Nonlocal interaction with zonal flows in the turbulence of drift and Rossby waves

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Theory of nonlocal weak turbulence of drift and Rossby waves[1,2] is extended to take into account the spectrum redistribution over scales due to nonlocal interaction with zonal flows. Corresponding generalization of the equation for turbulent spectrum evolution is obtained. According to this equation, the nonlocal interaction with zonal flows leads to a diffusion along mussel-like curves. These curves appear to coincide with the contours of spectral density of the new invariant found in Ref. 5.

The notion about nonlocal interaction in the turbulence of drift and Rossby waves was introduced in Refs. 1 and 2. Assuming that the evolution of drift turbulence spectrum is determined by the interactions with large scales and with the turbulence of zonal flows and not by near scales interactions (see the basis for such an assumption in Refs. 1 and 3), and assuming that the turbulence is weak, the following equation for spectrum evolution was derived in Ref. 2:

$$\frac{\partial n_{\vec{k}}}{\partial t} = I^0_{ls} + I^0_{zf}, \tag{1}$$

where

$$I^0_{ls} = \frac{\partial \Omega}{\partial k_x}\left(\frac{\partial}{\partial k_y}\left(S\frac{\partial n_{\vec{k}}}{\partial k_y}\right)_\Omega\right)_\Omega, \tag{2}$$

$$I^0_{zf} = Y[n(k_x,-k_y) - n(k_x,k_y)], \tag{3}$$

$$S = \left(\frac{\partial \Omega}{\partial k_x}\right)^{-2}\int_{p\ll 1,k} 4\pi\left[\left|V_{\vec{k},\vec{p},-\vec{k}-\vec{p}}\right|^2 n_{\vec{p}}\right]_{p_x=-\alpha p_y} p_y^2 dp_y, \tag{4}$$

$$\alpha = \frac{\partial \Omega}{\partial k_y}\Big/\frac{\partial \Omega}{\partial k_x},$$

$$Y = \left|\frac{\partial \Omega}{\partial k_y}\right|^{-1}\int_{|q_x|\ll k_x} 8\pi\left[\left|V_{\vec{k},\vec{q},-\vec{k}-\vec{q}}\right|^2 n_{\vec{q}}\right]_{q=(q_x,-2k_y)} dq_x,$$

$$\Omega = \Omega(\vec{k}) = k_x - \omega_{\vec{k}}.$$

Here n_k is the spectrum of wave action, $\mathbf{k} = (k_x,k_y)$ is a wave vector,

$\omega_{\mathbf{k}} = k_x/(1+\mathrm{k}^2)$ is the dispersion relation of drift and Rossby waves, the operator $(\partial/\partial k_y.)\ \Omega$ means the k_y derivative under the constant value of the function $\Omega = \Omega(\mathbf{k}); V_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3}$ is a matrix element, which can be different, depending on the particular physical situation.

The first term in Eq. (1), I^0_{ls}, corresponds to the main order in the expansion of nonlocal interaction with large scales over small values of wave vectors $\mathbf{p}$ of these scales, and the second term, I^0_{zf}, is the main contribution of the interactions with the turbulence of zonal flows (when expanding it over the small parameter q_x/k_x, where z_x and k_x are the wave vectors of the zonal flow turbulence and the turbulence under consideration, respectively).

According to Eqs. (1) and (2), the interaction with large scales leads to a spectrum redistribution independently of each of the curves $\Omega(\mathbf{k}) = \mathrm{const}$. The nonlocal interaction with the turbulence of zonal flows in the order retained in Eqs. (1) and (3) does not contribute to a spectrum redistribution among the scales with different values $|k_y|$; it influences only the evolution of asymmetric (relative to the k_x axis) spectra which gives rise to the symmetrization of these spectra. In other words, the spectrum redistribution on $|k_y|$ due to the interaction with zonal flows is a slower process than its symmetrization relative to the k_x axis; this redistribution is described by the next orders in the expansion over q_x/k_x. We shall show below that the rate of such redistribution can be greater than the one caused by the interaction with large scales.

Starting with the kinetic equation for waves[4]

$$\frac{\partial n_{\vec{k}}}{\partial t} = St[n],$$

$$St[n] = \int_{k_{1x},k_{2x}>0} (R_{012} - R_{102} - R_{210})\, dk_1 dk_2,$$

where

$$R_{012} = 2\pi |V_{\vec{k},\vec{k}_1,\vec{k}_2}|^2 \delta(\vec{k} - \vec{k}_1 - \vec{k}_2)\delta(\omega_{\vec{k}} - \omega_{\vec{k}_1} - \omega_{\vec{k}_2})$$
$$\times [n_{\vec{k}_1} n_{\vec{k}_2} - n_{\vec{k}} n_{\vec{k}_1} - n_{\vec{k}} n_{\vec{k}_2}],$$

and retaining the two main terms in the expansion over q_z/k_x (where $q_x = k_{1x}$ or $q_x = k_{2x}$, depending on which vector, $\mathbf{k}_1$ or $\mathbf{k}_2$, lies in the zonal flow region), we can obtain more exact [than in Eq. (1)] expression for the interaction with zonal flows:

$$I_{zf} = I^0_{zf} + I^1_{zf}, \tag{5}$$

where

$$I^1_{zf} = \frac{\partial \phi}{\partial k_x} \left(\frac{\partial}{\partial k_y} \left(Z \frac{\partial n_{\vec{k}}}{\partial k_y} \right)_{\phi} \right)_{\phi}, \tag{6}$$

$$Z = \left|\frac{\partial\Omega}{\partial k_y}\right|^{-1}\left(\frac{\partial\phi}{\partial k_y}\right)^{-1}\beta^{-2}\int_{|q_x|\ll k_x} 4\pi\left[\left|V_{\vec{k},\vec{q},-\vec{k}-\vec{q}}\right|^2 n_{\vec{q}}\right]_{q=(q_x,-2k_y)} q_x^2 dq_x, \tag{7}$$

$$\beta = \frac{3k_y^2 - 3k_x^2 + 3k_y^4 - 6k_x^2k_y^2 - k_x^4}{2k_xk_y(1+4k_y^2)}.$$

Here the operator $(\partial/\partial k_{y.})_\phi$ denotes the k_y derivative for a constant value of the function ϕ; the latter is a solution of the equation

$$\beta\frac{\partial\phi}{\partial k_x} - \frac{\partial\phi}{\partial k_y} = 0. \tag{8}$$

The integration in Eq. (4) is carried out over the small (compared to unity) values p_y, while in Eq. (7) the value q_x must be less than k_x, but may be of order one or even greater. Therefore, even for small values of the ratio n_q/n_p (n_q and n_p are characteristic values of turbulent spectra in the zonal flow region and in the region of large scales, respectively), the term I^1_{zf} can be greater than the term I^0_{ls}; in this case the redistribution of the spectrum due to the nonlocal interaction with zonal flows will be the dominant process, and in place of Eq. (1) one should write the following equation for the spectrum evolution:

$$\frac{\partial n_{\vec{k}}}{\partial t} = I^0_{zf} + I^1_{zf}, \tag{9}$$

where the quantities I^0_{zf} and I^1_{zf} are defined by Eqs. (3) and (6).

Equation (9) describes one-dimensional diffusion in k space along the lines of

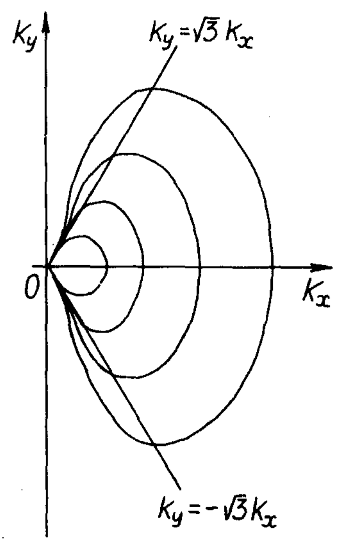

FIG. 1.

constant values of the function $\phi(\mathbf{k})$ satisfying Eq. (8). The important fact is that this function appears to be the same as the spectral density of the new invariant found in Ref. 5. This fact emphasizes the significance of such an invariant for the drift turbulence theory.

Contours of the solution of Eq. (8) are shown in Fig. 1. There is an interesting resemblance of Fig. 1 to a pattern on the surface of a mussel: the closed curves $\phi(\mathbf{k}) = \text{const}$ cross the point $\mathbf{k} = (0,0)$ on the same slope (equal to $3^{1/2}$).

It should be noted that because of the one-dimensionality of the redistribution due to the interaction with large scales and due to the nonlocal interaction with the turbulence of zonal flows, we should retain contributions of each of these processes if we want to take into account the two-dimensionality of total redistribution in the dominant process.

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