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Spin-charge separation and the Pauli electron

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The separation between the spin and the charge converts the quantum mechanical Pauli Hamiltonian into the Hamiltonian of the non-Abelian Georgi-Glashow model, notorious for its magnetic monopoles and confinement. The independent spin and charge fluctuations both lead to the Faddeev model, suggesting the existence of a deep duality structure and indicating that the fundamental carriers of spin and charge are knotted solitons.

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Usually, we expect that an electron behaves like a structureless pointlike elementary particle. However, recently this view has been challenged by theoretical proposals [1, 2] that aim to explain observed phenomena in strongly correlated environments such as high-temperature superconducting cuprates [2, 3] and fractional quantum Hall systems [4]. According to these proposals, when an electron propagates in isolation or interacts only with a very low density environment its spin and charge remain confined into each other. But in a dense material environment the strong many-body correlations between different electrons may force the spin and the charge to start acting independently.

If present, a spin-charge separation could have far-reaching practical consequences to spintronics [5] that develops devices which are driven by the spin properties of electrons. In a wider context [6], the spin-charge separation could possibly explain the behavior of elementary particles in dense environments such as Early Universe and the interior of compact stars. It might even become visible in the LHC-ALICE experiment at CERN.

Here we study the spin-charge separation in the context of non-relativistic spin-$\frac{1}{2}$ particles that are described by the standard three dimensional, second quantized Pauli-Maxwell model. We select this model since its predictions are likely to lead to experimentally observable consequences in condensed matter physics. Furthermore, as a field theory model it subsumes structures that are commonly present in more developed, relativistic quantum field theories. In obvious notation

$$\mathcal{L} = \psi^\dagger(i\partial_0 - eA_0 + \mu)\psi + \frac{1}{2m}|(i\partial_k - eA_k)\psi|^2 + \\ + \frac{ge}{2m}\psi^\dagger \boldsymbol{\sigma}\cdot\mathbf{H}\psi - \frac{1}{4}F_{\mu\nu}^2. \quad (1)$$

Here ψ is a two-component commuting spinor, a Hartree-type wavefunction that describes the nonrelativistic dynamics of interacting electrons in its totally antisymmetric subspace. For completeness we have also included a finite chemical potential μ, and a Zeeman term with a generic g-factor.

We wish to employ (1) to describe a quantum state where the spin and the charge become separated. For this we choose the wavefunction ψ to describe a *topologically* mixed state where both the spin-up and the spin-down components are present: The direction of the spin polarization is a variable, and specified by a three component unit vector field $\mathbf{s}(\mathbf{x})$. If this vector field approaches a position independent limit at large distances, $\mathbb{R}^3$ becomes effectively compactified into a three-sphere. By recalling that $\pi_3[\mathbb{S}^2] \simeq \mathbb{Z}$ we can then employ the nontrivial homotopy of $\mathbf{s}$ to characterize topological mixing in a spin ensemble, which we build from the following orthonormal spin-up ($\mathcal{X}_+$) and spin-down ($\mathcal{X}_-$) states

$$\mathcal{X}_+ = U[\mathbf{s}]\begin{pmatrix}1\\0\end{pmatrix}, \quad \mathcal{X}_- = U[\mathbf{s}]\begin{pmatrix}0\\1\end{pmatrix}. \quad (2)$$

Here $U[\mathbf{s}]$ is an element of $SU(2)$ and its relation to $\mathbf{s}$ is

$$\mathbf{s} = \mathcal{X}_+^\dagger\boldsymbol{\sigma}\mathcal{X}_+ = -\mathcal{X}_-^\dagger\boldsymbol{\sigma}\mathcal{X}_-\,, \quad (3)$$

so that up to a phase (3) defines $\mathcal{X}_\pm$ in terms of $\mathbf{s}$. The spin projection operator is simply $\frac{1}{2}\mathbf{s}\cdot\boldsymbol{\sigma}$ and $\mathcal{X}_\pm$ are indeed its $\pm\frac{1}{2}$ eigenstates. When $\phi_\pm$ denote the probability densities of the spin-up and spin-down components,

$$\psi = \phi_+\mathcal{X}_+ + \phi_-\mathcal{X}_- = U\begin{pmatrix}\rho_+ e^{i\Omega_+}\\ \rho_- e^{i\Omega_-}\end{pmatrix} = \rho U\Phi, \qquad (4)$$

is our topologically mixed wavefunction. Here $\rho_\pm$ are the charge densities of the spin-up and spin-down electrons and ρ is the total density. We normalize ψ so that the space integral of ρ coincides with the total number of electrons in the ensemble. In analogy with (3) we introduce the three-component unit vector $\mathbf{n} = \Phi^\dagger\boldsymbol{\sigma}\Phi$. In general its $\pi_3[\mathbb{S}^2] \simeq \mathbb{Z}$ homotopy class is similarly nontrivial. As a consequence our topologically mixed wavefunctions ψ are classified in terms of the $\pi_3[\mathbb{S}^2]$ homotopy classes of *both* $\mathbf{s}$ and $\mathbf{n}$.

We propose that (4) entails a separation between the spin and the charge in ψ. Indeed, under a Maxwellian $U_M(1)$ gauge transformation $\psi \to \exp\{i\beta\}\psi$ and $U[\mathbf{s}] \in \in SU(2)$ can not change under this $U_M(1)$ gauge transformation. Instead $\Phi \to \exp\{i\beta\}\Phi$ which identifies the components $\phi_\pm$ as sole carriers of electric charge.

For the spin, we consider the effect of a global spatial $SO(3)$ rotation on the spinor ψ. Since the direction of the spin polarization vector $\mathbf{s}(\mathbf{x})$ is variable, we implement a $SO(3)$ transformation that at a generic position $\mathbf{x}_0 \in \mathbb{R}^3$ determines a rotation by an angle γ_0 in the normal plane of $\mathbf{s}(\mathbf{x}_0) = \mathbf{s}_0$,

$$\psi(\mathbf{x}_0) \to e^{\frac{i}{2}\gamma_0\mathbf{s}_0\cdot\boldsymbol{\sigma}}\psi(\mathbf{x}_0) = \rho\left[e^{\frac{i}{2}\gamma_0\mathbf{s}_0\cdot\boldsymbol{\sigma}}\cdot U\right]\Phi. \qquad (5)$$

For $\gamma_0 = 2\pi$ we indeed have $\psi \to -\psi$ which is consistent with the fermionic nature of ψ. Since the spatial rotation acts on U by left-multiplication, we identify U as the carrier of the spin. This confirms that (4) is a decomposition of ψ into its independent spin and charge degrees of freedom.

Clearly, the decomposition (4) has also a local *internal* $SU(2)$ symmetry that leaves ψ intact but sends $U \to Ug$ and $\Phi \to g^{-1}\Phi$. This ensures that both sides of (4) describe four independent field degrees of freedom. For a given spin polarization direction, the internal symmetry transformation in general mixes the (relative) probabilities between the spin-up and spin-down components. Consequently in a given material environment the local internal $SU(2)$ symmetry must become (spontaneously) broken. Indeed, the material environment specifies nontrivial ground state expectation values $\langle\rho_\pm\rangle = \Delta_\pm$. This breaks spontaneously the internal $SU(2)$ symmetry into a local internal *compact* $U_I(1)$ symmetry in the direction of the diagonal Cartan subalgebra that sends

$$\mathcal{X}_\pm \to e^{\pm\frac{i}{2}\alpha}\mathcal{X}_\pm, \quad \phi_\pm \to e^{\mp\frac{i}{2}\alpha}\phi_\pm. \qquad (6)$$

This corresponds to (opposite) simultaneous local rotations in the normal planes of the two unit vectors $\mathbf{s}$ and $\mathbf{n}$ respectively. As a consequence we have a local $U_M(1)\times U_I(1)$ gauge symmetry: the $U_I(1)$ gauge transformation (6) when applied only to the charges $\phi_\pm$ provokes a *spatial* rotation (5) in the normal plane of the spin polarization direction at the angle $\gamma_0 = -\alpha$.

Note that in terms of the spin variables the internal $SU(2)$ gauge symmetry determines a *local* rotation of the spin quantization axis $\mathbf{s}(x)$. When the $\pi_3[\mathbb{S}^2]$ homotopy class of $\mathbf{s}$ is nontrivial, a unitary gauge condition that attempts to *globally* align the direction of the spin quantization axis e.g. with $\mathbf{s}(x) \equiv \hat{\mathbf{z}}$, overlooks the presence of topological defects in the homotopically nontrivial $\mathbf{s}(x)$.

We substitute (4) into (1) and obtain the following remarkable result:

$$\mathcal{L} = \frac{1}{2m}(\partial_k\rho)^2 + \rho^2(J_0+\mu) + \frac{\rho^2}{2m}J_k^2 + \frac{\rho^2}{8m}(D_k\mathbf{n})^2 + \frac{1}{16e^2}\left[\mathcal{F}_{\mu\nu} - 4\pi\left(\frac{\rho_+^2}{\rho^2}\tilde{\Sigma}^+_{\mu\nu} + \frac{\rho_-^2}{\rho^2}\tilde{\Sigma}^-_{\mu\nu}\right)\right]^2 + \frac{eg\rho^2}{4m}(\mathbf{H}\cdot\overleftrightarrow{\mathbf{M}}\cdot\mathbf{n}). \qquad (7)$$

Here $D_\mu = \partial_\mu + \mathbf{X}\times$ is the SO(3) covariant derivative,

$$\mathbf{X}_\mu = \mathbf{W}_\mu - 2\mathbf{J}_\mu\mathbf{n}, \qquad \tfrac{1}{2}\mathbf{W}_\mu\cdot\sigma = \mathbf{i}\mathbf{U}^\dagger\partial_\mu\mathbf{U}, \qquad J_\mu = -eA_\mu + i\Phi^\dagger\partial_\mu\Phi + \tfrac{1}{2}\mathbf{n}\cdot\mathbf{W}_\mu, \qquad (8)$$

$\mathbf{G}_{\mu\nu} = [D_\mu, D_\nu]$ is the $SO(3)$ field strength tensor, and

$$\mathcal{F}_{\mu\nu}(\mathbf{X},\mathbf{n}) = \mathbf{G}_{\mu\nu}\cdot\mathbf{n} - \mathbf{n}\cdot D_\mu\mathbf{n}\times D_\nu\mathbf{n} \qquad (9)$$

is the 't Hooft tensor [7] with $\mathbf{H}$ its magnetic part (that can also accommodate an external background field), and

$$\overleftrightarrow{\mathbf{M}} = \frac{1}{2}\mathrm{Tr}[U^\dagger\overleftarrow{\sigma}U\overrightarrow{\sigma}] \qquad (10)$$

is the spin quantization frame. Finally, $\Sigma^\pm$ describe the worldsheets of closed Abrikosov vortices [8]

$$\tilde{\Sigma}^\pm_{\mu\nu} = \frac{1}{2\pi}\partial_{[\mu,}\partial_{\nu]}\Omega_\pm, \qquad \Sigma_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\tilde{\Sigma}_{\alpha\beta}. \qquad (11)$$

Curiously (7) is essentially the $SO(3)$ Georgi-Glashow model [7] for the multiplet $(\mathbf{n},\mathbf{X}_\mu)$: The material background $\langle\rho_\pm\rangle = \Delta^\pm$ breaks the $SO(3)$ symmetry spontaneously into the $U_I(1)$ symmetry. This leaves the Cartan $\mathbf{n}\cdot\mathbf{X}_\mu$ as the sole propagating component of $\mathbf{X}_\mu$. The local $U_I(1)$ gauge symmetry is spontaneously broken by the mass gap for (spatial) J_μ, the $U_M(1)$ and $SO(3)$ invariant supercurrent that describes the gauge invariant content of $\mathbf{n}\cdot\mathbf{X}_\mu$. The unit vector $\mathbf{n}$ is also a propagating degree of freedom, acquiring a mass gap from the Zeeman term. Finally ρ too propagates, as a canonical variable with a conjugate variable that lurks in J_0. Thus (7) describes six independent physical degrees of freedom, conforming with the four variables in ψ and the two transverse polarizations in the Maxwellian A_μ. Due to the fourth (covariant derivative) term in (7) the off-diagonal components of $\mathbf{X}_\mu$ are gapped and non-propagating.

We first consider (7) in a uniform spin background, e.g. with $\mathbf{s} = \hat{\mathbf{z}}$ and $U = \mathbf{1}$, and in the absence of the Abrikosov vortices. Thus we can set $\mathbf{W}_\mu = 0$ and $\Omega^\pm = 0$. Since the supercurrent J_k is subject to the Meissner effect it can be overlooked in the infrared and we obtain from (7), (9) the Lagrangian for the charge degrees of freedom

$$\mathcal{L}_{\text{charge}} = \frac{1}{2m}(\partial_k\rho)^2 + \frac{\rho^2}{8m}(\partial_k\mathbf{n})^2 + \\ + \frac{1}{16e^2}(\mathbf{n}\cdot\partial_\mu\mathbf{n}\times\partial_\nu\mathbf{n})^2 + \frac{ge\rho^2}{4m}(\mathbf{H}\cdot\mathbf{n}). \qquad (12)$$

This is the Faddeev model [9] in interaction with the scalar field ρ, known to support stable knotted solitons with a self-linking number that coincides with the $\pi_3[\mathbb{S}^2]$ homotopy class of $\mathbf{n}$ [10]. This suggests that in the uniform spin limit the elementary excitations in (7) are knotted solitons of charge.

Curiously, we find that knotted solitons also describe the spin excitations. For this we consider a uniform charge distribution represented by a constant $\mathbf{n}$ which we align with the positive z-axis. We again look at large distance scales where the Meissner effect allows us to discard the supercurrent contribution. We then obtain from (3), (7), (8) the structure (12), with $\mathbf{n}$ replaced by $\mathbf{s}$.

The appearance of the Faddeev model both in the uniform spin and the uniform charge limits of (7) suggests that in the general case it describes the interactive dynamics of knotted solitons of spin and charge. In particular, we have a manifest spin-charge duality in line with Ref.[11]. Furthermore, since (7) is a descendant of the Pauli-Maxwell Lagrangian we also have a strong-weak coupling duality between the Landau-Fermi description of electron liquid and a description in terms of knotted spin and charge solitons: Since (1) lacks a kinetic term for a $U_I(1)$ gauge field, we can formally view it as the strong coupling limit. A compact $U(1)$ gauge theory is a confining theory, with a first-order deconfinement transition [12]. Since it is natural to expect that the coupling in a compact $U(1)$ theory increases with increasing energy, this explains why at very high energies (and low densities) the electron behaves like a pointlike particle despite its nontrivial internal structure. Similarly it explains why at low energies and/or in proper finite density environments the $U_I(1)$ coupling can become weak leading to a deconfinement of the spin and the charge with the ensuing decomposition of the electron into its constituents.

Since (7) relates to the $SO(3)$ Georgi-Glashow model, we expect that it supports a version of the 't Hooft-Polyakov magnetic monopole. But since the additional $U_I(1)$ symmetry breaking gaps the supercurrent J_k subjecting it to a Meissner effect, we arrive at the following proposal: As usual, the 't Hooft-Polyakov monopole appears as singularity in the $U_I(1)$ connection $\mathbf{n}\cdot\mathbf{X}_k$. But since the off-diagonal components of $\mathbf{X}_\mu$ do not propagate, the corresponding (bare) off-diagonal correlation length ξ_{off} is infinite, and the monopole has a pointlike non-Abelian core. Due to a Meissner effect in J_k the flux of the diagonal component is squeezed into two Abrikosov vortices $\Sigma^\pm$. This confines monopoles and anti-monopoles into magnetically neutral *monopolium* pairs. Indeed, the topological content of (7) coincides with that of compact Abelian Higgs model with two condensed Higgs fields. The vortices appear as singularities in the up and down components of the Pauli electron while non-trivial hedgehog-like spin configurations corresponds to the monopoles. In particular, at the core of vortex Σ^+ *resp.* vortex Σ^- the ρ_+ *resp.* ρ_- spin component vanishes while at the monopole core $\rho_+ = \rho_- = 0$.

Eventually, the off-diagonal component of $\mathbf{X}$ may acquire a *finite* mass proportional to $1/\xi_{\text{off}}$. This may happen due to quantum corrections, e.g. in analogy with high-T_c superconductors [2]. The off-diagonal $\mathbf{X}$ becomes then a propagating degree of freedom. In elementary particle physics such massive bosons are common, see e.g. [16]: When the Georgi-Glashow model is remote from its compact $U(1)$ limit the magnetic flux of a monopole becomes organized into two vortices, each carrying half of the total monopole flux. The ensuing structures are similar to the center vortices in non-Abelian gauge models [17] and quite different from the present $\Sigma^\pm$ vortices. Center vortices have a tendency to organize the Abelian monopoles into dipole-like and chain-like structures, which are also present in Abelian models with doubly charged matter fields [18].

The representation (7) suggests that the spin-up and spin-down components may condense independently. In particular, in general the presence of nontrivial condensates $\langle\rho_\pm\rangle \neq 0$ does not ensure that $\langle\rho_\pm e^{i\Omega_\pm}\rangle \neq 0$. Suppose that there is indeed a state where both components of spin become frustrated so that $\langle\rho_\pm e^{i\Omega_\pm}\rangle = 0$, while ξ_{off} remains finite. The vortices described by $\Sigma^\pm$ are then light, while the center vortices are heavy. As a consequence monopole dynamics is governed by the center vortices rather than by the $\Sigma^{\pm}$ vortices. In such a state it may then become possible to observe the monopole-anti-monopole chains proposed in [16] and [18]: a monopole is a point defect, where the flux of the vortex alternates.

There may also be a state where only one of the spin components condenses. Such a *partial* pseudogap phase is then a spin analog of the metallic/electronic superfluid phase in liquid hydrogen [15].

The Zeeman term in (7) yields a Josephson coupling between $\phi_{\pm}$ which allows e.g. for a frustration to spread between the spin components. Note that the Josephson coupling is absent exactly at points where $\mathbf{s}$ and $\mathbf{H}$ are parallel. By applying space-varying external magnetic field one can emulate phenomena familiar from physics of Josephson junctions. Both the partial pseudogap phase and the Josephson junctions between up-spin and down-spin components are observable consequences of the spin-charge separation.

In the Georgi-Glashow representation (7) confinement leads to area law in the expectation values of the non-Abelian and Abelian Wilson loops for $\mathbf{X}_\mu$ and its diagonal component, respectively. The Abelian loop

$$W_C = \exp\Bigl\{i \int_S d^2 s_{\mu\nu}\, \mathcal{F}_{\mu\nu}(\mathbf{X},\mathbf{n})\Bigr\} = \\ = \exp\Bigl\{ie \int_C d^2 x_\mu\, A_\mu\Bigr\} \cdot \exp\Bigl\{-iS_{WZ}(\mathbf{n})\Bigr\}, \qquad (13)$$

factorizes into contributions from the Maxwellian and charge fields, respectively. Since the former does not confine, confinement must manifest itself as a disorder in the latter. This leads to large values is the Wess-Zumino action,

$$S_{WZ} = \int_S d^2 s_{\mu\nu}\, \mathbf{n} \cdot \partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n}, \qquad (14)$$

and to the ensuing rapid, area-like decay of the Wilson loop (13). The disorder can originate both from the monopoles [7] and the vortices [17].

The limit of very strong external magnetic field incorporates the quantum Hall effect. For this, take $\mathbf{H}$ to contain an external magnetic background component aligned with the positive z-axis. We also restrict the dynamics into two spatial dimensions by suppressing fluctuations into the z-direction. In the limit of a very strong external field we then obtain

$$\hat{\mathbf{z}} \cdot \overleftrightarrow{\mathbf{M}} \cdot \mathbf{n} \to -1, \qquad (15)$$

and, as a result,

$$\mathbf{n} = \mathbf{n}_0(\mathbf{s}) = -\hat{\mathbf{z}} \cdot \overleftrightarrow{\mathbf{M}}(\mathbf{s}). \qquad (16)$$

This equation relates the charge variable $\mathbf{n}$ to the spin variable $\mathbf{s}$ and since $D_k \mathbf{n}_0 \equiv 0$,

$$\mathcal{F}_{\mu\nu}(\mathbf{W},\mathbf{n}_0) = 2\pi \frac{\rho_+^2 - \rho_-^2}{\rho^2} \left(\widetilde{\Sigma}^+_{\mu\nu} - \widetilde{\Sigma}^-_{\mu\nu}\right) + 2\pi \widetilde{\Sigma}^s_{\mu\nu}\,, \qquad (17)$$

where the spin vortex

$$\widetilde{\Sigma}^s_{\mu\nu} = \frac{1}{2\pi}[\partial_\mu, \partial_\nu]\omega\,, \qquad (18)$$

corresponds to a two-dimensional hedgehog

$$s_1 + i s_2 \propto e^{i\omega}\,, \qquad (19)$$

in the shadow of the polarization axis $\mathbf{s}$ at the plane which is perpendicular to the magnetic field $\mathbf{H} \propto \hat{\mathbf{z}}$. Indeed, we expect that in general topologically nontrivial spin vortices are present. But since the $\mathbf{n} \cdot \partial_\mu \mathbf{n} \times \partial_\nu \mathbf{n}$ contribution to the 't Hooft tensor $\mathcal{F}_{\mu\nu}$ is absent in the strong field limit, the magnetic monopoles disappear and (7) reduces to

$$\mathcal{L} \to \frac{1}{2m}(\partial_k \rho)^2 + \rho^2 (J_0 + \mu) + \frac{\rho^2}{2m} J_k^2 + \\ + \frac{1}{4e^2}\left[\partial_\mu J_\nu - \partial_\nu J_\mu + \pi \left(\widetilde{\Sigma}^+_{\mu\nu} + \widetilde{\Sigma}^-_{\mu\nu} - \widetilde{\Sigma}^s_{\mu\nu}\right)\right]^2. \qquad (20)$$

Since $\pi_3[\mathbb{S}^2] \sim \pi_2[\mathbb{S}^2]$ our topologically mixed states persist in two dimensions, and assuming that the Abrikosov vortices become aligned with the z-axis we can replace each of the string terms $\Sigma^{\pm}$ by the field strength of an Abelian Chern-Simons. After averaging over the Gaussian field J_k we find that (20) reproduces the familiar anyon approach to fractional quantum Hall effect including the Chern-Simons description of its fractional statistics [4]. In three dimensional the quasiparticles are then knotted solitons of spin and charge, tightly entangled around the (closed) Abrikosov vortices that describe the three dimensional fractional statistics [13] via an appropriate generalization of the Chern-Simons action.

The vector field J_k is subject to the Meissner effect with the ensuing quantization of magnetic flux. We integrate the supercurrent around a large circle in the normal plane that surrounds a vortex configuration, once in a clockwise direction. We assume an asymptotic London limit where the electron densities coincide with their constant background expectation values $\Delta_{\pm}$. Due to the Meissner effect the contribution from the supercurrent vanishes and we get for the magnetic flux [14]

$$\oint dl_k A_k = -\frac{2\pi}{e}\frac{1}{\Delta^2}\sum_{i=\pm} \Delta_i^2 N_i + \frac{1}{2e}\oint dl_k \mathbf{n} \cdot \mathbf{W}_k. \qquad (21)$$

Here $N_\pm$ are the circulations of the Abrikosov vortices in the spin-up and spin-down components of Φ. For a finite energy configuration we conclude from (1), (7) that for a partially polarized state $\Delta_\pm \neq 0$ and we have *(i)* the constraint $N_+ = N_-$ so that the 1^{st} term in (21) is $2\pi/e$ times an integer; *(ii)* $D_k\mathbf{n} = 0$ which implies a relation between spin and charge variables in the 2^{nd} term, that is

$$W_k^a \equiv -\epsilon^{abc}\mathbf{M}_{bn}\partial_k\mathbf{M}_{nc} = -\epsilon^{abc}n^b\partial_k n^c + \lambda_k n^a, \quad (22)$$

where $\lambda_k \equiv \mathbf{n}\cdot\mathbf{W}_k$ is the longitudinal part of $\mathbf{W}_k$. If the spin quantization frame $\overleftrightarrow{\mathbf{M}}$ is spatially constant, then $\mathbf{n}\cdot\mathbf{W_k} = 0$ and the spin-charge mixing term in (21) vanishes. We then recover the standard flux quantization even though (21) *a priori* allows for an arbitrary flux. Note that the presence of spin vortices in the last term of (20) suggests that the spin-charge mixing term may provide the flux quantization in units of $2\pi/2e$ even though the electron has charge e [14], and even if the off-diagonal components of $\mathbf{X}$-field are non-propagating with $\xi_{\text{off}} = 0$.

The asymptotic flux quantization does not exclude a fine local structure of the vortices. Despite global flux quantization, locally the vortices may split into separate spin-up and spin-down constituents with *a priori* arbitrary fractional fluxes. Furthermore, the flux obtains nontrivial contributions both from the differences in the relative charge densities $\Delta_\pm$ and from the spin-charge mixing. The spin-up and spin-down vortices are confined into each other by a logarithmic potential, in configurations which are subject to the Abrikosov quantization [14, 15]. We propose that in general, in a strongly correlated system the vortices become locally composed of spin-up and spin-down components. However, we suspect that the vortices with a double-core structure should be metastable due to the attraction of their cores.

Finally, the domain walls that connect the spin vortices described by Σ^s along the lines of the sign flips $\psi \to -\psi$ should confine these vortices into spatially finite regions. We expect that the spin-vortex structures as well as the double-core vortices should be visible in fractional quantum Hall experiments [4] and spintronic devices [5].

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