

QCD observables in $(e, e'p)$ scattering on nuclei in the color transparency regime

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It was found that a transverse size of the ejectile state emerging from the quasielastic $e+p$ scattering on nuclei does not decrease with the momentum transfer \mathbf{Q} . Nonetheless, the strength of nuclear attenuation vanishes with \mathbf{Q} , and the QCD mechanism of vanishing attenuation (color transparency) is identified. Strong correlation between the onset of color transparency regime and the asymptotic behavior of the charge form factors is found. Color transparency regime is elusive up to very large \mathbf{Q} .

The quasielastic $(e, e'p)$ scattering on nuclei is a widely discussed candidate reaction for observation of color transparency (CT)—vanishing final state interaction (FSI) at large momentum transfer \mathbf{Q} . The standard treatment of this reaction focuses on the assertion^{1,2} that the ejectile state is dominated by small size, $\rho \sim 1/\mathbf{Q}$, quark configurations, which have a small interaction cross section $\sigma(\rho) \propto \rho^2$ (Refs. 3 and 4) and, henceforth, weak FSI.

The subject of this paper is an approach to the CT regime. First, it is demonstrated that contrary to the reasoning of Refs. 1 and 2, the *true transverse size* of the ejectile state

$$|E\rangle = J_{em}|p\rangle = \sum_i |i\rangle \langle i| J_{em}|p\rangle = \sum_i F_{ip}(\mathbf{Q}) |i\rangle, \quad (1)$$

defined as $\rho_E^2 = \langle E| \rho^2 |E\rangle / \langle E|E\rangle$, does not decrease with \mathbf{Q}^2 . Second, we introduce the QCD observable $\Sigma_{ep} = \langle p| \hat{\sigma} |E\rangle / \langle p|E\rangle$, which quantifies the asymptotic strength of FSI, where $\hat{\sigma}$ is the diffraction scattering (cross section) operator. It is shown that, although $\rho_E^2 = \text{const}(\mathbf{Q})$, the strength of FSI Σ_{ep} does nonetheless vanish at large \mathbf{Q} , and the QCD mechanism for the vanishing of Σ_{ep} is identified.

This observable Σ_{ep} controls the approach of the nuclear transmission coefficient (nuclear transparency) Tr_A to the CT regime of $\text{Tr}_A = 1$:

$$\text{Tr}_A = \frac{d\sigma_A}{A d\sigma_N} \approx 1 - \Sigma_{ep} \frac{1}{2A} \int d^2\mathbf{b} T(b)^2. \quad (2)$$

Here \mathbf{b} is the impact parameter, $T(b) = \int dz n_A(\mathbf{b}, z)$ is the optical thickness of the nucleus, and $\int d^2\mathbf{b} T(b)^2 \propto A^2/R_A^2 \propto A^{4/3}$. Expansion (2) is valid at $1 - \text{Tr}_A \ll 1$, when FSI is weak and only the single rescattering of the ejectile is important. In the generic case Tr_A also contains the n -fold rescattering terms $\propto \langle p| \hat{\sigma}^n |E\rangle$.

In order to determine which aspect of QCD is tested by measuring Σ_{ep} , we start with the electron-pion scattering in the nonrelativistic quark model (NRQM), where [here $\mathbf{r}=(\vec{\rho},z)$ the $\vec{\rho}$ plane is normal to the momentum transfer \mathbf{Q}]

$$F_{em}(\mathbf{Q}) = \int d^2\vec{\rho} \int dz |\Psi(z, \vec{\rho})|^2 \exp\left(-\frac{i}{2} \mathbf{Q}z\right) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \varphi^*\left(\mathbf{k} + \frac{1}{2} \mathbf{Q}\right) \varphi(\mathbf{k}). \quad (3)$$

The proof of $\rho_E^2 = \text{const}(\mathbf{Q})$ is straightforward: In the NRQM $|E\rangle = \exp(\frac{i}{2} \mathbf{Q}z) |p\rangle$ and $\langle E|E\rangle = 1$. Since the interaction cross section $\sigma(\rho)$ and ρ^2 do not depend on z , we readily find $\langle E| \rho^2 |E\rangle = \langle p| \rho^2 |p\rangle$. Recall that by the nature of CT experiments the electroproduced ejectile state $|E\rangle$ is probed by its intranuclear FSI at very short proper time scales, when the intrinsic motion of quarks in nucleons can be neglected, i.e., the initial transverse separation of quarks $\vec{\rho}$ is obviously retained (notice a close similarity to Migdal's celebrated shakeup approximation introduced⁵ in 1939). Consequently, predictions^{6,7} of CT effects under the assumption that an ejectile is of vanishing initial size are erroneous.

Since $\hat{\sigma} \propto \rho^2$, the alternate measure of the strength of FSI is $\langle \rho^2 \rangle = \langle p| \rho^2 |E\rangle / \langle p|E\rangle$, which should not be confused with ρ_E^2 (the subsequent presentation follows, and supersedes, the author's preprint⁸)

$$\begin{aligned} \langle \rho^2 \rangle &= \frac{1}{F_{em}(\mathbf{Q})} \int d^2\vec{\rho} \rho^2 \int dz |\Psi(z, \vec{\rho})|^2 \exp(-i \frac{1}{2} \mathbf{Q}z) \\ &= \frac{1}{F_{em}(\mathbf{Q})} \int \frac{d^3\mathbf{k}}{(2\pi)^3} [\partial_{\mathbf{k}l} \varphi^*(\mathbf{k} + \frac{1}{2} \mathbf{Q})] [\partial_{\mathbf{k}l} \varphi(\mathbf{k})]. \end{aligned} \quad (4)$$

In the relativistic case of $\mathbf{Q}^2 \gg m^2$ one must use the Drell-Yan light-cone wave functions (LCWF). Assume that $p_z \rightarrow \infty$ and that \mathbf{Q} runs along the y axis. Then ρ_x is the transverse size which does not change under the Lorentz transformation from the light-cone frame in which the momentum transfer \mathbf{Q} is strictly transverse to the laboratory frame in which \mathbf{Q} is strictly longitudinal, so that one must compute $\langle \rho^2 \rangle = 2\langle \rho_x^2 \rangle = 2\langle \partial_x^2 \rangle$:

$$\begin{aligned} F_{em}(\mathbf{Q}) &= \int \frac{d^2k}{2\pi} \frac{dx}{x(1-x)} \varphi^*(M_f^2) \varphi(M_i^2) \\ &= \int_0^1 dx \int d^2\vec{\rho} \exp[-i(1-x)\vec{\rho}\mathbf{Q}] |\Psi(x, \vec{\rho})|^2 \end{aligned} \quad (5)$$

$$\langle \rho_x^2 \rangle = \frac{1}{F_{em}(\mathbf{Q})} \int \frac{d^2k}{2\pi} \frac{dx}{x(1-x)} \Psi^{*\prime}(M_f^2) \Psi'(M_i^2) \left[\frac{2k_x}{x(1-x)} \right]^2, \quad (6)$$

where x is a fraction of the (light-cone) momentum of the pion carried by the struck quark, and in (5) the invariant variables⁹ of LCWF are $M_i^2 = (m_q^2 + k^2)/x(1-x)$ and $M_f^2 = [m_q^2 + (\mathbf{k} + (1-x)\mathbf{Q})^2]/x(1-x)$. Evidently, the ρ_E^2 retention property holds also in the relativistic case.

Consider now the \mathbf{Q} dependence of the strength of FSI. With the Gaussian ansatz for the NRQM wave function the $d^2\vec{\rho}$ and dz integrations in Eqs. (3) and (4) fac-

torize, so that $\langle \rho^2 \rangle = R^2$ and does not depend on Q^2 . With the Gaussian LCWF $\varphi(M^2) = \varphi_0 \exp(-\frac{1}{2} R^2 M^2)$ the form factor (5) is dominated by the end-point contribution from

$$1 - x \sim 2m_q / Q, \quad (7)$$

and from (6) we find $\langle \rho^2 \rangle \sim \langle R^2 / (1-x) \rangle \propto R^2 Q / m_q$. The striking feature is that the CT law $\sigma(\rho) \propto \rho^2$ does not guarantee the vanishing of FSI, and neither Q -independent strength of FSI nor one that increases with Q contradicts any general principles. Indeed, in the expansion over the intermediate states

$$\langle p | \hat{\sigma} | E \rangle = \sum_i \langle p | \hat{\sigma} | i \rangle \langle i | J_{em}(Q) | p \rangle \quad (8)$$

the matrix elements of the diffractive operator $\langle p | \hat{\sigma} | i \rangle$ do not depend on the energy and/or on Q^2 . Furthermore, the larger is Q^2 the heavier are the electroproduced intermediate states $|i\rangle$, which have a progressively increasing radius and interaction cross section.

We shall now demonstrate that QCD as a theory of strong interactions *predicts*, nonetheless, that Σ_{ep} *vanishes* at large Q^2 . Besides, the law $\sigma(\rho) \propto \rho^2$, the origin of vanishing Σ_{ep} is the one-gluon exchange Coulomb interaction between the constituent (anti)quarks of the hadron, which is an indispensable property of QCD. We shall illustrate the basic idea of the proof starting with the Schrödinger equation in the momentum representation:

$$\varphi(Q) = \frac{1}{\varepsilon - Q^2/2m} \int \frac{d^3k}{(2\pi)^3} V(Q-k) \varphi(k). \quad (9)$$

The confining potential is the dominant one, and the short-range Coulomb interaction can be treated as weak perturbation. The dominant (confining) part of $\varphi(k)$ will then be a steep function of k^2 . On the other hand, the asymptotics of the $V(Q-k)$ at large Q is dominated by the QCD one-gluon exchange: $V(Q-k) \propto 1/(Q-k)^2$ [the logarithmic $\alpha_S(Q)$ factor is not significant for our purposes]. As a result, asymptotically $\varphi(Q) \propto 1/Q^4$, and with this Coulomb tail the asymptotics of the form factor (FF) will be dominated by contributions when one of the wave functions in Eqs. (3) and (4) will be in the Gaussian-like (confining) regime and the other will be in the Coulomb tail regime. In (3) we can then ignore k compared with Q in $\varphi(k + \frac{1}{2}Q)$, factor out $\varphi(\frac{1}{2}Q)$, and find the "QCD power-law" asymptotics $F_{em}(Q) \propto \varphi(\frac{1}{2}Q) \propto 1/Q^4$ (in the above analysis I have followed Ref. 10, and in the relativistic case $V(Q)$ should be substituted for the relativistic $\sim 180^\circ$ Coulomb scattering amplitude, which leads to $\varphi(M^2) \propto 1/M^2$). In the same "QCD dominated" regime, the ∂_{k_i} differentiations in (4) lead to the extra factor $\sim R^2 k^2 / Q^2$ in the integrand, and we indeed find $\langle \rho^2 \rangle \propto 1/Q^2$. It can thus be concluded that the vanishing strength of FSI in quasielastic ($e, e'p$) scattering and the power-law asymptotics of the electromagnetic FF¹⁰⁻¹² originate from exactly the same short-range QCD interaction in hadrons. In terms of expansions (1) and (8), the origin of vanishing FSI is not in the vanishing size ρ_E of the ejectile, rather weak FSI emerges after projecting the ejectile state onto the final state proton, and comes from cancellations between the diagonal, $|i\rangle = |p\rangle$, and off-diagonal, $|i\rangle \neq |p\rangle$, intranuclear rescatterings (see also the CT sum rule¹³).

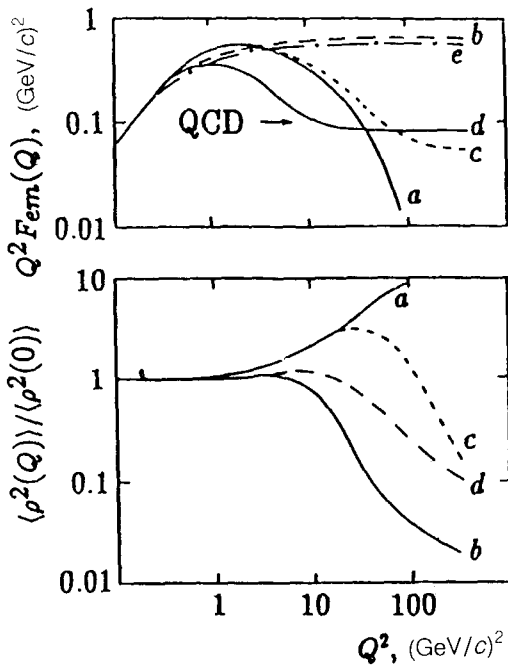


FIG. 1. Correlation of the asymptotic behavior of the electromagnetic form factor with the strength of the final-state interaction: top— $Q^2 F_{em}(Q)$, bottom—the strength of the final-state interaction measured by $\langle \rho^2(Q) \rangle / \langle \rho^2(0) \rangle$ vs Q^2 . The order of magnitude of the asymptotic normalization suggested by perturbative QCD is indicated by the arrow. The curves correspond to $[m_q = 150 \text{ (MeV/c)}^2]$: (a) the Gaussian wave function; (b) the Gaussian+Coulomb wave function, $m = 150 \text{ (MeV/c)}^2$, $\alpha = 0.1$; (c) the Gaussian+Coulomb wave function, $m = 150 \text{ (MeV/c)}^2$, $\alpha = 0.01$; (d) the Gaussian+Coulomb wave function, $m_q = 300 \text{ (MeV/c)}^2$, $\alpha = 0.01$; (e) the monopole form factor.

The most important effect of the one-gluon exchange QCD interaction in the relativistic case is that it eliminates a dominance of the end-point [Eq. (7)] contribution to the FF. The FF are dominated by finite, weakly Q -dependent values of $1-x$. Nevertheless, the presence of extra $x^2(1-x)^2$ in the denominator of the integrand of (6) enhances the sensitivity of the strength of FSI to the end-point contribution. Consequently, the onset of vanishing Σ_{ep} is slower than the onset of the power asymptotics of the FF.

Figure 1 shows the results from the light-cone toy model of the scalar pion with the scalar quarks, which incorporates the principal features of the QCD wave function:

$$\varphi(M^2) \propto \exp\left(-\frac{1}{2} R^2 M^2\right) + \alpha \frac{1}{(1 + R^2 M^2/2)^n}. \quad (10)$$

For our purpose, the presence of the $\alpha_S(M^2)^n$ factor in the Coulomb correction is not essential. To the first order in the Coulomb correction the $\propto 1/Q^2$ behavior of the pion FF corresponds to $n=1$.

Experimentally, the pion FF follows the ρ^0 -pole formula $F_{em}(\mathbf{Q})=1/(1+\mathbf{Q}^2/m_\rho^2)$ up to $^{14} \mathbf{Q}^2 \sim 10 \text{ (GeV}/c)^2$. Therefore, in all cases R^2 is adjusted to $\langle R_{ch}^2 \rangle = 6/m_\rho^2$. The Coulomb-admixture parameter α controls the large- \mathbf{Q} normalization $\Lambda^2 = \mathbf{Q}^2 F_\pi(\mathbf{Q})$, changing from $\Lambda^2 \rightarrow 0$ for the Gaussian LCWF to $\Lambda^2 \approx m_\rho^2$ for the ρ^0 -dominated monopole FF. [Since neither the running QCD coupling nor the higher-order QCD effects like the Sudakov¹⁵ FF (see the text below) are included, our toy model should not be extended to very large \mathbf{Q}^2 .] As a case in between we consider LCWF's, giving $\Lambda^2 \sim 8\pi\alpha_S f_\pi^2 \sim 0.1 \text{ (GeV}/c)^2$, as suggested by the perturbative¹² QCD. In the problem of interest it is natural to use the effective mass of quarks, m_q , somewhere in between the spectroscopic constituent mass of $\sim 300 \text{ MeV}/c^2$ and the current mass $\sim 10 \text{ MeV}/c^2$. Here we use $m_q = 150 \text{ MeV}/c^2$.

The principal results are:

1. With the purely Gaussian LCWF, $\alpha=0$, $R=1.90 \text{ (GeV}/c)^{-1}$, the FF $F_{em}(\mathbf{Q}) \propto \exp(-R^2 m_q \mathbf{Q})/\mathbf{Q}^3$ follows closely the monopole FF up to $\mathbf{Q}^2 \sim 5 \text{ (GeV}/c)^2$ and increases the strength of FSI.
2. With $\alpha=0.1$, $R=2.23 \text{ (GeV}/c)^{-1}$ we reproduce the monopole FF. Notice that even in this case $\alpha \ll 1$. The strength of FSI remains approximately constant up to $\mathbf{Q}^2 \sim 10-20 \text{ (GeV}/c)^2$, and then decreases rapidly with \mathbf{Q}^2 .
3. The FF with $\alpha=0.01$, $R=1.95 \text{ (GeV}/c)^{-1}$ follows the monopole FF up to $\mathbf{Q}^2 \sim 10 \text{ (GeV}/c)^2$ and has the large- \mathbf{Q} normalization close to the QCD prediction. The strength of FSI first increases to a saturation at $\mathbf{Q}^2 \sim 30-40 \text{ (GeV}/c)^2$ and then abruptly decreases beyond $\sim 100 \text{ (GeV}/c)^2$.
4. With the choice of $m_q = 300 \text{ MeV}/c^2$ [$\alpha=0.01$ and $R=2.72 \text{ (GeV}/c)^{-1}$] the strength of FSI remains approximately constant up to $\mathbf{Q}^2 \sim 10 \text{ (GeV}/c)^2$ and then begins to decrease. However, the large- \mathbf{Q} normalization of this FF decreases to the perturbative QCD value too rapidly, in conflict with the experiment of Ref. 14.

We conclude that the steeper is a decrease of the FF and the smaller is the large- \mathbf{Q} normalization, the slower is the onset of CT regime. With realistic LCWF models, FSI might remain strong up to very large \mathbf{Q}^2 . An interesting observation is that the strength of FSI is very sensitive to the quark structure of the hadron. The QCD asymptotics $\Sigma_{ep} \propto 1/\mathbf{Q}^2$ should be even more elusive in the electron-proton scattering.¹⁶

Besides the short-range, one-gluon exchange interaction between (anti)quarks, there is yet another QCD mechanism which filters the small-size components of hadrons in the ep scattering: the Sudakov¹⁵ FF. In the case of the elastic scattering one requires no radiation of gluons. For the quark-antiquark system of size $\vec{\rho}$, struck by an electron with momentum transfer \mathbf{Q} the mean number of probable radiated gluons N_g , which would take a fraction x_g of the hadrons momentum $x_{\min} < x_g < 1$, is¹⁷

$$N_g(\rho, \mathbf{Q}^2, x_{\min}) \approx \frac{16}{3} \int_{x_{\min}}^1 \frac{dx}{x} \int_0^{\mathbf{Q}^2} \frac{d^2 \mathbf{k}}{2\pi} \frac{\alpha_S(\mathbf{k}^2)}{2\pi} \frac{1 - \exp(-i\mathbf{k}\vec{\rho})}{\mathbf{k}^2}. \quad (11)$$

The probability of no-radiation, i.e., the Sudakov FF, can then be estimated as (for more rigorous derivation see Ref. 18) $F_S(\rho, \mathbf{Q}, x_{\min}) \approx \exp[-N_g(\rho, \mathbf{Q}^2, x_{\min})]$ and the Sudakov-modified charge FF (5) takes the form

$$F_{em}(\mathbf{Q}) = \int_0^1 dx \int d^2\vec{\rho} \exp[-i(1-x)\vec{\rho}\mathbf{Q}] |\Psi(x, \vec{\rho})|^2 F_S(\rho, \mathbf{Q}, 1-x). \quad (12)$$

The relative significance of the Sudakov suppression of the large ρ contribution depends on the LCWF. With the Gaussian LCWF we have the end-point [Eq. (7)] dominance, so that $2 \log(1/x_{\min}) = \log(\mathbf{Q}^2/m^2)$ and the Sudakov FF takes on the standard form

$$F_S(\rho, \mathbf{Q}, x_{\min}) \approx \exp\left\{-\frac{8}{27} \log\left(\frac{\mathbf{Q}^2}{m_q^2}\right) \log\left[\frac{\alpha_S(1/\rho^2)}{\alpha_S(\mathbf{Q}^2)}\right]\right\}. \quad (13)$$

Equation (13) holds before the onset of the Coulomb dominance regime, when x_{\min} is small but finite and depends only slightly on \mathbf{Q} . In this case the Sudakov suppression is a very slow function of \mathbf{Q} and of the size ρ and its effect on the rate of vanishing of Σ_{ep} is marginal (for the recent discussion of the Sudakov effects see also Ref. 19).

Conclusions: We have shown that the onset of weak FSI is closely related to the onset of the power-law asymptotics of electromagnetic FF. The smaller is the large- \mathbf{Q} normalization $\mathbf{Q}^2 F_{em}(\mathbf{Q}^2)$, the larger \mathbf{Q}^2 is needed for the onset of color transparency. The end-point contribution is more significant in the strength of FSI than in the FF. With realistic LCWF's we found the onset of the CT regime to be very late, which is consistent with the preliminary results of the NE-18 experiment at SLAC.²⁰

We have focused above on the asymptotic \mathbf{Q}^2 , assuming always a complete set of intermediate excited states $|i\rangle$. At moderately large \mathbf{Q}^2 , there is yet another reason for the elusive color transparency regime: Only finite number of intermediate states can be electroproduced and can contribute to the expansion (8) at a finite energy.¹³

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