Anomalous slowing of lower hybrid waves in toroidal plasma

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Waves in the frequency range $\sqrt{\omega_{\rm He}\omega_{\rm Hi}} < \omega < \omega_{\rm He}$ can undergo an anomalous slowing in a toroidal tokamak plasma. In the approximation of a cold plasma, there exists a ray-trajectory attractor. As this attractor is approached, the wave vector **k** and the wave field **E** increase without bound. An effective Čerenkov absorption of rf power occurs in a plasma with a nonzero temperature.

We would like to call attention to the fact that in a tokamak one can observe a feature in the propagation of a slow wave mode, at frequencies several times the frequency of the lower hybrid resonance, which leads to a rapid absorption of the waves, regardless of the plasma temperature. This feature is manifested as the existence of an attractor of the ray trajectories calculated without consideration of the thermal motion of the particles. This attractor is a circle centered on the symmetry axis of the system. It "attracts" trajectories from a finite volume of the plasma. As the attractor is approached asymptotically, there is an unbounded growth of the wave vector **k**. At a nonzero temperature, this growth leads to a damping of the waves.

The reason for this effect is the poloidal variation of the magnetic field over a magnetic surface. The effect is thus seen particularly vividly if the cross section of the plasma column is noncircular. In such a case, the effect is observed over a wide frequency range.

To illustrate the situation, we show in Fig. 1 examples of the ray trajectories of waves (projected onto the minor cross section of the tokamak) and the evolution of the longitudinal slowing of the waves, $N_{\parallel} = k_{\parallel} c/\omega$, along these trajectories; here $k_{\parallel}$ is the projection of k onto the resultant magnetic field B, and ω is the wave frequency. Calculations have been carried out for $T_e = T_i = 0$, $\omega = 6.8\omega_{LH}$ [$\omega_{LH} = \omega_{pi}/(1+\omega_{pe}^2/\omega_{He}^2)^{1/2}$ is the lower hybrid frequency], and for parameter values which otherwise correspond to one version of the ITERR design and which have been used previously in calculations on current drive by lower hybrid waves[1] ($R_0 = 5.8$ m, $a = 2$ m, $k = 2$, $n_0 = 1.5\times10^{14}$ cm^{-3}, $B_0 = 5.1$ T, and $I = 18$ MA; the notation is standard). The initial values of $N_{\parallel}$ and $N_{\rm pol} = k_{\rm pol} c/\omega$ ($k_{\rm pol}$ is the projection of **k** onto the poloidal magnetic field $B_{\rm pol}$) for the two trajectories shown are 3 and 10, respectively. We see from the figure that the projections of the trajectories converge on a single point, which is the trace of the attractor. In space, the length of a trajectory is infinite because of the motion in the toroidal direction. For a nonzero temperature, the trajectory essentially retains its shape out to distances at which the amplitude becomes small in comparison with its initial value. In this particular example, the attractor exists in the frequency interval 8.5–14.5 GHz. We will show below that not only the wave vector of the wave but also its electric field becomes singular on

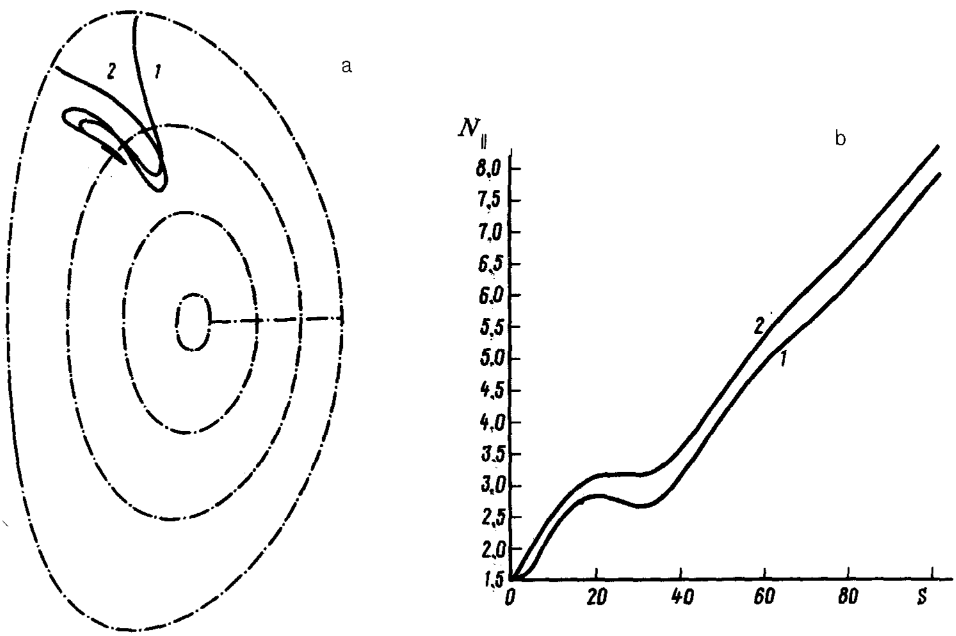

FIG. 1. a—Ray trajectories of waves, projected onto the minor cross section of the torus, for initial (launching) angles (1) $\theta = 1.57$ and (2) $\theta = 2.1$; b—variation of the longitudinal slowing along these trajectories in space.

the ray-trajectory attractor. The singularity is thus not a consequence of the use of the geometric-optics approximation. Its origin is explained on the basis of the analysis of Refs. 2 and 3, according to which the singularity stems from certain selected points of the parabolic line of the equation

$$\mathrm{div}\,\hat{\epsilon}\,\nabla\varphi = 0, \tag{1}$$

where φ is the scalar potential, and $\hat{\epsilon}$ is the dielectric tensor of the plasma at $T_e = T_i = 0$ (we are considering the two-dimensional problem in the plane of a poloidal cross section of the tokamak; this simplification is sufficient by virtue of the axial symmetry of the plasma configuration). The parabolic line separates the region in which Eq. (1) is hyperbolic from the region in which it is an elliptic equation. It is convenient to analyze the parabolic line by examining, in place of (1), the corresponding dispersion relation $\epsilon k_\perp^2 + \eta k_\parallel^2 = 0$, where $\epsilon = \epsilon_{xx}$ and $\eta = \epsilon_{zz}$ in the coordinate system with z axis running along the magnetic field. In a tokamak, under the condition $B_{\rm pol}/B \ll 1$, we would have $B_{\rm pol}/B \ll 1$, $k_\perp^2 \approx k_\rho^2 + k_{\rm pol}^2$ and $k_\parallel \approx k_{\rm pol} B_{\rm pol}/B + k_\varphi$, where k_ρ is the component of the poloidal component of k directed perpendicular to $B_{\rm pol}$, and k_φ is the toroidal component of the wave vector, so the dispersion relation becomes

$$k_\rho^2 = -\eta\,(B_{pol} k_{pol}/B + k_\varphi)^2/\epsilon - k_{pol}^2\,, \tag{2}$$

Just what type of equation Eq. (1) is depends on the coefficient $\Phi = -\eta B_{\rm pol}^2/(\epsilon B^2)$ of $k_{\rm pol}^2$. The region with $\Phi > 1$ is hyperbolic; two-dimensional waves with arbitrary large values of $k_{\rm pol}$ and k_ρ can exist in it. In the elliptic region ($\Phi < 1$), $k_{\rm pol}$ and k_ρ are bounded. The parabolic lines are thus evidently defined by the equalities

$$\epsilon = 0 \quad \text{and} \quad \Phi = 1 \, . \tag{3}$$

At the frequencies which we are considering here, the relation $\epsilon > 1$ holds throughout the plasma volume, so we are left with only the second condition in (3). According to Ref. 2, that point of the line $\Phi = 1$ at which this line is perpendicular to $B_{\rm pol}$ is a singular point of the wave equation (in a minor cross section of the tokamak). To determine the conditions under which such a point exists, we note that the function $\Phi(r,\theta)$ (r and θ are polar coordinates in the poloidal cross section, with origin at the magnetic axis of the discharge) vanishes at $r = 0$ and also at the boundary of the plasma column. In the intervening region, it has a maximum $\Phi_0(\theta)$ as a function of r. At low frequencies the relation $\Phi_0 \gg 1$ holds, and there are two parabolic lines: one near the axis and one at the periphery. These lines make a small angle with $B_{\rm pol}$ everywhere, and there is no singular point. As the frequency is increased, Φ_0 decreases. At a certain $\omega = \omega_1 \sim \omega_{pe} a/(R_0 q_0)$ (q_0 is the safety factor), the minimum value of $\Phi_0(\theta)$ along the direction of the angle θ becomes equal to unity. The two parabolic lines connect, and a singular point appears. This singular point exists up to a frequency $\omega = \omega_2$, at which the maximum value of Φ_0 along the angle becomes equal to 1, and the hyperbolic region disappears. Figure 2 illustrates this situation with two

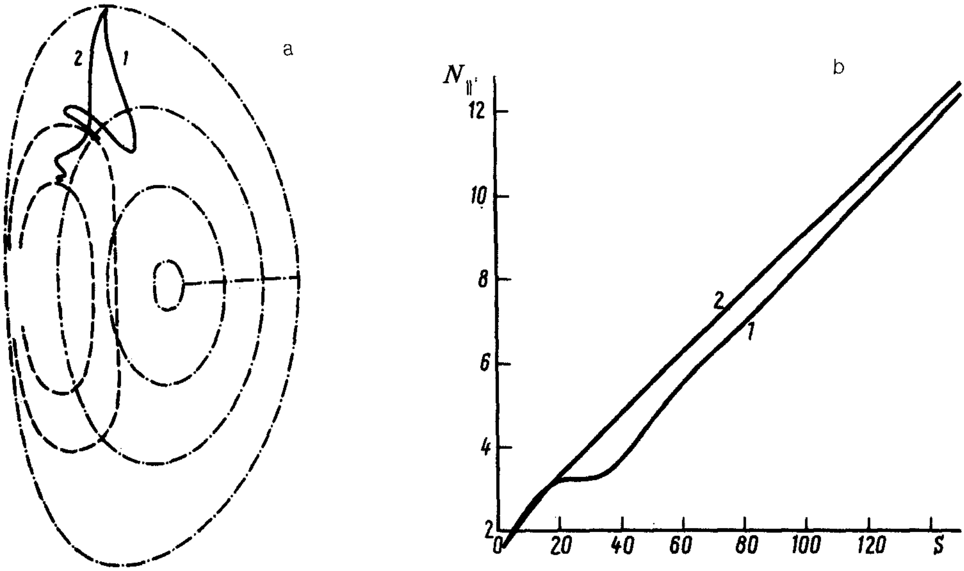

FIG. 2. a—Ray trajectories of waves, projected onto the minor cross section of the tokamak, for frequencies (1) $f = 9.2$ GHz and (2) $f = 11.4$ GHz; b—variation of the longitudinal slowing along these trajectories in space.

parabolic lines (dashed lines) for various wave frequencies. The solid lines show the projections of the ray trajectories onto the minor cross section of the tokamak for the corresponding frequencies.

To see how the solution of the wave equation behaves near the attractor, we introduce a local coordinate system with origin at the singular point, with x axis running tangent to the parabolic line, and with z axis running along the outward normal to it. For small values of x and y we can thus set $\Phi - 1 = x^2/l^2 = \lambda y/l$ where l and λ are constants, and the other coefficients in (2) are assumed to be independent of the coordinates. In this approximation, potential wave equation (1) becomes

$$\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\partial \varphi}{\partial \zeta}\left[(\xi^2 + \lambda \zeta)\frac{\partial \varphi}{\partial \zeta}\right] - 2ip\frac{\partial \varphi}{\partial \zeta} + p^2 \varphi = 0, \tag{4}$$

where $\varphi(\xi,\zeta)$ is a potential $\xi = x/l$, $\zeta = y/l$, and $P = \sqrt{|\eta_0|/\epsilon}k_\varphi l$. Equation (4) has been studied in detail[3] for the case $p = 0$. For $p \neq 0$, it is again possible to find a system of natural modes, which decay as $|\xi| \to \infty$:

$$\varphi_n(\xi,\zeta) = \int_0^\infty t^{-3/4+i\nu} \exp[-i\{(1 - i\lambda/\beta^2)\zeta^2/2 - p^2/\lambda t + t\zeta\}] \cdot H_n(\beta\sqrt{t}\xi)dt, \tag{5}$$

where H_n is the Hermite polynomial ($n = 0,1,...$), $\beta = (1 - \lambda^2/16)^{1/4}$ (we are assuming $\lambda < 4$), and $v = [(2n+1)\beta^2 - 2p]/\lambda$.

We can get a clear picture of the behavior of functions (5) by examining them in the case $\xi = 0$. We immediately find from (5) that we have $\varphi \to \zeta^{-3/4 + iv}$ as $\zeta \to 0$ in this case. The degree of the singularity is such that it results in a finite absorption of energy at the singular point at the dissipation approaches 0. As this point is approached, the amplitude of the field decreases by a factor of $\exp(\pi,\nu)$, signifying an essentially complete attenuation. Analysis of integral (3) shows that a mode with large indices $n > p/\beta^2 - 1/2$ is localized in the hyperbolic region, while a mode with $n < p/\beta^2 - 1/2$ describes waves which are coming in from the elliptic region. The existence of an infinite set of independent solutions with a singularity at the point $x = y = 0$ shows that an extremely wide class of solutions of Eq. (1) (sepcifically, integrals of this equation which depend on a single arbitrary function) are singular at this point.

We do not have room here to discuss the practical aspects of this phenomenon. We simply note that it is similar to an ordinary hybrid resonance. In particular, the lower hybrid resonance is associated with a singular point on the parabolic line $\epsilon = 0$. The behavior of the ray trajectories and of the solutions of Eq. (1) near it is completely analogous to that described above.

[1]Yu. F. Baranov and A. R. Esterkin, Meeting on Current Drive and Heating in ITER, Garching, 1989. ITER-IL-6-9-S-16.
[2]A. D. Piliya and V. I. Fedorov, Zh. Eksp. Teor. Fiz. **60**,. 389 (1971) [Sov. Phys. JETP. **33**, 210 (1971)].
[3]A. D. Piliya and V. I. Fedorov, in *Radio-Frequency Heating of Plasmas*, IPF Akad. Nauk SSSR, Gorki, 1983, p. 281.

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