Many-electron dimples on a thick helium film

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A theory of a many-electron dimple on a thick helium film of thickness d is derived. The stability diagram is found. There is a sharp deviation from a monotonic behavior on the plot of the total dimple energy W versus d as the latter varies monotonically near the minimum of the stability diagram. The results of the calculations are used to interpret some experiments by Andrei [Phys. Rev. Lett. **52**, 1449 (1984)].

In some interesting experiments on the mobility of electrons on a thick$^{1)}$ helium film on a sapphire substrate, Andrei[1] observed a sharp negative spike in the mobility as the thickness of the helium film was reduced monotonically. Among various possible explanations for this effect, Andrei[1] rejected as inapplicable the hypothesis that many-electron dimples arise on the helium film. The main argument was based on calculations by Peeters[2] of the minimum critical electron density n_k on a helium film. According to Peeters,[2] the minimum density n_k for a film on a metal substrate is $n_k \simeq 10^8$ cm^{-2}. If the substrate instead has a dielectric constant $\epsilon = 10$, this quantity increases to $n_k \simeq 6.5 \times 10^8$ cm^{-2}. The experiments of Ref. 1 were carried out under the conditions $n \sim 10^8$ cm^{-2} and $\epsilon \simeq 19$, according to Ref. 2, those conditions guaranteed the stability of the charged helium film.

However, the results of Ref. 2 need to be corrected. In the first place, a finite dielectric constant ($\epsilon = 10$ instead of infinity) cannot change the minimum critical electron density by nearly an order of magnitude. The combination $(\epsilon^2 - 1)/\epsilon^2$ which appears in the definition of n_k in Ref. 2 differs from one by 1% at $\epsilon = 10$. The change in n_k from the "metallic" limit should be of the same scale. Second, the calculations of Ref. 2 ignored the influence of retardation effects on the strength of the van der Waals forces, which control the position of the minimum for the critical electron density on the helium film. As we will show below, retardation effects lower the stability of the film.

It is thus worthwhile to examine the behavior of a many-electron dimple on a thick helium film. This problem, which has essentially not been solved before now, is interesting in its own right, and it is probably pertinent to the experiments of Ref. 1.

We assume a dimple on a helium film of thickness d above a massive metal body (the sapphire in the experiments of Ref. 1, with a dielectric constant $\epsilon = 18$, is essentially equivalent to a metal for our purposes). A finite film thickness d affects the parameters of a many-electron dimple by three mechanisms. First, there is a change in the effective compressing field E:

$$E = E_\perp + F_d/e, \quad F_d = \frac{e^2}{4d^2}, \tag{1}$$

where $E_\perp$ is the external electric field, which pushes electrons toward the film surface, and F_d is the image force which is set up by the interactions of electrons with the substrate in the limit $\epsilon \rightarrow \infty$.

Second, the film thickness d appears in the definition of the Coulomb energy of a dimple. While this energy is given by[3]

$$V_c = c_0 \frac{Q^2}{R}, \quad Q = eN, \quad c_0 = 15.73 \tag{2}$$

above massive helium, where R is the radius of the charged spot at the center of the dimple, and Q is the total charge of the dimple, it should have the following structure on a helium film with $d/R \ll 1$:

$$V_c = 2dQ^2/R^2. \tag{3}$$

This expression corresponds to the energy of a plane capacitor.

An interpolation of the asymptotic expressions for V_c in (2) and (3) leads to the following general expression for the energy V_c, which incorporates the role of the parameter d/R in a qualitatively correct way over a wide range of this parameter:

$$V_c = \frac{c_0 Q^2}{R\left(1 + \frac{c_1 R}{d}\right)}, \quad c_1 = \frac{1}{2} c_0 . \tag{4}$$

Third, the definition of the effective capillary constant $\tilde{\kappa}$ changes. The details of this modification can be seen by examining the definition of $\tilde{\kappa}$ in (5a).

We can accordingly write the following expression for the total energy W of a many-electron dimple on a helium film, which contains an adjustable radius R, whose value is found from the minimum of W at fixed values of N, $E_\perp$, and d:

$$W = \frac{Q^2 E^2}{8\pi\alpha} \exp\left(\frac{\tilde{\kappa}^2 R^2}{2}\right) Ei\left(-\frac{\tilde{\kappa}^2 R^2}{2}\right) + V_c(Rd) \tag{5}$$

$$\tilde{\kappa}^2 = \rho\tilde{g}/\alpha, \quad \tilde{g} = g + \frac{3f}{\eta\rho d^4} \frac{1}{1 + \frac{d}{d_*}} . \tag{5a}$$

We have written the energy in this form by analogy with its definition in the theory of a many-electron dimple on massive helium,[3] taking into account modifications (1) and (4). Here $\tilde{\kappa}$ is the effective capillary constant, ρ and α are the density and surface tension of liquid helium, f is the van der Waals energy, which has a scale value $f \sim 60$ K for the (liquid helium)-sapphire pair, and d_* is an effective length which arises in the theory of van der Waals forces when retardation effects are taken into account. The calculations of Ref. 2 correspond to $d_* \rightarrow \infty$. The quantity $Ei(x)$ is the integral exponential function; $V_c(R,d)$ is taken from (4), and E from (1).

An energy of the type in (5) was discussed by Degani and Hipolito.[4] However, they ignored the effect of the parameter d/R on the Coulomb energy of the dimple; i.e.,

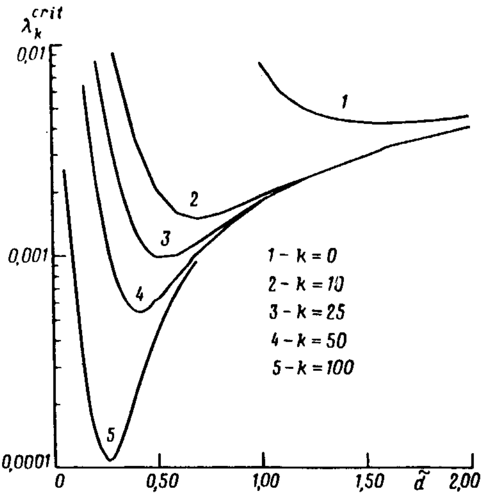

FIG. 1. Diagram for a many-electron dimple on a helium film. The definitions of the λ_k^{crit} and $\tilde{d}$ axes and of the parameter k are given in the text [see (6), (7), and the related discussion in the text]. The dimples are stable above each of curves 1–5.

they used V_c from (2) instead of V_c from (4). For this reason, the results of Ref. 4 for many-electron dimples on a film of liquid helium are incorrect in the most interesting region, $d/R<1$.

Introducing a dimensionless electric field γ and a dimensionless film thickness $\tilde{d}$,

$$E_1^2=\lambda\kappa_0\alpha,\quad \kappa_0^2=\rho g/\alpha,\quad \tilde{d}=d/d_0 \tag{6}$$

$$d_0^4=3f/\rho g\simeq 10^{-4}\ \text{cm},$$

we first find the stability diagram for the many-electron dimple, using the condition

$$W(\lambda_k^{\text{crit}})\simeq 0. \tag{7}$$

Figure 1 shows a corresponding plot of $\lambda_k^{\text{crit}}(\tilde{d})$ for various values of the parameter $k=d^*/d_0$. We see that the value of k has a significant effect on the diagram. The case $k=0$ corresponds to the case in which retardation effects are ignored. It is also worth mentioning that the minimum values of λ_k^{crit} are smaller than the typical compressing fields used in Ref. 1. In the dimensionless units of (6), the fields of Ref. 1 have a scale value $\lambda\simeq 10^{-2}$.

The next plot shows the dimple energy W versus the dimensionless film thickness at a fixed value of λ near the experimental values of Ref. 1, for various values of the number of electrons in the dimple, N. The value of N was optimized by requiring that the energy be at a scale higher than the experimental temperature, $\simeq 1$ K. Significantly, the position of the minimum of the curve of $W^{\text{opt}}(\tilde{d})$ is insensitive to the value of N. This result means that many-electron dimples with various values of N, which

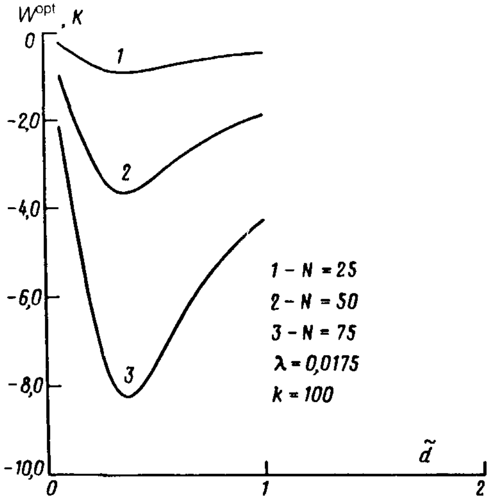

FIG. 2. The total energy W in (5) as a function of $\tilde{d}$ in (6) for various values of N. The value of N was optimized by the requirement $W(N) < \sim 10$ K.

arise when the film loses its stability, have a maximum binding energy at a value of $\tilde{d}$, the same for all N, which is approximately equal to that measured in Ref. 1.

Figure 3 shows that the energy W is a strong function of the amplitude of the compressing field; this fact was also noted in the experiments of Ref. 1.

In summary, we reach the conclusion that there may be a relationship between the properties of many-electron dimples on a thick helium film and the dip in the

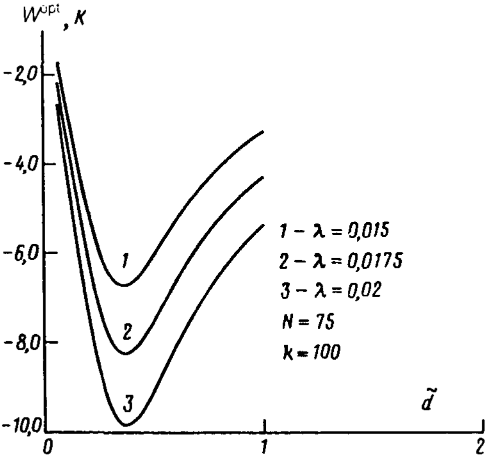

FIG. 3. Plot of $W(\tilde{d})$ for various values of λ near the values of this parameter which were used in Ref. 1.

electron mobility observed in Ref. 1 during passage through the region 10^{-5} cm.

[1)] A "thick" film here is one whose thickness d is in the neighborhood $d \sim d_{\min}$, where $d_{\min}$ is a characteristic thickness corresponding to a minimum stability of the charged helium film.

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[3] V. B. Shikin and Yu. P. Monarkha, *Two-Dimensional Charged Systems in Helium*, Nauka, Moscow, 1989, p. 83.
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Translated by D. Parsons