Anomalous orthogonality catastrophe for Luttinger liquid with repulsion

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The orthogonality catastrophe for Luttinger liquid has been studied. Surprisingly, it was found that the backscattering from the external potential causes an enhancement of the infrared divergence for repulsive internal interaction and changes the singularity from the ordinary logarithmic to a power-law singularity.

Interest in the properties of Luttinger liquid has recently been renewed, mainly in response to the proposal of non-Fermi-liquid behavior of high-T_c superconductors[1] and in view of the development of microfabrication technology for inorganic one-dimensional semiconductor compounds ("quantum wires"). In particular, the photoemission,[2] the Fermi-edge singularities,[3] and the influence of the core hole dynamics on them have recently been studied for the Luttinger liquid.

As is well known, the singularities in the x-ray response of metals[4] and some other phenomena, e.g., the Kondo effect, have a common origin: the orthogonality catastrophe.[5] It is accordingly worthwhile to directly investigate the orthogonality catastrophe phenomenon for the Luttinger liquid. The Hamiltonian of the problem is

$$H = H_{\rm Lutt} + V,$$

$$H_{\rm Lutt} = \sum_k \{k(a_{1k}^+ a_{1k} - a_{2k}^+ a_{2k}) + U(k)\rho_1(k)\rho_2(-k)\}, \quad (1)$$

$$V = \theta(-t)e^{t/\tau}\sum_k \{V(k)[\rho_1(k)+\rho_2(k)] + V(2k_F+k)a_{1k}^+ a_{2k} + \text{h.c.}\},$$

where $a_{1,2}$ are the operators for the right and left fermions, $\rho_{1,2}$ are the corresponding density operators, $U(k)$ is internal interaction, and $V(k)$ is the external potential which is adiabatically turned on for a time τ. One must calculate the overlap $\langle 0|V\rangle$ between the wave functions at $T=-\infty$ and $t=0$, which correspond to the ground states with and without the external potential in the limit $\tau\to\infty$.

The interaction with the external potential [Eq. (1)] involves two processes: the forward scattering from the external potential governed by its zero-momentum Fourier component $V(0)$ and the backward scattering governed by $V(2k_F)$. Ignoring the backscattering, we obtain the Hamiltonian, quadratic in the density operators, which satisfies the boson algebra.[6] This problem can be trivially solved by means of a Bogolyubov transformation. In particular, the overlap integral is

$$\log|\langle 0|V\rangle_{f.s.}| = -(\delta_{\rm eff}/2\pi)^2 \log(\epsilon\tau), \quad (2)$$

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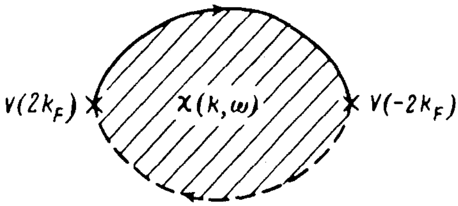

FIG. 1. Diagram for the overlap, which is in the second order in V and which contains all internal interactions in the polarization loop. The full (dashed) curve represents right (left) fermions.

where $\epsilon \sim \epsilon_F$ is a high energy cutoff, and δ_{eff} is the scattering phase which is modified by the internal interaction (due to the renormalization of the Fermi velocity and due to the strength of the potential): $\delta_{\text{eff}} = \delta_0/\sqrt{1-\alpha}; \delta_0 = V(0)/v_F, \ \alpha = U(0)/\pi v_F$.

Equation (2) can be easily understood. Physically, the orthogonality catastrophe is caused by the creation of electron-hole excitations with small energies in the process of adaptation of the Fermi surface to an external potential. Accordingly, the density of states of the electron-hole excitations, rather than the (single-particle) non-Fermi-liquid behavior itself, is relevant to the orthogonality catastrophe. In the absence of backscattering, only the creation of electron-hole excitations with a total momentum close to zero is allowed. These excitations are sound waves and they remain well-defined quasiparticles even if the internal interaction is taken into account. That is why the response of the Luttinger liquid to an external potential in this case is qualitatively the same as for the free fermion gas. The above consideration can be regarded as a simple physical background for the results obtained in Ref. 3, where the backscattering was ignored.

Let us now consider the backscattering effect which leads to the creation of electron-hole excitations with a total momentum close to $\pm 2k_F$. These excitations, in contrast with the sound waves, are extremely sensitive to the internal interaction. The model equation (1) is no longer exactly solvable. We must therefore expand the log of the overlap integral in the external potential (see Fig. 1 and Ref. 4, for example):

$$\log|\langle 0|V\rangle| = -\frac{1}{4\pi}\int \frac{d\omega}{\omega^2+(1/\tau)^2}\int \frac{dp}{2\pi}|V(p)|^2 \text{Im}\,\chi(p,\omega), \tag{3}$$

where $\chi(x,t) = -i\langle T\{\rho(x,t)\rho(0)\}\rangle$ is the exact correlation function of the total density. Near $p = \pm 2k_F$ it can be calculated either by the bosonization method or extracted from Ref. 7 using the relation $\text{Im}\,\chi = \text{sign}\,\omega\ \text{Im}\,\chi_R$, where the retarded function χ_R was found in Ref. 7. It turns out that

$$\text{Im}\,\chi(2k_F+p,\omega) \propto \theta(\omega^2-c^2p^2)(\omega^2-c^2p^2)^{-g},$$

where c is the renormalized Fermi velocity, and $g = 1 - \sqrt{(1-\alpha)/(1+\alpha)}$. This means that the density of states of these electron-hole excitations is proportional to ω^{1-2g}, but not to ω, as in the usual case.[4] Thus, the backscattering contributes an additional factor to the overlap integral:

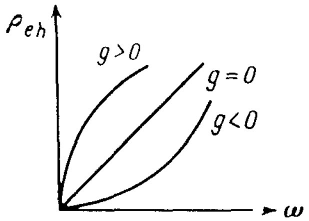

FIG. 2. The density of states, $\rho_{eh} \propto \Sigma_{p\sim 2k_F} \operatorname{Im} \chi$, of the electron-hole excitations relevant to the orthogonality catastrophe (for different internal interactions g).

$$\log|\langle 0|V\rangle_{b.s.}| = -\gamma |V(2k_F)/2\pi v_F|^2 (\epsilon\tau)^{2g}, \tag{4}$$

where the constant $\gamma = 2\sqrt{\pi}(c/\tau)^{2g}\Gamma(3/2-g)/\Gamma^2(1-g)$, and τ is the range of the potential $U(k)$.

For the repulsive interaction $(g>0)$ the orthogonality catastrophe not only survives but is even enhanced. The type of the singularity changes from a logarithmic to a power-law singularity. In previous studies such a change was not observed.[8] This is a special feature of internal correlations in the one-dimensional Fermi system and it is caused by the enhancement of the density of states of electron-hole excitations near $\pm 2k_F$ (Fig. 2). Note that only the backscattering is crucial in this problem. In contrast, for attraction $(g<0)$ the backscattering does not contribute to the orthogonality catastrope and it remains logarithmic due to the forward scattering processes [Eq. (2)]. Strictly speaking, Eq. (4), which is a perturbative result, makes sense as an intermediate asymptotic relation. It is a very difficult and amusing mathematical problem to compute the true limit[9] $\tau \to \infty$.

It is noteworthy that the function $\operatorname{Im} \chi$ is connected by the Kramers–Kronig relation with $\operatorname{Re} \chi$ and, therefore, the anomalous orthogonality catastrophe [Eq. (4)] is "dual" to the anomalous Peierls susceptibility, $\operatorname{Re} \chi(2k_F,\omega) \propto \omega^{-2g}$, found in Ref. 7.

In application of the orthogonality catastrophe to the problem of motion of a core hole the fermionic system feels the change of the potential: $\Delta V = V(x) - V(x-a)$ (the hole tunnels from the point $x=0$ to the point $x=a$). Since the zero Fourier component ΔV vanishes, only the backscattering is responsible for the orthogonality and the overlap integral is

$$\log|\langle 0|V\rangle| = -\gamma |V(2k_F)/2\pi v_F|^2 4\sin^2(k_F a)(\epsilon\tau)^{2g}.$$

It is interesting that for attraction $(g<0)$ the core hole does not feel the orthogonality at all. For repulsion $(g>0)$ the orthogonality is generally very strong, but it disappears for $a = \pi n/k_F$. It seem that the core hole prefers to delocalize within some self-consistent "Peierls drop." This problem will be discussed in more details elsewhere.[9]

The unusual power-law singularity in the overlap integral obtained above requires a reexamination of all the associated phenomena: the x-ray response, the motion of a core hole, and the Kondo effect.[9]

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