The possibility of the existence of phases in one-dimensional systems

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It is shown that in one-dimensional systems it is possible to observe phase transitions into states of continuous degeneracy, which are characterized by a power-law behavior of the correlation functions.

It was pointed out by Landau (Ref. 1, § 163) that there can be no coexistence of phases in one-dimensional systems. Therefore, neither first-order phase transitions nor phases with discrete degeneracy can occur in these systems. In states with a continuous degeneracy the fluctuations of the degeneracy parameter diverge, as a rule, even at zero temperature. Here the situation is completely analogous to the behavior of smectics in the three-dimensional case, and of crystals or superfluids in the two-dimensional case. There, however, the correlation of the fluctuations is not exponential, and this property distinguishes the state from a simple liquid; therefore, the opinion that has persisted until now, that these states are possible only in systems with limited dimensions, is not correct. In the one-dimensional case, for example, the pair correlation of density fluctuations in a crystal falls off exponentially at large distances. This communication discusses the characteristics of two states (crystalline and superfluid) which distinguish them from a normal one-dimensional liquid, and which suggest that a phase transition in principle can occur in a one-dimensional system.

A one-dimensional crystal is described by the density $\rho_0[x-u(x)]$ (compare with a two-dimensional crystal, Ref. 1, § 138), where ρ_0 is a periodic function and u is the displacement vector. The Lagrangian of the phonon mode reduces to the expression

$$\frac{\rho}{2}\{(\partial_t u)^2 - c^2(\partial_x u)^2\}, \tag{1}$$

where c is the speed of sound and ρ is the average density. In accordance with expression (1), the mean square modulus of the Fourier component of the displacement vector in the long-wavelength region is

$$< |u_k|^2 >= \frac{\hbar}{\rho c k} n_k, \tag{2}$$

where n_k is the Planck function of the distribution of thermal phonons. The mean square fluctuations of the displacement diverges, but the mean square deformation of the crystal is finite and decreases with decreasing temperature, as for ordinary crystals.

Let us explain the behavior of the four-point correlator $\langle \Delta\rho_1 \Delta\rho_2 \Delta\rho_3 \Delta\rho_4 \rangle$, where $\Delta\rho_i = \rho(x_i) - \rho$, $\rho(x_i)$ is the density at a particular instant of time at the point x_i and where the distances $x_1 - x_2 = b_1$ and $x_3 - x_4 = b_2$ are of the order of the crystallographic lattice constant a, and the quantity $L = (x_1 + x_2 - x_3 - x_4)/2$ is considerably larger. We expand the function ρ_0 in a Fourier series $\rho_0(x) = \rho + \Sigma \rho_f \exp(ifx)$, where $f = n\pi/a$, $n \neq 0$ is an integer, and $\rho_f = \rho^*_{-f}$. The relevant contributions to the correlation function come from the following terms:

$$\rho_f \rho_{-f} \rho_q \rho_{-q} < \exp i\{f[x_1 - x_2 + u_1 - u_2] + q[x_3 - x_4 + u_3 - u_4]\} > .$$

The difference in the displacements at distances of the order of the crystallographic period is small. Introducing the deformations $\partial_x u$, we have

$$|\rho_f|^2 |\rho_q|^2 \exp i(fb_1 + qb_2) < \exp i(b_1 \partial_x u_1 + b_2 \partial_x u_3) > . \tag{3}$$

Then, averaging (cf. Ref. 1, § 138), summing over wave vectors, and subtracting the part $\langle \rho_1 \rho_2 \rangle \langle \rho_3 \rho_4 \rangle$ that is unrelated to the correlations at large distances, we find

$$\Sigma |\rho_f^2 \rho_q^2| \exp[ifb_1 + iqb_2 - \frac{1}{2}(b_1^2 + b_2^2) < (\partial_x u)^2 >]$$
$$\times \{\exp[-b_1 b_2 < \partial_x u_1 \partial_x u_3 >] - 1\}.$$

The correlation of the fluctuations in the deformation $\langle \partial_x u_1 \partial_x u_3 \rangle$ is

$$\int k^2 |u_k^2| e^{ikL} \frac{dk}{2\pi} = \frac{\hbar}{2\pi\rho c} \int_{-\infty}^{+\infty} \frac{k \cos kL}{\exp(\hbar c k/T) - 1} dk$$
$$= \frac{\hbar}{2\pi\rho c} \{L^{-2} - (\pi T/\hbar c)^2 \sinh^{-2}(\pi T L/\hbar c)\}.$$

Ignoring small corrections, we have in the limit $L \gg \hbar c/T$ the expression

$$< \Delta\rho_1 \Delta\rho_2 \Delta\rho_3 \Delta\rho_4 > - < \Delta\rho_1 \Delta\rho_2 >< \Delta\rho_3 \Delta\rho_4 >$$
$$\approx -\frac{\hbar}{8\pi\rho c} \Sigma |\rho_f^2 \rho_q^2| \cos fb_1 \cos qb_2 \frac{b_1 b_2}{L^2}. \tag{4}$$

Although the contribution to the asymptotic behavior of the correlator is due to thermal fluctuations, it is temperature independent at distances that are large compared to the wavelength of the thermal phonons. To take into account the zero-point oscillations, it is necessary to have information on the spectrum at atomic wavelengths. In a model of a one-dimensional crystal with a single atom in a cell and only nearest-neighbor interactions, the phonon spectrum is given by the expression $\epsilon(k)$

$= \hbar c a^{-1}|\sin ka|$. Here the contribution of the zero-point oscillations to the correlator (5) is proportional to the expression

$$L^{-1}\sin(\pi L/2a)F(b_1,b_2), \tag{5}$$

where $F(b_1,b_2)$, a function of b_1 and b_2, oscillates over distances a. It is easy to show that the correction of order L^{-2} due to zero-point motion exactly cancels contribution (5) and remains the only part that has the form $L^{-2}\cos(\pi L/2a)$. The expansion of L^{-1} is obtained by integration by parts. The trick that is usually used (see, e.g., the derivations of Eq. (87.6) and the problem of § 87 in Ref. 2) is not justified, and leads to incorrect results.

It should be noted that in a one-dimensional crystal with a single atom in the unit cell there are no such things as vacancies or interstitials. If there is a large number of atoms in the unit cell, these defects are meaningful, and their behavior at finite temperatures leads to exponential behavior for any correlators, since each defect degrades the coherence of the structure.

The Lagrangian of a one-dimensional superfluid is

$$\frac{1}{2}(\rho\partial_t\phi-\phi\partial_t\rho)-\epsilon(\rho)-\frac{1}{2}\rho(\partial_x\phi)^2+\mu_0\rho, \tag{6}$$

where ϵ is the energy per unit volume in the uniform state, ϕ is the velocity potential, which is related to the phase of the order parameter of the superfluid, $\Psi=\rho^{1/2}\exp[i\varphi(x)]$, by the relation $\phi=(\hbar/m)\varphi$, where m is the mass of the particle in the Bose condensate, and μ_0 is the Lagrangian multiplier that guarantees conservation of the number of particles. The fluctuations are determined by the averages

$$<|\phi_k|^2>=\frac{\hbar c}{\rho k}\{n_k+\frac{1}{2}\},\ \ <|\rho_k|^2>=\frac{\hbar\rho k}{c}\{n_k+\frac{1}{2}\} \tag{7}$$

[cf. Ref. 2, Eq. (87b5)]. The correlator of the density fluctuations is

$$<\Delta\rho(0)\Delta\rho(x)>\sim\frac{\hbar\rho}{c}\int_0^q k\{n_k+\frac{1}{2}\}\cos kx dk\approx\frac{\hbar\rho q}{2cx}\sin qx,\ \ T\ll\hbar cq. \tag{8}$$

Here $q\sim p/m$ is the point of termination of the spectrum in the superfluid. The correlation function $\langle\Psi^*(0)\Psi(x)\rangle$ falls off exponentially, but the correlator $\langle\Psi^*(x_1)\Psi(x_2)\Psi^*(x_3)\Psi(x_4)\rangle$ for the same arrangement of points, as in the example cited above, exhibits a power-law behavior

$$\begin{aligned}
&<\Psi^*(x_1)\Psi(x_2)\Psi^*(x_3)\Psi(x_4)>-<\Psi^*(x_1)\Psi(x_2)><\Psi^*(x_3)\Psi(x_4)>\\
&=\rho^{-2}\{<\exp i(\varphi_1-\varphi_2+\varphi_3-\varphi_4)>-<\exp i(\varphi_1-\varphi_2)><\exp i(\varphi_3-\varphi_4)>\}\\
&=\rho^{-2}\exp\left[-\frac{1}{2}<(\varphi_1-\varphi_2)^2>-\frac{1}{2}<(\varphi_3-\varphi_4)^2>\right]\\
&\quad\{\exp[-<(\varphi_1-\varphi_2)(\varphi_3-\varphi_4)>]-1\}\\
&\approx-\rho^{-2}<(\varphi_1-\varphi_2)(\varphi_3-\varphi_4)>\sim\frac{mc}{\hbar q}\sin qb_1\sin qb_2\frac{\sin qL}{qL}.
\end{aligned} \tag{9}$$

Let us now consider a feature of one-dimensional superfluidity. The structure of the order parameter, in addition to an anomalous one-frequency function Ψ—or in the case of superconductivity, a two-frequency function— is also determined by anomalous functions of a large number of particles. In the three-dimensional case any of these functions (and then *a fortiori*, all of the functions together) can have singularities at the vortex lines and in the two-dimensional case at a point. In the one-dimensional case these topological properties are not present. Therefore, if in the course of the fluctuations the function Ψ goes to zero at some point, there will be no consequences for the rest of the anomalous functions; i.e., the coherence near this point is not destroyed.

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[1]L. D. Landau and E. M. Lifshitz, *Statistical Physics* [in Russian], Part 1, Nauka, Moscow (1976) [English translation: *Statistical Physics, two volumes 3rd edition*, Pergamon Press, Oxford (1980)].
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