Magnetization of a frustrated 2D Heisenberg antiferromagnet: analogy with the fractional quantum Hall effect

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The magnetization M is found as a function of the field H for a frustrated 2D Heisenberg antiferromagnet on a 4×4 square lattice through an exact diagonalization of the Hamiltonian. The set of plateaus on the $M=M(H)$ curve confirms the analogy between an antiferromagnet and the fractional quantum Hall effect.

The discovery of high-T_c superconductivity has attracted increased interest in the properties of a 2D antiferromagnet, particularly its frustrated phase, whose ground state is believed to be a spin-liquid state. In an effort to describe this state and the low-lying excitations, Kalmeyer and Laughlin[1] have proposed using some wave functions which have been used successfully to explain the fractional quantum Hall effect. Since that study, the analogy between the two systems (a 2D frustrated antiferromagnet and the fractional quantum Hall effect) and between the basic properties of these systems (e.g., the gap in the spectrum of excitations and the fractional statistics) has been studied in detail.[2,3] Whether this analogy holds for the antiferromagnet as a whole, rather than simply for its ground state, is not yet clear. In particular, does this analogy extend to the states with a nonzero magnetization to which the antiferromagnet makes transitions in an external magnetic field?

Let us examine a 2D frustrated spin-1/2 Heisenberg antiferromagnet at absolute zero in an external magnetic field H strong enough to flip a substantial fraction of, or even all of, the spins to the field direction:

$$\mathcal{H} = J_0\sum_{\vec{i},\vec{e}} \vec{S}_{\vec{i}}\vec{S}_{\vec{i}+\vec{e}} + J_1\sum_{\vec{i},\vec{d}} \vec{S}_{\vec{i}}\vec{S}_{\vec{i}+\vec{d}} - H\sum_{\vec{i}} S^z_{\vec{i}}. \tag{1}$$

The spins are at the sites of a square lattice; J_0 is the exchange interaction between nearest-neighbor spins; the vector e connects spins at opposite ends of a side of a square; J_1 is the exchange interaction between the next-nearest spins; and the vector **d** connects the spins at opposite ends of a diagonal of a square.

Kalmeyer and Laughlin[1] analyzed a 2D antiferromagnet on a triangular lattice. That system has a frustration by virtue of the structure of the lattice itself. For an antiferromagnet on a square lattice, there is a long-range Néel order in the ground state. For this system, the frustration is introduced by the interaction of the next-nearest spins. The frustration disrupts the long-range order.

In their effort to describe the spin Hamiltonian, Kalmeyer and Laughlin obtained

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boson creation and annihilation operators through the use of Holstein–Primakoff transformations. In other papers,[2] the spins have been represented in terms of electron creation and annihilation operators. The ground state and excitations of the system of bosons or electrons have then been studied by variational methods on a lattice. The Laughlin functions, which have been used previously for the fractional quantum Hall effect, are used as the variational wave functions. In any of these representations, the magnetization operator of the spin system, $M = \Sigma_i S_i^z$, becomes the number operator of the particles which behave as electrons under conditions of the fractional quantum Hall effect. A magnetic field H applied to an antiferromagnet thus corresponds to the chemical potential μ in the system of electrons under conditions of the fractional quantum Hall effect, and the magnetization, when thought of as a function of the magnetic field, $M(H)$, corresponds to the number of particles $N(\mu)$, which depends on the chemical potential.

Upon a change in the chemical potential, the properties of a system of 2D electrons under conditions of the fractional quantum Hall effect (Ref. 4, for example) change periodically, depending on whether the chemical potential is in the gap in the spectrum of excitations or in a gap between neighboring Landau levels. When the filling of the Landau levels, ν, becomes equal to certain rational numbers, the system is in a state of an incompressible fluid. One might thus expect that again in the case of an antiferromagnet a variation of the magnetic field would put the system in states with different magnetizations, and the properties of these states would vary periodically. The states themselves could be described by wave functions corresponding to the states of the fractional quantum Hall effect with certain rational values of ν. The curve of $N(\mu)$ for the electrons under conditions of the fractional quantum Hall effect has a plateau when μ lies in the gap in the spectrum of excitations. One might thus expect that the curve of $M(H)$ for an antiferromagnet would also have a set of plateaus.

We have calculated the magnetization of an antiferromagnet in a strong field for a 4×4 lattice with periodic boundary conditions, through an exact diagonalization of Hamiltonian (1). Since the magnetization operator

$$M = \sum_i S_i^z$$

commutes with the Hamiltonian, it is sufficient to find the energies of the ground states in the absence of a field ($H = 0$) in the subspaces of states with certain values of M. We considered states having various symmetries under translation, rotation through 90°, and reflection of the lattice. In each subspace of states, we then found the ground state, its energy, and other characteristics, by an iterative method. The parameter $r = J_1/(J_0 + J_1)$, which is a measure of the degree of frustration, was varied over the interval $0 \leqslant r \leqslant 1$. At $r = 0$, there is a long-range Néel order in the ground state. At $r = 1$, the system can be partitioned into two noninteracting sublattices, within each of which there is a long-range order. At the point $r \cong 0.35$ ($J_1 \cong 0.55\, J_0$), where the frustration is at a maximum, a phase transition occurs.[5,6]

Figure 1 shows curves of $M(H)$ for two characteristic cases. In Fig. 1a, there is no frustration ($r = 0$), while in Fig. 1b the frustration is near its maximum ($r = 0.35$). The steps in each case correspond to the flip of a single spin (a unit increase in the magnetization). These steps become rounded as the dimensions of the system are

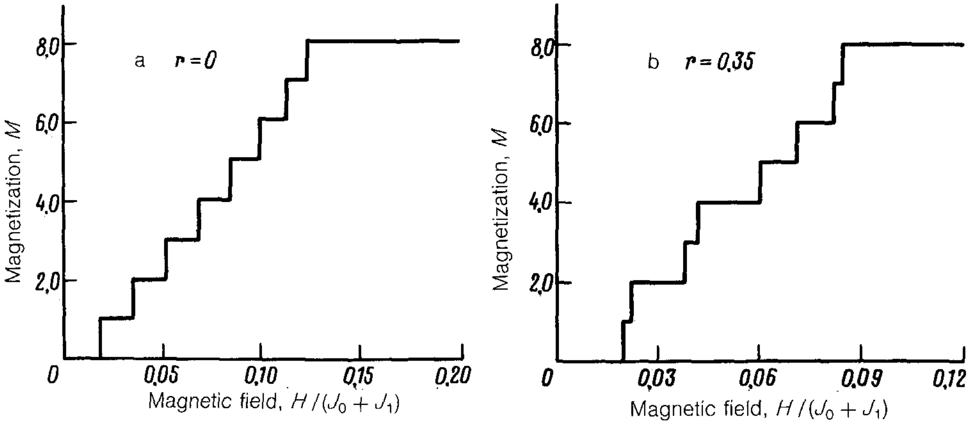

FIG. 1. Field dependence of the magnetization of an antiferromagnet. a—No frustration (there is no exchange interaction between next-nearest spins); b—with a nearly maximum frustration, in the case in which elongated plateaus appear.

increased. In the $r = 0$ case, the steps are regular, while at $r = 0.35$ some of the steps have become larger, and others smaller. In the limit of a system with large dimensions, this trend would correspond to the appearance of plateaus on a smooth curve.

We thus see that certain states with a definite magnetization ($M = 2.4$) are highly stable, and the system will remain in such states in the face of fairly strong variations in the external field. This property is analogous to the incompressibility of the states of the fractional quantum Hall effect at certain rational values of ν. The sizes of the plateaus depend on the parameter r, reaching a maximum at maximum frustration. These plateaus arise when the magnetization has values equal to 1/4, 1/2, and 3/4 of its maximum value. This comment apparently also applies to the large dimensions of the system. If we modify the system, however, and add to Hamiltonian (1) an interaction between more remote spins, then plateaus may arise at other values of M.

It can thus be concluded that the magnetization of a frustrated antiferromagnet has a set of plateaus in strong fields. The dimensions and positions of these plateaus depend on the type of lattice and the degree of frustration. The frustration plays an important role. The analogy between a frustrated antiferromagnet and the fractional quantum Hall effect runs deep. It is not limited to the energy region near the ground and low-lying excited states of the antiferromagnet. These results suggest that Kalmeyer and Laughlin's approach can also be used to find a description of the states of an antiferromagnet with large values of M.

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