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Universal behavior of $CePd_{1-x}Rh_x$ Ferromagnet at Quantum Critical Point

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The heavy-fermion metal $CePd_{1-x}Rh_x$ can be tuned from ferromagnetism at $x = 0$ to non-magnetic state at some critical concentration x_c. The non-Fermi liquid behavior (NFL) at $x \simeq x_c$ is recognized by power low dependence of the specific heat $C(T)$ given by the electronic contribution, susceptibility $\chi(T)$ and volume expansion coefficient $\alpha(T)$ at low temperatures: $C/T \propto \chi(T) \propto \alpha(T)/T \propto 1/\sqrt{T}$. We also demonstrate that the behavior of normalized effective mass M^*_N observed in $CePd_{1-x}Rh_x$ at $x \simeq 0.8$ agrees with that of M^*_N observed in paramagnetic $CeRu_2Si_2$ and conclude that these alloys exhibit the universal NFL thermodynamic behavior at their quantum critical points. We show that the NFL behavior of $CePd_{1-x}Rh_x$ can be accounted for within frameworks of quasiparticle picture and fermion condensation quantum phase transition, while this alloy exhibits a universal thermodynamic NFL behavior which is independent of the characteristic features of the given alloy such as its lattice structure, magnetic ground state, dimension etc.

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The nature of the non-Fermi liquid (NFL) behavior observed in heavy-fermion (HF) metals is still hotly debated. It is widely believed that the observed behavior is determined by quantum phase transitions which occur at quantum critical points, while the proximity of a system to quantum critical points creates its NFL behavior brought about by the corresponding thermal and quantum fluctuations suppressing quasiparticle excitations [1, 2]. A quantum critical point (QCP) can arise by suppressing the transition temperature T_c of a ferromagnetic (or antiferromagnetic) phase to zero by tuning some control parameters other than temperature, such as pressure, magnetic field, or doping x as it takes place in the case of the HF ferromagnet $CePd_{1-x}Rh_x$ [3, 4] or the HF metal $CeIn_{3-x}Sn_x$ [5]. QCPs are of great interest due to their singular ability to influence the thermodynamic properties of materials producing the NFL behavior. The NFL behavior around QCPs manifests itself in various anomalies. One of them is power in T variations of the specific heat $C(T)$, thermal expansion $\alpha(T)$, magnetic susceptibility $\chi(T)$ etc. [1, 2, 6].

Measurements on $CePd_{1-x}Rh_x$ show that around concentration $x = x_c \simeq 0.9$ the suppression of the ferromagnetic phase takes place, so that this alloy is tuned from ferromagnetism at $x = 0$ to non-magnetic state at QCP with the critical concentration x_c [3, 4]. Studies of the NFL behavior revealed in the HF metal $CePd_{1-x}Rh_x$ [3, 4] are of great interest since this alloy is a three dimensional ferromagnet. Basing on the theory of critical fluctuations which claims that these are responsible for the corresponding NFL behavior[1, 2, 6], one can assume that the NFL behavior demonstrating by $CePd_{1-x}Rh_x$ is to be different from that of $CeNi_2Ge_2$ exhibiting a paramagnetic ground state [7] or from that of the antiferromagnetic cubic HF metal $CeIn_{3-x}Sn_x$ [5]. Obviously the corresponding critical fluctuations taking place at QCPs in the mentioned three different HF metals are different, therefore one cannot expect to observe a universal behavior demonstrating by these metals, while the traditional theory has no grounds to consider these QCPs as a single QCP. Moreover, the distinctive features between ferromagnetic, antiferromagnetic and paramagnetic systems suggest intrinsic differences in their QCPs resulting in the difference of their thermodynamic properties, and the theory predicts that magnetic, thermal and transport properties of these systems have to be different [1 – 4, 6, 8].

At the critical concentration x_c, measurements on $CePd_{1-x}Rh_x$ show that the specific heat $C(T)/T \propto \propto 1/\sqrt{T}$, while around that concentration C/T and $\chi(T)$ coincide in their temperature dependence, $C(T)/T \propto \propto \chi(T) \propto 1/\sqrt{T}$ [3, 4]. Moreover, as we shall see it proved to be $\alpha(T)/T \propto 1/\sqrt{T}$ and this NFL behavior of the thermal expansion coefficient coincides with that of $\alpha(T)$ observed in the HF metals $CeNi_2Ge_2$ [7] and $CeIn_{3-x}Sn_x$ [3, 4]. The observed power laws and re-

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lationships between them can be hardly accounted for within scenarios based on the QCP occurrence with quantum and thermal fluctuations [1−3, 9] when quasiparticles are suppressed, for there is no reason to expect that $C(T)$, $\chi(T)$, $\alpha(T)$ and other thermodynamic propreties are affected by fluctuations in a correlated fashion.

These demonstrate that the fluctuations are not responsible for the observed behavior, and if they are not, what kind of physics determines the NFL behavior? Fortunately, the direct observations of quasiparticles in $CeCoIn_5$ have been reported recently [10]. On the other hand, when the electronic system of HF metals undergoes the fermion condensation quantum phase transition (FCQPT), the fluctuations are strongly suppressed and cannot destroy the quasiparticles which survive down to lowest temperatures and we can safely suggest that quasiparticles are responsible for the NFL behavior observed in HF metals [9, 11−13].

In this letter we show that the NFL behavior of the thermal expansion coefficient $\alpha(T)/T \propto 1/\sqrt{T}$ observed in $CePd_{1-x}Rh_x$ coincides with that of $\alpha(T)$ observed in both $CeNi_2Ge_2$ and $CeIn_{3-x}Sn_x$. While the NFL behavior of the ferromagnet $CePd_{1-x}Rh_x$ related to the uniform temperature dependence of $C(T)/T \propto \chi(T) \propto \alpha(T)/T \propto 1/\sqrt{T}$ can be accounted for within the framework of quasiparticle picture and FCQPT. We demonstrate this alloy is of great interest as it exhibits the universal NFL thermodynamic behavior at its QCP. This behavior is independent of the characteristic features of the given alloy such as its lattice structure, magnetic ground state, dimension etc. We also conclude that numerous QCPs assumed to be responsible for the NFL behavior of the thermal expansion coefficient and other thermodynamic properties observed in HF metals can be substituted by the only QCP related to FCQPT.

To study the low temperature universal features of HF metals, we use a model of homogeneous HF liquid with effective mass M^* in order to avoid the complications associated with the crystalline anisotropy of solids. This is possible since we consider the universal behavior related to the power-law divergences of observable values like the effective mass, thermal expansion coefficient, specific heat etc. These divergences are determined by small (as compared to those from unit cell of the corresponding reciprocal lattice) momenta transfer so that the contribution from larger momenta can be safely ignored.

To describe the effective mass M^* as a function of temperature and applied magnetic fields B when the system approaches FCQPT from the disordered side, $x \to x_{FC}$, we use the Landau equation connecting the effective mass $M^*(T,B)$ with the bare mass M and Landau interaction amplitude $F(\mathbf{p}_1,\mathbf{p}_2,x)$ [14]

$$\frac{1}{M} = \frac{1}{M^*(T,B)} + \int \frac{\mathbf{p_F}}{p_F^2}\frac{\partial F(\mathbf{p_F},\mathbf{p},x)}{\partial \mathbf{p_F}} n(\mathbf{p},T,B)\frac{d\mathbf{p}}{(2\pi)^3}, \tag{1}$$

where $n(\mathbf{p},T,B)$ is the quasiparticle distribution function

$$n(\mathbf{p},T,B) = \left\{1 + \exp\left[\frac{(\varepsilon(\mathbf{p},T,B)-\mu(B))}{T}\right]\right\}^{-1}. \tag{2}$$

Here both the single-particle energy $\varepsilon(\mathbf{p},T,B)$ and chemical potential $\mu(B)$ depend on temperature and magnetic field. It follows from Eq. (2) that at $B \to 0$ and $T \to 0$ the distribution function $n(\mathbf{p},T,B) \to \theta(p_F - p)$ with $\theta(p_F - p)$ being the step function and we obtain from Eq. (1) that [14−16]

$$M^*(x) = \frac{M}{1 - N_0F^1(p_F,p_F,x)/3} \simeq A + \frac{B}{x - x_{FC}}. \tag{3}$$

Here N_0 is the density of states of a free electron gas, p_F is Fermi momentum, $F^1(p_F,p_F)$ is the p-wave component of Landau interaction amplitude, A and B are constants. Since Landau Fermi liquid (LFL) theory implies the number density in the form $x = p_F^3/3\pi^2$, we can rewrite the amplitude as $F^1(p_F,p_F,x) = F^1(x)$. When $x \to x_{FC}$, $F^1(x)$ being a function of x achieves some value at which the denominator tends to zero so that the effective mass diverges at $T = 0$ as seen from Eq. (3).

At first let us consider the dependence of the effective mass on temperature. Upon using Eq. (3) and introducing the function $\delta n(\mathbf{p},T) = n(\mathbf{p},T) - \theta(p_F - p)$, we transform Eq. (1) and it takes the form

$$\frac{1}{M^*(T)} = \frac{1}{M^*(x)} - \int \frac{\mathbf{p_F}}{p_F^2}\frac{\partial F(\mathbf{p_F},\mathbf{p},x)}{\partial \mathbf{p_F}} \delta n(\mathbf{p},T)\frac{d\mathbf{p}}{(2\pi)^3}. \tag{4}$$

We integrate the second term on the right hand side of Eq. (4) over the angle variable and use the notation

$$F_1(p_F,p,x) = Mp_F\int \mathbf{p_F}\frac{\partial F(\mathbf{p_F},\mathbf{p},x)}{\partial \mathbf{p_F}}\frac{d\Omega}{(2\pi)^3}, \tag{5}$$

and substitute the variable p by z, $z = (\varepsilon(p) - \mu)/T$. Since in HF metals the band is flat and narrow, we use the approximation $(\varepsilon(p) - \mu) \simeq p_F(p - p_F)/M^*(T)$ and taking into account Eqs. (4) and (5) finally obtain

$$\frac{M}{M^*(T)} = \frac{M}{M^*(x)} - \alpha_1\int_0^\infty \frac{F_1(p_F,p_F(1+\alpha_1 z),x)dz}{1+e^z} + \alpha_1\int_0^{1/\alpha_1} F_1(p_F,p_F(1-\alpha_1 z),x)\frac{dz}{1+e^z}. \tag{6}$$

Here the factor $\alpha_1 = TM^*(T)/p_F^2$. The Fermi momentum p_F is defined from the relation $\varepsilon(p_F) = \mu$. We first assume that $M^*(x)$ is finite and $\alpha_1 \ll 1$. Then upon omitting terms of the order of $\exp(-1/\alpha_1)$, we expand the upper limit of the second integral on the right hand side of Eq. (6) to ∞ and observe that the sum of the second and third terms represents an even function of α_1. These are the typical expressions with Fermi-Dirac functions as integrands and can be calculated using standard procedures [17]. We conclude that at $T \ll T_F \sim p_F^2/M^*(x)$ the sum represents a T^2-correction to $M^*(x)$ and the system demonstrates the LFL behavior [18]. When $x \to x_{FC}$ the effective mass diverges and both $T_F \to 0$ and $1/\alpha_1 \to 0$, while the temperature interval over which the LFL behavior takes place is vanishing [18]. In that case, Eq. (6) becomes homogeneous and the second integral on the right hand side can be omitted. As a result, we can estimate that

$$M^*(T) \propto \frac{1}{\sqrt{T}}. \qquad (7)$$

Equation (6) shows the universal power low behavior of the effective mass which does not depend on the inter-particle interaction. We illustrate this behavior by calculations using a model functional

$$E[n(p)] = \int \frac{\mathbf{p}^2}{2M}\frac{d\mathbf{p}}{(2\pi)^3} + \frac{1}{2}\int V(\mathbf{p}_1 - \mathbf{p}_2) \times \times n(\mathbf{p}_1)n(\mathbf{p}_2)\frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi)^6}, \qquad (8)$$

with the inter-particle interaction

$$V(\mathbf{p}) = g_0\frac{\exp(-\beta_0|\mathbf{p}|)}{|\mathbf{p}|}. \qquad (9)$$

We normalized the effective mass by M, $M^* = M^*(T)/M$, temperature T by the Fermi energy ε_F^0, $T = T/\varepsilon_F^0$ and use the dimensionless coupling constant $g = g_0 M/2\pi^2$ and $\beta = \beta_0 p_F$. FCQPT takes place when the parameters reach their critical values, $\beta = b_c$ and $g = g_c$, in our case $b_c = 3$ and $g_c = 6.7176$. In Fig.1, the evolution of the low temperature entropy is shown. The calculated behavior of $S(T)/T \propto M^*(T) \propto 1/\sqrt{T}$ is in accord with Eq. (7).

Now consider the thermal expansion coefficient $\alpha(T)$ given by [17]

$$\alpha(T) = \frac{1}{3}\left(\frac{\partial(\ln V)}{\partial T}\right)_P = -\frac{1}{3V}\left(\frac{\partial(S/x)}{\partial P}\right)_T. \qquad (10)$$

Here, P is the pressure and V is the volume. The compressibility $K(x)$ is not expected to be singular at

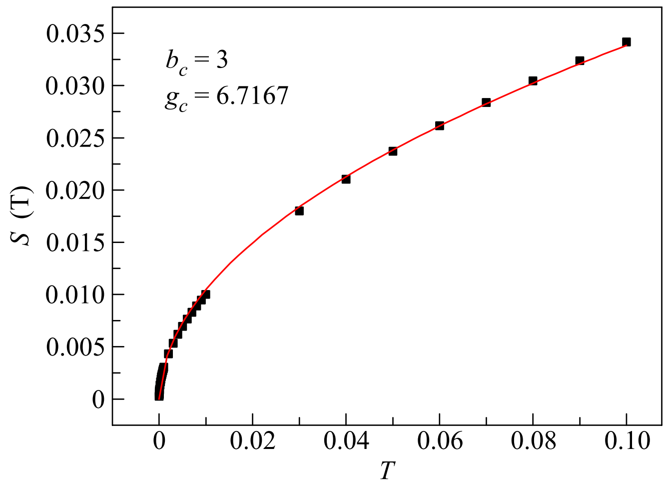

Fig.1. Calculated entropy $S(T)$ as a function of temperature. Continuous line represents $S(T) = c_1\sqrt{T}$, where c_1 is a parameter. Solid squares are the results of calculations based on functional (8)

FCQPT and is approximately constant [19]. Inserting into Eq. (10) the entropy $S(T) \propto \sqrt{T}$, we find that

$$\alpha(T) \simeq \frac{M^* T}{p_F^2 K} \propto \sqrt{T}. \qquad (11)$$

On the other hand, the specific heat

$$C(T) = T\frac{\partial S(T)}{\partial T} \propto \sqrt{T}. \qquad (12)$$

As a result, at $T \to 0$ the Grüneisen ratio $\Gamma(T)$ tends to some constant value rather than diverges as in the case when the electronic system is on the ordered side of FCQPT [20, 21]

$$\Gamma(T) = \alpha(T)/C(T) = \text{const}. \qquad (13)$$

Since the magnetic susceptibility $\chi(T) \propto M^*(T)$ and both the Sommerfeld coefficient $C(T)/T \propto M^*(T)$ and $\alpha(T)/T \propto M^*(T)$ we conclude that at $T \to 0$

$$\frac{C(T)}{T} \propto \chi(T) \propto \frac{\alpha(T)}{T} \propto M^*(T) \propto \frac{1}{\sqrt{T}}. \qquad (14)$$

At this point, we consider how Eq. (14) and the behavior of the effective mass given by Eq. (7) correspond to experimental observations obtained on $CePd_{1-x}Rh_x$. Measurements of the thermal expansion coefficient $\alpha(T)$ on $CePd_{1-x}Rh_x$ with $x = 0.87$ and $x = 0.90$ [3] are shown in Fig.2. It is seen that the approximation $\alpha(T) = c_1\sqrt{T}$ for composition $x = 0.90$ is in good agreement with facts over two orders of magnitude in the temperature range from 6 K down to 100 mK, and measurements on $CeNi_2Ge_2$ [7] and $CeIn_{3-x}Sn_x$ [5] demonstrate

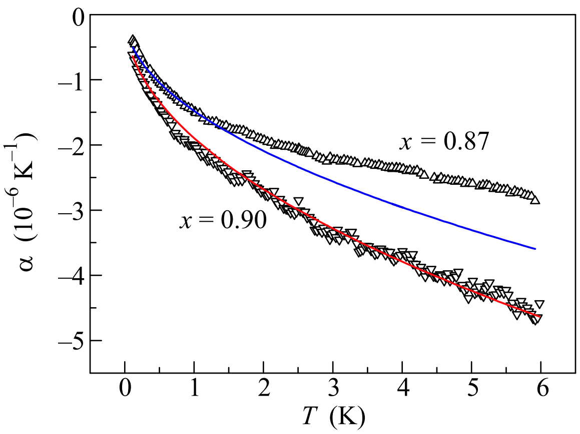

Fig.2. The thermal expansion coefficient $\alpha(T)$ as a function of temperature in the interval $100\,\text{mK} \le T \le 6\,\text{K}$. Continuous curves are fits for $x = 0.87$ and $x = 0.90$ data [3] based on Eq. (7) and represented by function $\alpha(T) = c_1\sqrt{T}$ with c_1 being a fitting parameter

the same behavior. While $CePd_{1-x}Rh_x$ is a three dimensional ferromagnet [3, 4], $CeNi_2Ge_2$ exhibits a paramagnetic ground state [7] and $CeIn_{3-x}Sn_x$ is antiferromagnetic cubic metal [5]. We conclude that the observed uniform behavior of the thermal expansion coefficient of these metals is determined by quasiparticles and FCQPT rather than by different magnetic quantum critical points and corresponding fluctuations. Measurements of the specific heat $C(T)$ on $CePd_{1-x}Rh_x$ with $x = 0.87$ and $x = 0.90$ show a power law T dependence. These are described by a $C(T)/T = AT^{-q}$ formula with the exponent $q \simeq 0.5 - 0.4$ and A is a constant, around that concentration $C(T)/T$ and $\chi(T)$ coincide in their T^{-q} temperature dependence [3, 4]. We conclude that the results given by Eq. (14) agree with facts.

Consider the case when the concentration x deviates from the critical value x_{FC} so that the system is moved to the disordered side of FCQPT and temperature T_F becomes finite. As a result, at $T \ll T_F$ the system exhibits the LFL behavior with the effective mass being approximately constant, $M^*(T) \simeq M^*(x) + a_1(T/T_F)^2$, where the term $a_1(T/T_F)^2$ represents correction to $M^*(x)$. If $x = x_{FC}$ then the application of magnetic field making T_F finite restores the LFL behavior with the effective mass depending on B as [22, 23]

$$M^*(B) \propto (B - B_{c0})^{-2/3}, \qquad (15)$$

where B_{c0} is the critical magnetic field which drives both a HF metal to its magnetic field tuned QCP and the corresponding Néel temperature toward $T = 0$. In some cases $B_{c0} = 0$, for example, the HF metal $CeRu_2Si_2$ is characterized by $B_{c0} = 0$ and shows neither evidence of the magnetic ordering, superconductivity nor the LFL behavior down to the lowest temperatures [24].

At $T \sim T_F$, the effective mass depends mainly on temperature [23, 18]

$$M^*(T) \propto T^{-2/3}. \qquad (16)$$

Then, at elevated temperatures $T \gg T_F$, the behavior of the effective mass is given by Eq. (7) [18]. Therefore at $T \lesssim T_F$ the behavior of the effective mass can be described by a simple function [25]

$$\frac{M^*(B,T)}{M^*(B)} \approx \frac{1 + c_2 y^2}{1 + c_3 y^{8/3}}, \qquad (17)$$

which represents an approximation to solutions of Eq. (1) that agrees with Eqs. (15) and (16). Here $y = (T/B - B_{c0})$, c_2 and c_3 are fitting parameters. Since the effective mass reaches its maximum value M^*_M at some $y = y_M$ [18, 23] we define a normalized effective mass as $M^*_N(T,B) = M^*(T,B)/M^*_M$. Taking into account Eq. (17) and introducing the variable $z = y/y_M$ we obtain the function

$$M^*_N(z) \approx \frac{1}{M^*_M}\frac{1 + c_2 z^2}{1 + c_3 z^{8/3}}, \qquad (18)$$

which describes a universal behavior of the effective mass $M^*_N(z)$. In the case of finite $M^*(x)$, Eq. (18) is valid at $T \sim T_F$ if $M^*(T,B)/M^*(x) \ll 1$ because the term $1/M^*(x)$ on the right hand side of Eq. (4) being a small correction to the effective mass can be omitted [26]. It is seen from Eq. (18) that $M^*_N(z)$ reaches its maximum value at $z = 1$, $M^*_N(z = 1) = 1$.

The effective mass $M^*(T,B)$ can be measured in experiments on HF metals. For example, as it follows from Eq. (14) $M^*(T,B) \propto C/T$ and $M^*(T,B) \propto \chi_{AC}(T)$ where $\chi_{AC}(T)$ is the magnetic susceptibility. If the corresponding measurements are carried out at fixed value of magnetic field B (or at fixed value of the concentration x and $B = 0$) then as it follows from Eq. (17) the effective mass reaches the maximum at some temperature T_M. Upon normalizing both the effective mass by its peak height at each field B and the temperature by T_M, we observe that all the curves should demonstrate a scaling and collapse on the single curve given by Eq. (18).

As shown in Fig.3, the behavior of the normalized susceptibility $\chi^N_{AC}(z) = \chi_{AC}(T/T_M,B)/\chi_{AC}(1,B) = M^*_N(z)$ obtained in measurements on the HF paramagnetic $CeRu_2Si_2$ [24] is in accord with the approximation given by Eq. (18). Since the crossover temperature T^* from the regime given by Eq. (16) to the regime given by Eq. (7) is proportional to the magnetic field, $T^* \propto B$ [18, 25], we expect that the temperature range over which the scaling takes place shrinks

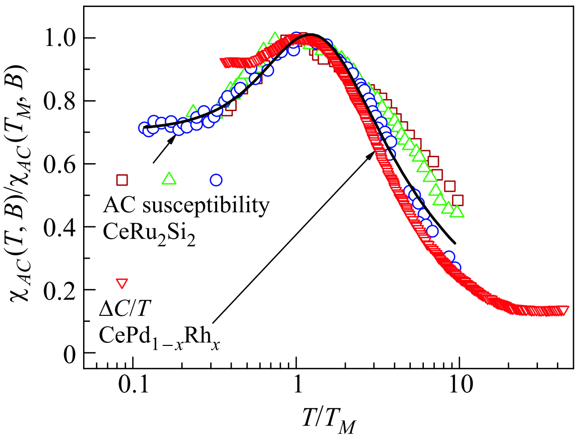

Fig.3. Normalized magnetic susceptibility $\chi_{AC}(T,B)/\chi_{AC}(T_M,B)$ for $CeRu_2Si_2$ in magnetic fields 0.20 mT (squares), 0.39 mT (upright triangles) and 0.94 mT (circles) is plotted against normalized temperature T/T_M [24]. The susceptibility reaches its maximum value $\chi_{AC}(T_M,B)$ at $T = T_M$. Normalized $4f$ electron contribution $(\Delta C(T)/T)/(\Delta C(T_M)/T_M)$ to the specific heat of $CePd_{1-x}Rh_x$ with $x = 0.80$ versus normalized temperature T/T_M is shown by downright triangles [4]. Here T_M is the temperature at the peak of $\Delta C(T)/T$. The solid curve traces the universal behavior of the normalized effective mass determined by Eq. (18)

when the applied magnetic field B is diminished. It is seen from Fig.3 that the deviation of the data corresponding to the smallest value of the magnetic field $B = 0.20\,\mathrm{mT}$ (shown by squares) is largest at the elevated normalized temperature, while the slope of this curve tends to that of curve described by Eq. (7). At small temperatures as seen from Fig.3, both the curve given by Eq. (18) and the effective mass determined by Eq. (15) agree perfectly with facts collected in measurements on $CeRu_2Si_2$ whose electronic system is placed at FCQPT [25]. As to the normalized $4f$ contribution $(\Delta C(T)/T)/(\Delta C(T_M)/T_M) = M^*_N(T)$ (shown by downright triangles in Fig.3) to the specific heat of $CePd_{1-x}Rh_x$ with $x = 0.80$ [4], the scaling takes place up to relatively large temperatures because the deflection of the $x = 0.8$ from the critical concentration $x_{FC} \simeq 0.9$ is big, while $T^* \propto B \propto |x_{FC} - x|$ [26]. As a result, at diminishing temperatures the scaling is ceased at relatively high temperatures as soon as the LFL behavior sets in.

In summary, we have shown that the NFL behavior of the thermal expansion coefficient $\alpha(T)$ observed in $CePd_{1-x}Rh_x$ at the critical concentration $x_c \simeq 0.9$ coincides with that of $\alpha(T)$ observed in both $CeNi_2Ge_2$ exhibiting the paramagnetic ground state and antiferromagnetic cubic HF metal $CeIn_{3-x}Sn_x$. We have also shown that the behavior of the normalized effective mass M^*_N observed in $CePd_{1-x}Rh_x$ at $x \simeq 0.8$ agrees with that of M^*_N observed under the application of magnetic field in paramagnetic $CeRu_2Si_2$ and concluded that these alloys exhibit the universal NFL thermodynamic behavior at its QCP. The outlined behavior is independent of the characteristic features of the given alloys while numerous CQPs assumed to be responsible for the NFL behavior of different HF metals can be substituted by the only QCP related to FCQPT.

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