Coexistence of weak and strong Langmuir turbulence

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A parametrically excited plasma turbulence is studied at moderate pump amplitudes. In the course of a weak-turbulence spectral shift there can be a self-focusing of waves and a local increase in the amplitude of these waves. This increase would then lead to a collapse of the plasma waves.

It is generally believed that at moderate turbulence levels, with characteristic values of the nonlinear growth rate small in comparison with the sound frequency, a plasma turbulence is weak. In this case the evolution of the waves can be described in terms of quasiparticles (plasmons) with the help of kinetic equations for the waves. The primary nonlinear process in an isotropic plasma is a decay of a Langmuir plasmon into another Langmuir plasmon and sound:

$$\omega_k = \omega_{k1} + \Omega_{k-k_1}. \tag{1}$$

In an isothermal plasma, this process gives way to an induced scattering by ions:

$$\omega_k = \omega_{k_1} + |k - k_1| v_{T_i}. \tag{2}$$

Process (1) or (2) shifts plasmons into the long-wavelength part of the spectrum with essentially no energy loss. In the long-wave part of the spectrum, a modulational instability and a collapse of plasma waves occur. The collapse results in a dissipation of energy.

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This turbulence picture is proposed in Refs. 1 and 2 and has been observed in numerical simulations.[3,4] It is possible, however, that this picture of turbulence does not actually occur experimentally. As was shown in Refs. 2 and 5, the steady-state spectra of a weak turbulence in an isothermal plasma (i.e., in the case in which the nonlinear growth rates are small in comparison with damping of the sound) are of a jet nature, being concentrated on surfaces or lines in k space or even consisting of a set of quasimonochromatic waves. Distributions of this sort are unstable with respect to the spontaneous onset of a modulation of the turbulence level.[6] A local increase in the wave intensity resulting from the modulational instability may lead to the formation of collapsing cavities, and there may be an absorption of energy without a spectral shift to the small-k region.

This effect could have substantial macroscopic manifestations, e.g., the appearance of accelerated electrons at comparatively low pump levels.

However, the instability described above may be suppressed because of its drift nature in k space and the finite size of the shift interval. The only way to find a picture of what occurs is to work by numerical simulation, as we have done in the present study.

We restrict the discussion to the parametric excitation of waves in an isotropic plasma. In this case,[2,5] waves parallel to the external electric field $E = E_0 \cos\omega_0 t$ are excited. The turbulence that arises can be described by the equations[6]

$$i\dot{\psi}_{k+} + \frac{3}{2}\omega_p r_d^2 \Delta_\perp \psi_k + \int T_{kk'} (\psi_{k'})^2 dk' \psi_k = i(\gamma_p(k) - \gamma_k)\psi_k. \tag{3}$$

Along our approach, long-wavelength spatial perturbations directed transverse with respect to the jet are described in the r representation, and k is the wave vector along the jet. No spatial variation arises along the jet because of the large width of the spectrum in this direction. Here γ_p is the pump growth rate, and γ_k is the rate of the damping, which is usually collisional.

The matrix element $T_{kk'}$ is given by[2,5,7]

$$T_{kk'} = \frac{\omega_p^2}{4\pi n T_e} G\left(\frac{\omega_k - \omega_{k'}}{|k - k'|}\right), \quad G = \frac{\epsilon_e}{\epsilon} - 1. \tag{4}$$

Here ϵ is the longitudinal part of the dielectric constant, ϵ_e is its electron component, and $\omega_k = \omega_p(1 + 3/2k^2 r_d^2)$. The matrix element $T_{kk'}$ can be approximated well by the expression[6]

$$G\left(\frac{\Omega}{k}\right) = \frac{T_e}{T_e + T_i} \frac{k^2 c_s^2}{\Omega^2 - k^2 c_s^2 + 2i\gamma_s \Omega_s}, \tag{5}$$

where c_s and γ_s are the velocity and damping rate of the ion sound.

In the homogeneous situation, Eq. (3) reduces to a kinetic equation for waves, which describes an induced scattering by ions:

$$\frac{\partial n_k}{\partial t} + (\gamma_k - \gamma_p(k))n_k = \mathrm{Im} \int T_{kk'} n_{k'} dk' n_k, \quad n_k = |\psi_k|^2. \tag{6}$$

The wave distribution along the jet in the case of parametric excitation is known[5] to consist of a set of sharp peaks, separated from each other by a distance equal to the spectral-shift step:

$$\Delta k \sim k_{diff} \sim \frac{1}{3} r_d^{-1} \sqrt{\frac{m}{M}}.$$

It thus becomes possible to simplify Eq. (3), by going over to equations for the amplitudes of these peaks [an analog of the satellite approximation for Eq. (6); Ref. 4]. In dimensionless variables we have the chain of equations

$$\gamma_p + \rightarrow t, \quad \frac{k}{k_{diff}} \rightarrow k, \quad \frac{T_0 |\psi|^2}{\gamma_p} \rightarrow |\psi|^2, \quad \nu = \frac{\gamma_k}{\gamma_0} \frac{2}{3} \frac{r_\perp^2}{r_d^2} \frac{\gamma_0}{\omega_p} \rightarrow r_\perp^2,$$

$$i\frac{\partial \psi_0}{\partial t} + \Delta \psi_0 + |\psi_0|^2 \psi_0 = i(1 - \nu - T|\psi_1|^2)\psi_0,$$

$$i\frac{\partial \psi_1}{\partial t} + \Delta \psi_1 + |\psi_1|^2 \psi_1 = i(-\nu + T(|\psi_0|^2) - |\psi_2|^2))\psi_2, \tag{7}$$

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$$i\frac{\partial \psi_n}{\partial t} + \Delta \psi_n + |\psi_n|^2 \psi_n = i(-\nu + T(|\psi_{n-1}|^2 - |\psi_{n+1}|^2))\psi_n.$$

Here T is the ratio of the maximum of the induced-scattering matrix element $\mathrm{Im} T_{kk'}$ to the static value $|T(0)| = [\omega_p^2/4n(T_e + T_i)]$. Using approximate expression (5), we can write $T \sim \Omega_s/2\gamma_s$. With $T_i \sim T_e$ we have $T \sim 1$. This ratio increases rapidly with increasing difference between the ion and electron temperatures of the plasma.

Each of Eqs. (7) has the form of a nonlinear Schrödinger equation, in which a collapse occurs in a finite time, giving rise to field singularities that contain a finite energy (Ref. 8, for example). The interaction between peaks leads to a shift of the energy and may suppress the collapse. Since the time scale for the two processes are comparable in the case $T \sim 1$, the only way to reach an understanding of the behavior of the system is to resort to numerical simulation.

Under actual physical conditions, the shift interval is narrow, no more than ten steps, and there is an energy sink at small k due to Langmuir collapse. To simulate this sink, we use running boundary conditions in our numerical calculations:

$$\psi_n = \psi_{n-1}.$$

The number of peaks was varied up to ten, and noisy initial conditions were used. The case of primary interest is that in which the pump is well above the threshold, so we initially carried out calculations with $\nu = 0$. The evolution of the waves depends strongly on the parameter T. Figure 1 shows the time evolution of the maximum amplitude of the individual peaks. We see that a collapse occurs. It typically occurs not in the directly excited beam but in the scattered peaks.

At $T = 1$, the first scattered peak collapses. With increasing T, waves are shifted a

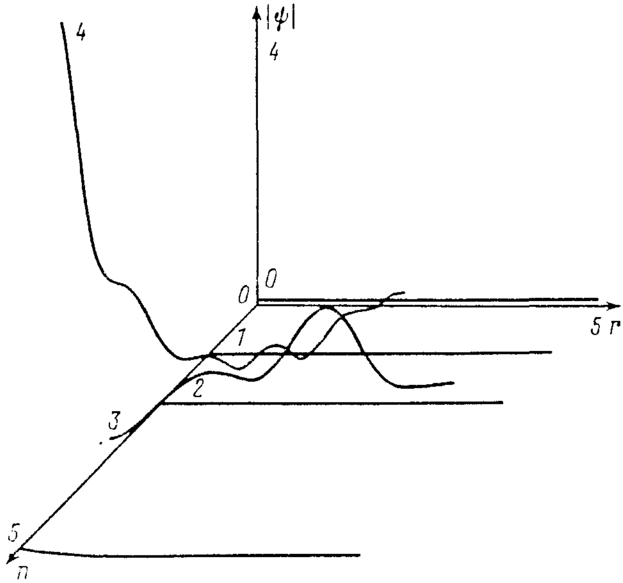

FIG. 1. Spatial distribution of the amplitudes of various modes at a time preceding collapse ($T = 1.4$).

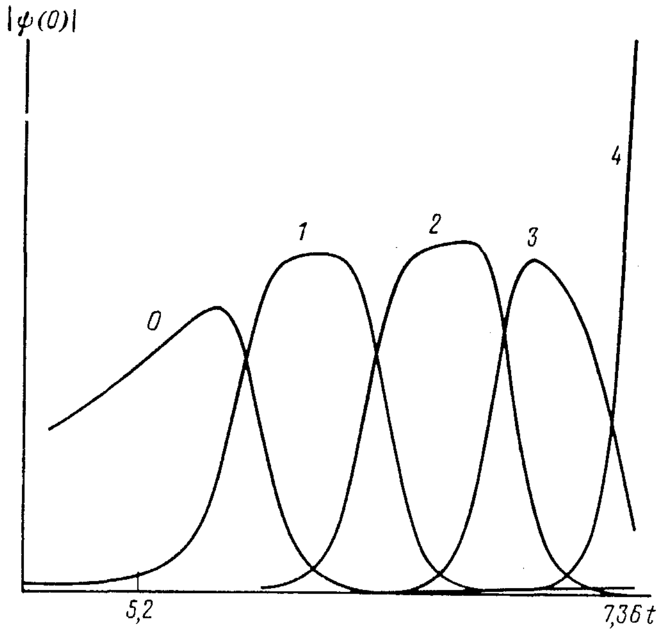

FIG. 2. Time evolution of the amplitudes of various modes at the center of the packets for the value $T = 1.4$. There is a successive transfer of energy from mode to mode, in accordance with a weak-turbulence description.

greater distance along the spectrum, and the collapse occurs after multiple scattering. Figure 1 corresponds to the value $T = 1.4$. Figure 2 shows the spatial distribution of the wave intensity in the various satellites. When the intensity of a collapsing satellite becomes significantly greater than the intensities of the neighboring satellites, the interaction with the latter can be ignored, and the field growth can be described by the well-studied nonlinear Schrödinger equation. Consequently, a finite amount of energy is absorbed in the collapse, and the collapse serves as an effective dissipation mechanism.

In practice, the number of spectral-shift steps $kr_d/\sqrt{m/M}$ cannot be greater than 10. Consequently, even at $T \sim 3$, which corresponds to $T_e \sim T_i$, the modulational instability cannot occur, according to our calculations, and the spectral shift toward small values of k, into the collapse region, is described by weak-turbulence theory.[5,6] This shift takes the form of a periodic splitting off of wave pulses (solitons), as can be seen clearly in numerical simulations.[2] If we assume that the peaks start with a steady-state uniform distribution, our calculations show that this distribution is stable at $T > 3$ and for an interval ~ 10 peaks. In this case the perturbations have time to reach the boundary of the interval before a collapse occurs.

Equations (7) also describe a self-focusing of light during multiple stimulated scattering. Our results show that, in general, the predominant effect should be a self-focusing of scattered light rather than of the light at the fundamental frequency.

In summary, we have shown that a weak plasma turbulence can coexist with a strong plasma turbulence in the case of parametric wave excitation in a plasma with $T_e \gtrsim 3T_i$. In a plasma with hotter ions, a self-focusing and a collapse of waves occur in the course of the spectral shift.

This phenomenon might (for example) substantially alter the turbulence frequency spectra that are measured. The results of a simulation of spectra in a case with a nonlinear damping due to self-focusing will be published separately.

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