How rare are the decays $Z \to \gamma J/\Psi$ and $Z \to \gamma \Upsilon(1S)$?

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The single-loop approximations $Z \to c\bar{c} \to \gamma\gamma^*$ and $Z \to b\bar{b} \to \gamma\gamma^*$ in the standard electroweak model are compared with the approximations of the vector-dominance model, $Z \to \gamma J/\Psi \to \gamma\gamma^*$ and $Z \to \gamma\Upsilon(1S) \to \gamma\gamma^*$, in the region of real γ rays. A sum rule is found as a result. This rule leads to $BR(Z \to \gamma J/\Psi) = 10^{-5}$ and $BR(Z \to \gamma\Upsilon(1S)) = 3 \times 10^{-5}$. These figures are two orders of magnitude higher than the values generally expected.

Let us calculate the amplitude for $Z \to \gamma(k_1)\gamma^*(k_2)$ according to a single-loop triangle diagram with intermediate heavy quarks ($Z \to c\bar{c} \to \gamma\gamma^*$ or $Z \to b\bar{b} \to \gamma\gamma^*$) for $0 \leqslant k_2^2 = E^2 \leqslant 4m_q^2$ ($k_1^2 = 0$) in the rest frame of the Z boson:

$$T(Z \to \gamma\gamma^*) = M^2 E\left(1 - \frac{E^2}{M^2}\right) t_q(E, M) \cdot$$

$$\cdot \left\{\frac{E}{M}(\vec{n} \cdot \vec{e}(Z))(\vec{n} \cdot [\vec{e}(\gamma^*) \cdot \vec{e}(\gamma)]) + (\vec{n} \cdot \vec{e}(\gamma^*))(\vec{n} \cdot [\vec{e}(\gamma) \cdot \vec{e}(Z)]\right\}, \qquad (1)$$

where M is the mass of the Z boson, $M^2 = (k_1 + k_2)^2$; $\vec{n} = \vec{k}_1/|\vec{k}_1|$; $\vec{e}(Z)$, $\vec{e}(\gamma^*)$, and $\vec{e}(\gamma)$ are 3D polarization vectors; $q = c, b$ is a quark; m_q is the mass of the quark; and $t_q(E,M)$ incorporates three identical loops corresponding to three colors and is given by

$$t_q(E,\ M) = \sigma_q \frac{e^3}{4} \frac{e_q^2 3}{\cos\theta_W \sin\theta_W} \frac{1}{4\pi^2} \{ \frac{M^2}{M^2 - E^2}$$

$$\times \left[i\rho(\pi + i \ln\frac{1+\rho}{1-\rho}) + 2\sqrt{-\beta^R} \arctan\frac{1}{\sqrt{-\beta^2}} \right] - \frac{m_q^2}{M^2 - E^2}$$

$$\times [(\pi + i\ln\frac{1+\rho}{1-\rho})^2 - 4(\arctan\frac{1}{\sqrt{-\beta^2}})^2] + 1 \Big\} \frac{1}{M^2 - E^2}, \tag{2}$$

$$\rho^2 = 1 - 4m_q^2/M^2, \qquad \beta^2 = 1 - 4m_q^2/E^2.$$

$$\sigma_c = 1, \qquad \sigma_b = -1, \qquad e_c = 2/3, \quad e_b = 1/3.$$

In other regions of E^2 (and M^2), $t_q(E,M)$ is found by analytic continuation.[1] For $Q^2 = -E^2 \geqslant 0$, for example, one finds

$$\sqrt{-\beta^2} \to -i\beta, \quad \arctan\frac{1}{\sqrt{-\beta^2}} \to i\frac{1}{2}\ln\frac{\beta+1}{\beta-1}. \tag{3}$$

In the region $0 \leqslant Q^2 = -E^2 \ll M^2$ the expression

$$t_q(E,M) = \sigma_q \frac{e\alpha}{4} \frac{e_q^2 3}{\cos\theta_W \sin\theta_W} \frac{1}{M^2} \ (i - \frac{2}{\pi}\ln\frac{M}{m_q} + \frac{1}{\pi} + \frac{\beta}{\pi}\ln\frac{\beta+1}{\beta-1}) \tag{4}$$

describes the amplitude for $Z \to q\bar{q} \to \gamma\gamma^*$ within higher-order corrections of QCD and the standard electroweak theory, i.e., within corrections on the order of $\alpha_s(4m_q^2)/\pi$, $\alpha_s(M^2)/\pi$, and α/π.

In the region $0 \leqslant E^2 \leqslant m_V^2$, on the other hand, the model of a dominance of vector mesons $V = \rho$, ω, ϕ in the electric current of light quarks is a generally accepted working model. This model has also earned a good reputation in the case of the electromagnetic current of c quarks: $V = J/\Psi$ (Ref. 2).

Let us merge single-loop approximation (1) with the approximation of the vector-dominance model:

$$T(Z \to \gamma\gamma^*) = M^2 E(1 - \frac{E^2}{M^2}) t_V(E,M)$$

$$\times \{ \frac{E}{M}(\vec{n}\cdot\vec{e}(Z))(\vec{n}[\vec{e}(\gamma^*)\cdot\vec{e}(\gamma)]) + \vec{n}\vec{e}(\gamma^*))(\vec{n}\cdot[\vec{e}(\gamma)\cdot\vec{e}(Z)]) \},$$

$$t_V = \frac{m_V^2}{m_V^2 - E^2} \frac{e}{f_V} T_V(M,\ m_V), \tag{5}$$

where $E^2 = 0$. As a result, we find the sum rule

$$T_V(M,\ m_V) = \sigma_q \frac{\alpha f_V}{4} \frac{e_q^2 3}{\cos\theta_W \sin\theta_W} \frac{1}{M^2} (i - \frac{2}{\pi}\ln\frac{M}{m_q} + \frac{3}{\pi}) = f_V T_q, \tag{6}$$

which links the integral of the jump in the amplitude with respect to intermediate hadronic states (the V meson) with the integral of the jump in the amplitude with respect to $q\bar{q}$ states in the E channel.

An unusual feature of this sum rule is the presence of an imaginary quantity on its right-hand side [see (6)]. This imaginary quantity stems from the jump in the amplitude with respect to intermediate $q\bar{q}$ states in the M channel. It gives a good description [within corrections on the order of $\alpha_s(M^2)/\pi$ of the jump with respect to intermediate hadronic states.

The width of the decay $Z\to\gamma V$ is

$$\Gamma(Z\to\gamma V) = \frac{1}{24\pi}(1-\frac{m_V^2}{M^2})^3(1+\frac{m_V^2}{M^2})M^3 m_V^2 |T_V(M,\ m_V)|^2$$

$$\simeq \frac{1}{24\pi}M^3 m_V^2 |T_V(M,\ m_V)|^2$$

$$= \frac{1}{24\pi}M^3 m_V^2 |T_q|^2 f_V^2 = \frac{1}{24}\alpha^2 \frac{f_V^2}{4\pi}\frac{e_q^4 9}{4\cos^2\theta_W \sin^2\theta_W}\frac{m_V^2}{M}[1+\frac{1}{\pi^2}(2\ln\frac{M}{m_q}-3)^2]. \quad (7)$$

To determine $f_V^2/4\pi$, we use experimental data[3] on

$$\Gamma(V\to e^+e^-) = \frac{4\pi}{3}\frac{m_V}{f_V^2}\alpha^2. \quad (8)$$

As a result, we find

$$f_{J/\Psi}^2/4\pi = 11.7; \qquad f_{\Upsilon(1s)}^2/4\pi = 125. \quad (9)$$

Using (7), (9), $m_c = 1.55$ GeV, $m_b = 5$ GeV, and $\Gamma_Z = 2.53$ GeV (Ref. 3), we find

$$BR(Z\to\gamma J/\Psi) = 10^{-5}, \quad BR(Z\to\gamma\Upsilon(1S)) = 3\times10^{-5}. \quad (10)$$

This result is two orders of magnitude greater than what is generally expected.[4,5] I therefore do not share the pessimism of Cocolicchio and Dittmar[5] regarding the possibility of observing a high luminosity of the decays $Z\to\gamma J/\Psi$ and $Z\to Z\to\gamma V$ at LEP. The angular distribution expected in the reaction $e^+e^-\to Z\to\tilde{\gamma}V$ is

$$W(\theta) = \frac{3}{8}\frac{1+\cos^2\theta+2m_V^2/M^2\sin^2\theta}{1+m_V^2/M^2} \cong (1+\cos^2\theta), \quad (11)$$

where θ is the angle between the momentum of the γ ray and the axis of the beams.

[1]N. N. Achasov, Phys. Lett. B **222**, 139 (1989).
[2]V. A. Novikov *et al.*, Nucl. Phys. B **165**, 55, 67 (1980).
[3]Particle Data Group, Phys. Lett. B **239**, 1 (1990).
[4]G. Guberina *et al.*, Nucl. Phys. B **174**, 317 (1980); J. H. Kuhn, Acta Phys. Pol. B **12**, 347 (1981).
[5]D. Cocolicchio and M. Dittmar, CERN-TH. 5753/90, 1990.

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