Gupta-Bleuler operator quantization in superstring theory

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For a fermion superstring, the amplitude of the interaction of massless physical states with null-spurions is finite and nonzero. A qualitative explanation is proposed for this result.

In superstring theory, as in any gauge theory, two quantization methods are used: quantization on the basis of a functional integral (see the review by D'Hoker and Phong[1] and the bibliography there) and operator quantization.[2] For operator quantization in a covariant gauge, one uses not the canonical formalism for gauge theories[3] but the Gupta-Blueler formalism. For the latter method, there exists no general proof of the equivalence to canonical quantization and to the functional integration method. A necessary condition for equivalence is a splitting off of the spurion states with a zero norm which arise in the course of the Gupta-Bleuler quantization from the physical sector. In string theory, however, it is known[4] that spurion states split off from physical states within a total derivative with respect to modular parameters. In a theory with "frozen" modular degrees of freedom, the canonical and Gupta-Blueler quantizations are therefore definitely not equivalent. There is the opinion that in a (super-) string theory in which the modular parameters are dynamic parameters, and an average is taken over them, the corresponding amplitude either vanishes[4,5] or (in the case of a boson string) can be caused to vanish through a suitable choice of the counterterms on surfaces of a lower type.[6,7]

The Gupta-Bleuler quantization is known[1,2] to be equivalent to the Becchi-Rouet-Stora-Tyutin (BRST) formalism, in which the BRST transformation operator Q is constructed after the primary constraints are resolved.[8] By virtue of the nilpotence of Q (in the critical dimension $d = 10$), operators of the type $V' = [Q, V'']$ correspond to null-spurions. The calculation of the correlation functions of V' with physical vertices V_i is based on the equality[1]

$$\langle V_1 \dots V_n V' \rangle = \int_{sM} (d\, m_P) \sum_{K=1}^{r} \frac{\partial}{\partial m_K} W_K + \text{c.c.},$$

$$W_K = \prod_{L=1}^{K-1} \delta(\langle \mu_L | \hat{B} \rangle) \delta'(\langle \mu_K | \hat{B} \rangle) \prod_{L=K+1}^{r} \delta(\langle \mu_L | \hat{B} \rangle)$$

$$\times \overline{\prod_{M=1}^{r} \delta(\langle \mu_M | \hat{B} \rangle) \hat{V}_1 \dots \hat{V}_n} \hat{V}'' Z(X^*, B^*, C^*)|_{*=0} \,. \qquad (1)$$

Here m_p are complex coordinates of the supermodular space sM, $r = \dim sM$, μ_K are Beltrami superdifferentials, C and B are ghost and antighost superfields, Z is a generating functional, and X^*, B^*, and C^* are external currents for the corresponding fields.

In the cases of a torus and a sphere, terms arising from the commutation of Q with the ghost zero modes also appear on the right side of (1). For the choice of V' below, however, these terms are zero. In the case of boson strings, there are also terms associated with the limit of coinciding vertices. In the literature on boson strings, the discussion usually concerns specifically these terms.[6,7,10,2] They are zero in the case of a superstring, as was shown in Ref. 5.

If we take V'' to be the operator $V_4'' = \int d^2z\sqrt{g}b(z)V_4(z)$ where $b(z)$ is an antighost field, and V_4 corresponds to the vertex of a massless state with a momentum p_4; and if we consider a four-point amplitude with physical massless vertices V_1, V_2, V_3, then we find the following result for the single-loop contribution from (1):

$$\langle V_1 V_2 V_3 V' \rangle = \int_M d^2m \frac{\partial}{\partial m}\tilde{A}(m,\bar{m}) + \text{c.c.} = i\oint_{\partial M} d\bar{m}\tilde{A}(m,\bar{m}) + \text{c.c.} = \oint_{\partial F} \frac{d\bar{\tau}}{2\tau_2}A(\tau,\bar{\tau}) + \text{c.c.} = \oint_{\partial F} \frac{d\tau_1}{\tau_2}A(\tau,\bar{\tau}) \,. \tag{2}$$

Here $\tau = \tau_1 + i\tau_2$ is the torus lattice parameter; ∂M and ∂F are the boundaries of the fundamental region of moduli in terms of the variables m and τ, respectively; and $\tilde{A}(m,\bar{m}) = A(\tau,\bar{\tau})$, where

$$A(\tau,\bar{\tau}) = \frac{1}{2\tau_2^3}\int d^2z_1 d^2z_2 d^2z_3 \left|\frac{F_{12}F_{34}}{F_{13}F_{24}}\right|^{-s/2}\left|\frac{F_{23}F_{14}}{F_{13}F_{24}}\right|^{-t/2}$$

is the correlation function of four massless vertex operators $V_1,\ldots,V_4$ before averaging over the moduli[1] [here $F_{ij} = F(z_i,z_j;\tau)$ are known functions, which can be expressed in terms of Θ (Jacobi functions), and s and t are kinematic invariants]. The boundary ∂F is known[1] to consist of the two rays $\partial F_{1,3} = \{\tau:\tau_1 \pm \frac{1}{2};\tau_2 \geqslant \sqrt{3}/2\}$, which run parallel to the imaginary axis and to an arc of a unit circle $\partial F_2 = \{\tau:|\tau|^2 = 1, |\tau_1| \leqslant \frac{1}{2}\}^2$. It follows from the form of (2) that the rays ∂F_1 and ∂F_3 make no contribution. As a result, we find the final integral of the positive function:

$$\langle V_1 V_2 V_3 V' \rangle = \oint_{\partial F_2} \frac{d\tau_1}{\tau_2}A(\tau,\bar{\tau}) = \int_{-1/2}^{1/2} \frac{d\tau_1}{\sqrt{1-\tau_1^2}}A(\tau_1,\tau_2 = \sqrt{1-\tau_1^2}) > 0. \tag{3}$$

In the case of a fermion superstring, quantized in the Gupta-Blueler manner, the null-spurion states thus do not split off from the physical states. Since the parts of the boundary ∂F_1 and ∂F_3 do not contribute, and the asymptotic behavior in the limit $\tau_2 \to \infty$ is inconsequential, a corresponding (finite) result is found for a boson string. In that case, however, the picture is complicated by divergences in the case of coinciding vertex arguments, corresponding to tachyon and dilaton "tadpoles" and the mass renormalization. These divergences can be canceled by suitable local counterterms,[6,7,10] but a finite nonlocal contribution remains. For a superstring there are no divergences; the result is finite and nonlocal; and the contribution of lowest approximation is zero. It is thus not possible to cause the correlation function of the spurion with physical vertices to vanish.

For a qualitative discussion of the result, we restrict our comments to the boson part of the superstring. Physical states are distinguished by the condition $Q|\chi> = 0$, which corresponds[2] to a set of conditions determined by constraints of the first kind, $L_n|\chi> = 0$ $(n>0)$. It is understood here that the constraints of the first kind are generators of gauge transformations. It is known,[11,12] however, that on surfaces of types $p\leqslant 1$ the elements of the Virasoro algebra $L_2,...,L_{3p-2}$ (in the case $p=1$, the element L_2) generate transformations of modular parameters. In other words, there are generally no generators of gauge symmetries on conformal theories with a nontrivial topology of a constraint of the first type. This circumstance disrupts the equivalence of the canonical quantization and the Gupta-Bleuler quantization, leading to a situation in which the null-spurion states do not split off from the physical states and instead lead to integrals of a derivative with respect to moduli. It is not a simple matter to achieve a systematic canonical covariant quantization in operator terms. In the functional approach, on the other hand, it is extremely convenient to distinguish gauge and nongauge degrees of freedom, regardless of the topology of the surface. As was shown in Ref. 13, a Polyakov string is equivalent to a Mandelshtam string, i.e., is unitary. We are left with the question of the anomaly in the complete Lagrange BRST transformations. It has been shown that there is an anomaly in the conformal gauge,[14] but it is possible to construct globally defined covariant gauges in which there are no anomalies.

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