Breathers and envelope solitons at a domain wall in a uniaxial ferromagnet

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The MHD equations for a domain wall in a uniaxial ferromagnet are reduced for a study of small-amplitude nonlinear spin-wave excitations localized at the wall. Solutions describing breather waves are found. The conditions under which these waves can exist in a steady state in an alternating magnetic field are found.

A domain wall in a ferromagnet is a convenient setting for studying nonlinear excitations with soliton-like properties.[1,2] Most of the research in this direction has dealt with topologically stable magnetic formations: Bloch lines.[3–6] Breathers and envelope solitons of spin waves localized at domain walls have received less attention, particularly on the theoretical side. Breathers participate directly in the annihilation and creation of Bloch lines, and they are also of interest in their own right.[5,7] Some numerical simulations have been carried out with nontopological solitons in a domain wall of a uniaxial ferromagnet.[5] Since the magnetodynamic equations used to describe these entities are not completely integrable, it is difficult to derive an analytic theory for them.[8] Below we use the case of a magnetic material with a large uniaxial anisotropy to reduce the initial equations for small-amplitude excitations to a nonlinear diffusion equation of the nonlinear-Schrödinger-equation type, with some additional perturbing terms which reflect dissipation and pumping. These equations make it possible to find approximate solutions which describe breather waves and to study dynamic processes in domain walls in which these waves are involved, to study the formation of dissipative structures, and more.

Spin-wave processes in a domain wall in a uniaxial ferromagnet with a large anisotropy can, as we know, be described well by the Slonczewski equations, which can be written in the form

$$\begin{cases} \partial_t\psi - \partial_x^2 q + b^2 q = h_z - \alpha\partial_t q, & (1) \\ \partial_t q + \partial_x^2\psi - \frac{1}{2}\sin 2\psi = \alpha\partial_t\psi + h_x \sin\psi, & (2) \end{cases}$$

where ψ is the azimuthal angle which characterizes the excursion of the magnetization from the plane of the domain wall, which is parallel to the ZX plane, q is the coordinate of the displacement of the center of the wall along the Y axis, the parameter b determines the force pinning the domain wall in the $q=0$ plane, h_x and h_z are the normalized magnetic fields acting along the X and Z axes, and α is the Hilbert damping constant. The variables are normalized as in Ref. 6.

We assume that the unexcited ground state of the domain wall corresponds to a vector

$$\vec{a} = \begin{pmatrix} \psi_0 \\ q_0 \end{pmatrix} = 0,$$

and we assume that the oscillation amplitude in the nonlinear wave is small, i.e., $|\psi| \ll \pi$. We seek a solution of Eqs. (1) and (2) as an expansion in a harmonic series:

$$\vec{a} = \begin{pmatrix} \psi \\ q \end{pmatrix} = \sum_{1=-\infty}^{+\infty} \sum_{n=1}^{\infty} \epsilon^n \vec{a}_1^{(n)} \exp[il(kx - \omega t)] + \text{c.c.}, \qquad (3)$$

where ϵ is the small parameter of the expansion (at the end of the calculations we set $\epsilon = 1$), ω is the oscillation frequency in the wave, and k is the wave number. We assume that the dissipation and the magnetic fields are weak; i.e., we assume $h_z,\ h_x\psi,\ \alpha\omega\psi \leqslant \epsilon^3$. We also assume that the amplitudes of the harmonics vary slowly in time and space, so we have $\psi_l^{(n)},\ q_l^{(n)} = F(\tau,\zeta)$, where $\tau = \epsilon^2 t,\ \zeta = \epsilon(x - \lambda t)$. The hierarchy of smallness parameters which we are assuming here allows us to go over from oscillation equations (1), (2) to evolution equations for the envelope amplitudes.

Substituting (3) into (1), (2), we find a system of equations

$$\hat{T}_l \vec{a}_l^{(n)} = \vec{f}_l^{(n)}$$

$$\hat{T}_l = \begin{pmatrix} -i\omega l & b^2 + k^2 l^2 \\ -1 - k^2 l^2 & -i\omega l \end{pmatrix}. \qquad (4)$$

Here we have $\det\hat{T}_{l\neq 1} \neq 0$, and $\det T_l = 0$, since the frequency ω satisfies the dispersion relation $\omega^2 = (b^2 + k^2)(1 + k^2)$. The vectors $\bar{f}_l^{(n)}$ contain small corrections for the dissipation, the pumping, and the spatial and temporal derivatives with respect to the "slow" variables. Going through the procedure of eliminating the secular terms in these equations systematically,[9] we find from (4) that $\lambda = \partial\omega/\partial k$ is the group velocity of the wave. For the amplitude of the first harmonic we find the evolution equation

$$-i\partial_\tau\psi_1^{(1)}+\frac{1}{2}\omega_{kk}\partial_\varsigma^2\psi_1^{(1)}+\sqrt{\frac{b^2+k^2}{1+k^2}}|\psi_1^{(1)}|^2\psi_1^{(1)}=\frac{i\alpha}{2}\psi_1^{(1)}(1+b^2+2k^2)$$

$$-\frac{i}{2}h_z e^{-i\omega t}+\frac{1}{2}h_x(\psi_1^{(1)*}e^{-2i\omega t}+\psi_1^{(1)})\sqrt{\frac{b^2+k^2}{1+k^2}}, \tag{5}$$

where $\omega_{kk}=\partial_k^2\omega(k)$. Here $q_1^{(1)}\psi_1^{(1)}=[(b^2+k^2)/i\omega]^{-1}$.

This nonlinear Schrödinger equation with perturbing terms has been analyzed in detail. The nonlinear Schrödinger equation has soliton solutions, which describe spin-wave packets that are localized at the domain wall. In the limit of small wave vectors ($k\ll b$), single-soliton solutions of the nonlinear Schrödinger equation correspond to standing wave packets of the breather type of our original system of equations, (1), (2), of the following type:

$$\psi=\frac{\Psi_0\sin(\omega t-\Psi_0^2bt/8+\beta)}{\mathrm{ch}[\Psi_0b(x-x_0)/2\sqrt{1+b^2}]}, \tag{6}$$

$$q=-\frac{\Psi_0\cos(\omega t-\Psi_0^2bt/8+\beta)}{b\,\mathrm{ch}[\Psi_0b(x-x_0)/2\sqrt{1+b^2}]}, \tag{7}$$

where β is the phase, and x_0 is the position of the center of the soliton. The approximate solution which we have found for the breather can be used as a reference point for constructing a perturbation theory if we need to deal with an interaction of the breather with a Bloch line, with a collision of breathers, or with other processes involving a breather. As an example, we will examine the conditions for the steady-state existence of a breather in a domain wall in the case with dissipation and with an alternating magnetic field $h_z(t)=h_z^0\exp(i\omega_0t)$, $h_x=0$.

Working from a perturbation theory for solitons of the nonlinear Schrödinger equation,[10,11] we find the following evolution equations from (5) for the amplitude $\Psi_0(\tau)$ and the phase $\chi(t)=\int_0^t(b-\Psi_0^2b/8-\omega_0)d\tau$:

$$\begin{cases}-\partial_\tau\Psi_0=\alpha(1+b^2)\Psi_0-\pi h_z^0\sin\chi,\\ \partial_\tau\chi=b-\Psi_0^2\frac{b}{8}-\omega_0.\end{cases} \tag{8}$$

It follows from these equations that a steady state, with $\partial_\tau\Psi_0=\partial_\tau\chi=0$, is possible only under the conditions $\omega_0=b(1-\Psi_0^2/8)$, $h_z^0>(\alpha\Psi_0/\pi)(1+b^2)$. Using the normalization of the parameters,[6] we can rewrite these conditions as $\omega_0=4\pi M_\gamma b(1-\Psi_0^2/8)$, $H_z>4M\alpha\Psi_0(1+b^2)$, where M is the magnetization, and γ is the magnetomechanical ratio.

We can find a numerical estimate of the feasibility of satisfying these conditions for rare-earth iron garnet films, in which Bloch lines are studied. Using $4\pi M=200$ G, $\alpha=0.1$, and $\gamma=2.8\times10^6$ Hz/G for $\Psi_0\sim1$, we find that the amplitude of the pump field must be at least 8 Oe at a frequency of 28 MHz. The oscillation amplitude of the domain wall, found from the expression $q_0 = \Psi_0\Delta/b$, where Δ is the thickness of the wall, would be about half a micron with $\Delta = 0.03\ \mu m$. These estimates show that visual observation of breathers at domain walls in iron garnet films would be possible in principle.

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[1]V. S. Gornakov, L. I. Dedukh, V. I. Nikitenko, and V. T. Synogach, Zh. Eksp. Teor. Fiz. **90**, 2090 (1986) [Sov. Phys. JETP **63**, 1225 (1986)].
[2]M. V. Chetkin, I. V. Parygina, V. B. Smirnov, and S. N. Gadetskiĭ, Zh. Eksp. Teor. Fiz. **97**, 337 (1990) [Sov. Phys. JETP **70**, 191 (1990)].
[3]A. V. Nikiforov and É. B. Sonin, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 325 (1984) [JETP Lett. **40**, 1119 (1984)].
[4]A. K. Zvezdin and A. F. Popkov, Zh. Eksp. Teor. Fiz. **91**, 1789 (1986) [Sov. Phys. JETP **64**, 1059 (1986)].
[5]J. J. Zebrovski, Phys. Rev. B **39**, 7205 (1989).
[6]A. K. Zvezdin, A. F. Popkov, and I. P. Yarema, Zh. Eksp. Teor. Fiz. **98**, 1070 (1990) [Sov. Phys. JETP **71**, 597 (1990)].
[7]Y. S. Kivshar and B. A. Malomed, Rev. Mod. Phys. **61**, 767 (1989).
[8]A. P. Malozemoff and J. C. Slonczewski, *Magnetic Domain Walls in Bubble Materials*, Academic Press, Orlando, 1979.
[9]N. Taniuti and N. Yajima, J. Math. Phys. **10**, 1369 (1969).
[10]J. P. Keener and D. W. McLaughlin, Phys. Rev. A **16**, 777 (1977).
[11]A. Newell, in: *Solitons* (ed. R. K. Bullough and P. J. Caudrey), Springer-Verlag, New York, 1980.

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