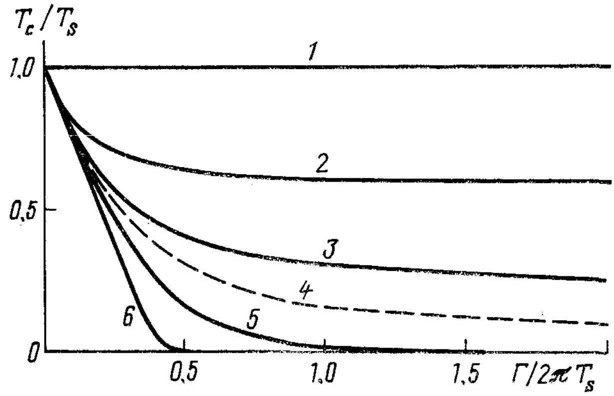

FIG. 1. Critical temperature T_c versus the transmission of an NS contact, Γ, for $c_N = c_S = 0.5$; $2\pi T_s/\omega_D = 0.2$ for various values of the parameter $A = \lambda_N^{-1} - \lambda_S^{-1}$. Solid lines: *1*—$A = 0$; *2*—$A = 1.4$; *3*—$A = 6.9$; *5*—$A = -6.9$; *6*—$A = -3$. Dashed line *4*—$A = \infty$ ($\lambda_N = 0$).

result predicted by the theory[1]). We see that in all the cases of a meaningful decrease of the parameter Γ, the rate at which the electrons are transferred between N and S gives rise to a monotonic increase in T_c.

When the transfer rates are low $\Gamma \ll \pi T_c$, the subsystems N and S are weakly linked. The order parameter Δ nucleates at $T = T_c$ in the S region ($\lambda_S > \lambda_N$) and percolates into N to the extent that the contact is transparent. In this limit it follows from (5) that $T_c = T_S - \pi\Gamma_S/4$; i.e., the rate at which the pairs escape, Γ_S, through the SN boundary functions as the depairing factor.

In another limiting case $\Gamma \gg \pi T_c$, a rapid transfer of electrons between N and S leads to a collectivization and renormalization of the interactions responsible for the superconductivity. In this case the NS contact is characterized by a single transition temperature and Eq. (5) can be written in a simpler form

$$\ln(2\gamma\omega_D/\pi T_c) = \lambda_{eff}^{-1} = (c_S\lambda_S^* + c_N\lambda_N^*)^{-1}, \tag{7}$$

where λ_{eff} is the effective electron–electron coupling in the NS system, and λ_S^* and λ_N^* are the renormalized coupling constants at the S and N edges of the contact, given by

$$\lambda_i^* = \lambda_i\left[1 - \frac{c_j(\lambda_i - \lambda_j)\ln(1+\omega_D/\Gamma)}{1 - \lambda_j \ln(1+\omega_D/\Gamma)}\right]^{-1}, \quad i \neq j. \tag{8}$$

If the transfer is very fast $\Gamma \gg \omega_D$, the logarithms in (8) are inconsequential, i.e., $\lambda_S^* \simeq \lambda_S$ and $\lambda_N^* \simeq \lambda_N$, and (7) reproduces the equation for the critical temperature, which was obtained in Ref. 2 and which corresponds to the lowest values of T_c of the NS contact.

With a decrease in the transfer rate, where $\pi T_c \ll \Gamma \ll \omega_D$, Eq. (8) predicts a logarithmic increase of the effective interaction in the S region ($\lambda_S^* > \lambda_S$) and, vice versa, it predicts its decrease in the N region ($|\lambda_N^*| < |\lambda_N|$). This gives rise to higher values of T_c that those reported in Ref. 2. Interestingly, the superconductivity in the

NS system can occur even when there is a strong repulsion on the N side of the contact ($|\lambda_N| \gg \lambda_S$, curve 6 in Fig. 1) if $|\lambda_N|^{-1} \ll \ln(\omega_D/\Gamma) \ll \lambda_S^{-1}$. In this case $\lambda_S^* \simeq s/c_S$ and $\lambda_N^* \simeq -[c_S \ln(\omega_D/\Gamma)]^{-1}$, while the condition under which superconductivity can occur is a less strict condition, $c_N < \lambda_S \ln(\omega_D/\Gamma)$, than that imposed by de Gennes[2] ($c_N < \lambda_S/|\lambda_N|$ at $\Gamma \gg \omega_D$).

From the physical standpoint, this result is similar to the suppression of Coulomb repulsion due to the retardation, an effect widely known in the theory of superconductivity (see, for example, Ref. 5). The point here is that when $\pi T_c \ll \Gamma \ll \omega_D$, the averaging of the electron–electron coupling to $(c_N\lambda_N + c_S\lambda_S)$ is done only for electrons from a narrow 2Γ layer near the Fermi energy. For the remaining electrons from the $2\omega_D$ layer the transfer through the NS boundary is a slower process than phonon transfer. As a result, a part of the interaction, which has a broader range in terms of energy, will be modified. Since $\lambda_N < (c_N\lambda_N + c_S\lambda_S) < \lambda_S$, the effective attraction in the S region will increase, while the interaction in the N region, in contrast, will decrease.

Substituting $\Gamma' = 2\Gamma$ for Γ, we can easily generalize Eqs. (5)–(8) in the case of N/S superlattices (alternating N and S layers with thicknesses d_N and d_S, respectively). An important point is that we can go from the case $\Gamma' > \omega_D$ to the case $\Gamma' < \omega_D$, which causes T_c to increase, not necessarily by reducing the transparency σ of the boundary, but by simply increasing the thickness of the layers d_N and d_S, while holding the value σ constant. The increase in T_c of the N/S superlattices as their period is increased $(d_N + d_S)$, which could not be explained previously (see Ref. 3 and the bibliography cited there), now can be explained on the basis of the increase of the effective electron–electron attraction which is described by Eqs. (7) and (8).

In addition, de Gennes's theory[2] gives the same value of T_c for contacts and for superlattices. At the same time, it follows from Eqs. (5)–(8) that since $\Gamma' > \Gamma$, the critical temperature of the N/S superlattice must be slightly lower than that of the NS contact for the same thicknesses of the layers d_N and d_S. An asymptotic agreement of these temperatures is found only in the limiting cases $\Gamma \ll \pi T_c$ and $\Gamma \gg \omega_D$. Our comparative analysis of T_c of the NS contacts and superlattices has been confirmed by recent experimental studies.[6]

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