Nonequivalence of perturbative infrared properties of gluon propagator in various gauges

O. K. Kalashnikov

P. N. Lebedev Physics Institute, Academy of Sciences of the USSR, Moscow

(Submitted 10 April 1990)
Pis'ma Zh. Eksp. Teor. Fiz. **51**, No. 12, 601–604 (25 June 1990)

At nonzero temperatures the perturbative infrared limit of the two-loop gluon polarization tensor behaves in qualitatively different ways in covariant α gauges (the false infrared pole in the gluon propagator is not eliminated) and in the gauge $A_4 = 0$, in which the magnetic mass $m_M^2 \neq 0$ arises.

Direct calculations in non-Abelian gauge theories carried out by the standard quantization rules (Ref. 1, for example) requires the additional incorporation in the calculation of several gauge conditions and a certain number of ghost (fictitious) fields. It is shown (but usually at a formal level) that observable physical quantities do not depend on the particular choice. In many cases the choice of gauge conditions is totally arbitrary and is usually made for convenience in direct calculations or for some other reasons which are not inherent in the theory itself. This arbitrariness in the choice of a calculation scheme is completely acceptable in research on quantities which are clearly gauge-invariant, but this is often not the case. In addition, along with the gauge-invariant quantities one studies quantities about whose gauge dependence no assertion can be made at the outset. In the implementation of such calculation schemes (i.e., schemes involving a study of quantities whose gauge invariance is dubious or unproved), the procedure for choosing a gauge is not always obvious, and in many cases it leads to some extremely nontrivial consequences. One could cite examples in which the choice of different gauge conditions would lead to qualitative differences in the behavior of the quantities under study (in a perturbation-theory approach, at least), and one cannot always tell at the outset how to ensure that the results will be consistent if only their quantitative behavior is postulated.

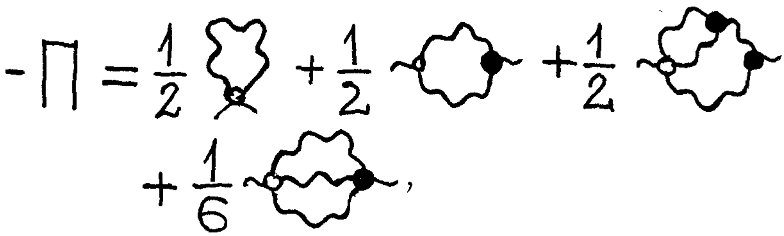
Our example involves a study of the infrared properties of non-Albelian gauge theories (in particular, the infrared asymptotic behavior of the gluon polarization tensor) in covariant α gauges and in the $A_4 = 0$ gauge. The truth condition is specified by the results of various kinds of nonperturbative calculations,[2,3] in which it is asserted that the infrared limit of the gluon polarization tensor (this limit is being studied) is finite (i.e., the magnetic mass of the gluon is finite), although it is possible that the corresponding numerical coefficient also depends on the particular computation method used. Effectively, only the $A_4 = 0$ gauge supports the nonperturbative predictions, and there is nothing of the sort in calculations in covariant gauges, in which the calculations themselves, in addition to the points discussed above, have several specific difficulties of their own.[4,5]

The $A_4 = 0$ gauge. By virtue of the simple Slavnov-Taylor identities, this gauge is the best choice for all nonperturbative calculations. In a perturbation theory, in contrast, this gauge presents many difficulties, and it requires tedious and laborious calculations. The reason for these difficulties is the complicated (and extremely specific) tensor structure of the "seed" propagator:

$$D_{ij}(\mathbf{k}, k_4) = \frac{1}{k^2}\left(\delta_{ij} + \frac{k_i k_j}{k_4^2}\right). \tag{1}$$

As a result, there is a catastrophic breeding of terms in the calculation of higher-order diagrams, and "false" poles appear at the points with $k_4 = 0$. A number of papers by various authors have been devoted to overcoming this difficulty (and to discussing other distinctive features of the $A_4 = 0$ gauge[6]), but all these difficulties are inconsequential in a calculation of the leading infrared asymptotic behavior of the gluon polarization tensor. A heuristic redefinition of several integrals (Ref. 3, for example) and simple algebraic transformations to single them out in their "pure" form are sufficient for such calculations.

We represent the gluon polarization tensor in the $A_4 = 0$ gauge by the set of four diagrams



(2)

where all the lines and filled points correspond to exact propagators and vertex functions of SU(N) gluodynamics. All the structural blocks in series (2) depend on p_4 and $|\mathbf{p}|$ separately. The infrared asymptotic behavior of these functions is found by taking the limit $p_4 = 0$ and then $|\mathbf{p}| \to 0$. An important point is that the step of taking this limit and the step of carrying out the direct calculations of the sums and integrals can be interchanged.

The infrared limit of the single-loop gluon polarization tensor was derived in the $A_4 = 0$ gauge in Refs. 7 and 8:

$$\frac{1}{2}\,\Pi_{ii}^{(1)}(k_4 = 0,\ |\mathbf{k}| \to 0) = -\frac{5}{16}\,g^2 N |\mathbf{k}| T. \tag{3}$$

The derivation involved a direct calculation of the corresponding diagrams of series (2). The anomalous behavior of expression (3) (in particular, its "wrong" sign) leads (as in covariant gauges[4,5]) to a fictitious pole in the gluon propagator in the infrared region of momenta:

$$D_{ij}(k_4 = 0, |\mathbf{k}| \to 0) = \delta_{ij} \,/\, [\mathbf{k}^2 - \frac{5}{16} g^2 N |\mathbf{k}| T]\ . \tag{4}$$

The position of the pole depends on the choice of the gauge vector n_i [for the $A_4 = 0$ gauge, $n_\mu = (0,1)$], but this pole does not disappear, no matter how this vector is chosen.[9] A magnetic mass of the gluon, which corresponds to a finite infrared limit of the trace of the polarization tensor,

$$m_M^2 = \frac{1}{2}\Pi_{ii}(k_4 = 0, \ |\mathbf{k}| \to 0), \tag{5}$$

does not arise in this approximation, but even in second-order perturbation theory we have $m_M^2 \neq 0$ (and this result sets the $A_4 = 0$ gauge radically apart from the α gauge).

It is extremely laborious to calculate the two-loop gluon polarization tensor in the $A_4 = 0$ gauge (which is determined by a set of eight diagrams), so certain assumptions are required even for a study of its infrared limit. In particular, it is necessary to use differential Slavnov-Taylor identities

$$\Gamma_3(p, -p, 0)^{abc}_{ijl} = (-igf^{abc})\frac{\partial D^{-1}_{ij}(p)}{\partial p_l} \tag{6}$$

in intermediate calculations. It then becomes possible to show[10] that [if, of course, identity (6) is exact] the magnetic mass of the gluon calculated in the first two nonperturbative diagrams of series (2) vanishes. The validity of identity (6) in the leading perturbation-theory order was demonstrated in Ref. 11, and its validity is beyond doubt, at least within the framework of perturbation theory. In each order of perturbation theory, only the last two nonperturbative diagrams of series (2) contribute to the expression for m_M^2. The magnetic mass of the gluon corresponding to this case was found in Ref. 3:

$$m_M^2 = (9g^4N^2/64\pi^2)T^2\ln(g^{-2}). \tag{7}$$

Expression (7) is radically different from the corresponding single-loop result (3), which for the $A_4 = 0$ gauge is not the characteristic result for m_M^2 and should be thought of instead as demonstrating the validity of assertion (6) and the vanishing of the "exact" expression for m_M^2 in the first two nonperturbative diagrams of series (2).

Relativistic α gauge. Because of the simple tensor structure of the gluon propagator, the relativistic α gauge is the most convenient gauge for any perturbative calculations (in particular, for studying the infrared properties of gluodynamics), but in this gauge we do not find an equivalence of the corresponding calculations and the expressions derived above. In the α gauge we do not have identity (6), and the equation analogous to (6) is quite complicated, ill-suited for an analysis of diagram series (2). Incidentally, in the α gauge the latter series is supplemented by yet another nonperturbative diagram (Ref. 12, for example), which incorporates the contribution of fictitious (ghost) fields, complicating all the calculations even further.

The infrared limit of the single-loop polarization tensor was originally found in the α gauge in Refs. 4 and 5:

$$\frac{1}{2}\Pi^{(1)}_{ii}(p_4 = 0, \ |\mathbf{p}| \to 0) = -\frac{g^2N|\mathbf{p}|T}{64}(9 + 2\alpha + \alpha^2). \tag{8}$$

This limit is not qualitatively different from expression (3). This agreement (which is itself extremely attractive and has been exploited in many places) is only superficial, however, since even in the two-loop approximation the results of the calculations in these gauges are completely different.

In the covariant α gauge the perturbative expression for $\Pi^{(2)}$ is extremely complicated, being determined by a set of 13 diagrams which arise from iterations of the nonperturbative single-loop and two-loop diagrams of modified series (2). The infrared behavior (i.e., their behavior at a finite $|\mathbf{p}|$) has not yet been found, but the infrared limit of the trace of $\Pi^{(2)}$ has been calculated in several recent papers.[12] In particular, it has been shown that

$$\lim \Pi_{ii}^{(2)}(p_4 = 0, |\mathbf{p}| \to 0) = 0 \tag{9}$$

and that this result holds for any choice of the gauge parameter α.

Consequently [according to (8) and (9)], in calculations in covariant gauges the perturbative magnetic mass of the gluon is apparently identically zero. This is a qualitative contradiction of the results calculated in the $A_4 = 0$ gauge. We are then led to ask just which of the gauges is the most physical and the most reliable for a study of the gauge-invariant properties of the theory. In this case we hope that the $A_4 = 0$ gauge is the gauge most nearly equivalent to the nature of the phenomenon under study, but this conclusion still requires an independent test.

[1]D. M. Gitman and I. V. Tyutin, *Canonical Quantization of Fields with Constraints*, Nauka, Moscow, 1986.
[2]D. J. Gross *et al.*, Rev. Mod. Phys. **53**, 43 (1981); O. K. Kalashnikov and E. Casado, Acta Phys. Pol. B **19**, 741 (1988).
[3]É. Kh. Veliev and O. K. Kalashnikov, Kratkie Soobshcheniya po Fizike **3**, 32 (1986).
[4]R. Jackiw and S. Templeton, Phys. Rev. D **23**, 2291 (1981).
[5]O. K. Kalashnikov and V. V. Klimov, Yad. Fiz. **33**, 848 (1981) [Sov. J. Nucl. Phys. **33**, 443 (1981)].
[6]V. M. Belyaev and V. L. Eletskiĭ, Pis'ma Zh, Eksp. Teor. Fiz. **50**, 49 (1989) [JETP Lett. **50**, 55 (1989)]; Yad. Fiz. **51**, 267 (1990) [Sov. J. Nucl. Phys. (to be published)].
[7]K. Kajantie and J. Kapusta, Ann. Phys. (N.Y.) **160**, 477 (1985).
[8]O. K. Kalashnikov, Preprint No. 265, Lebedev Physics Institute, 1984.
[9]O. K. Kalashnikov *et al.*, Fortschr. Phys. **31**, 613 (1983).
[10]T. Furusawa and K. Kikkawa, Phys. Lett. B **128**, 218 (1983); O. K. Kalashnikov, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 337 (1984) [JETP Lett. **39**, 405 (1984)].
[11]O. K. Kalashnikov, Yad. Fiz. 1990 (in press).
[12]O. K. Kalashnikov, Kratkie Soobshcheniya po Fizike **1**, 54 (1990); O. K. Kalashnikov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **51**, 293 (1990) [JETP Lett. **51**, 333 (1990)].

Translated by D. Parsons