Electron states in a planar 2D ferromagnet: combined spin-orbit symmetry and half-integer orbital angular momentum of an electron

R. R. Ramazashvili, Jr.
L. D. Landau Institute of Theoretical Physics, Russian Academy of Sciences, 117940, Moscow

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The electron states in a planar 2D ferromagnet with a spatially nonuniform distribution of the order parameter are analyzed. When a magnetic field is imposed on the system, electron states with a half-integer orbital angular momentum arise near topological defects (vortices). The shape and temperature dependence of the ESR intensity in the vortex states are discussed.

Let us examine the electron states in a ferromagnet with an easy-plane order. We consider states which arise near topological excitations: vortices[1] in the field of directions of the magnetic moment. Since the energy of a vortex filament is proportional to its length, this effect should be observed in thin ferromagnetic films (with a thickness of a few monolayers) if the temperature is low. It is thus legitimate to restrict the analysis to the 2D case.

The Hamiltonian of an electron in the nonuniform field of ferromagnetically ordered spins is

$$H=\frac{p^2}{2m}-\alpha[\vec{\sigma}\mathbf{n}(\mathbf{r})], \tag{1}$$

where the two-component unit vector $\mathbf{n}(\mathbf{r})$ describes the local direction of the magnetic moment.

A vortex configuration with a topological charge Q is described by the field

$$\mathbf{n}(\mathbf{r})=(\cos Q\varphi,\sin Q\varphi,0), \tag{2}$$

where φ is the polar angle in the plane of motion of the electron.

The Hamiltonian of an electron in the field of a vortex is symmetric under the combination of an orbital rotation through an angle $\delta\varphi$ and a spin rotation through an angle $-Q\delta\varphi$ around the $\hat{z}$ axis. The corresponding symmetry operator is the modified orbital angular momentum.

$$L_z=-i\partial_\varphi-\frac{Q}{2}\sigma_z,$$

which takes on half-integer eigenvalues if Q is odd.

The imposition of a magnetic field causes the ferromagnetic moments to rotate along the field. If there is a strong easy-plane anisotropy, however, the rotation angle is small, and a vortex is not destroyed. In the case of a quadratic anisotropy the energy of the ferromagnetic moment in a magnetic field directed along the anisotropy axis, $\hat{z}$, is described by

$$E=\frac{1}{2}\beta n_z^2-\gamma H n_z .$$

The equilibrium value $n_z=\gamma H/\beta$ is small if the magnetic field is not too strong.

Let us consider a state at a vortex with a topological charge Q. The rotation of the ferromagnetic moments does not disrupt the symmetry of the electron Hamiltonian under the combined spin-orbit rotation. In the axial gauge, the states in a magnetic field are characterized by a radial quantum number and the projection of the orbital angular momentum onto the normal to the plane.[2] According to (2), the matrix elements of the vortex texture are diagonal in the radial quantum number, and they are zero except for transitions involving a change in the orbital angular momentum by an amount Q. Hamiltonian (1) then transforms into a matrix of second order in the $\hat{z}$ projection of the combined orbital angular momentum, and it can be diagonalized easily:

$$\epsilon=\frac{\epsilon_l+\epsilon_{l-Q}}{2}\pm\sqrt{\alpha^2(1-n_z^2)+\left\{\alpha n_z+\mu_B H+\frac{\epsilon_{l-Q}-\epsilon_l}{2}\right\}^2} . \tag{3}$$

Here

$$\epsilon_l=\hbar\omega_c\left(n+\frac{|l|-l}{2}+\frac{1}{2}\right)$$

is the energy of the state with radial quantum number n and orbital angular momentum l, μ_B is the Bohr magneton, H is the magnetic field, and ω_c is the cyclotron frequency. According to (3), the Zeeman splitting of a state at a vortex is

$$\Delta\omega=2\sqrt{\alpha^2(1-n_z^2)+\{\alpha n_z+\mu_B H+Q\omega_c/2\}^2} , \tag{4}$$

while that for a state far from a vortex is

$$\Delta\omega=2\sqrt{\alpha^2(1-n_z^2)+\{\alpha n_z+\mu_B H\}^2} . \tag{5}$$

According to (4) and (5), the vortex states could have a noticeable effect only at values of α small in comparison with the magnetic field. However, the ESR peak from vortex states must be greatly broadened, since at low temperatures there are only "molecules," i.e., combinations of vortices with opposite topological charges.[1] A bound state with a half-integer orbital angular momentum may exist at one vortex of the molecule if this molecule is larger than the magnetic length.

$$l_H=\sqrt{\hbar c/eH} ,$$

which determines the scale radius of the electron wave function.

The combining of vortices into molecules leads to a splitting of spectrum (3) by virtue of the exchange interaction. In a molecule of size R, the splitting is given in order of magnitude by

$$\delta E \sim \hbar\omega_c \exp\left\{ -\left(\frac{R}{l_H}\right)^2 \right\}, \tag{6}$$

so the ESR peak from vortex states is broadened by an amount on the order of ω_c, because of vortex molecules with sizes on the order of the magnetic length. As the temperature is lowered, the number of vortex pairs falls off exponentially, so the intensity of the peak due to vortex pairs should also fall off exponentially.

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[1] V. L. Berezinskiĭ, Zh. Eksp. Teor. Fiz. **59**, 907 (1970) [Sov. Phys. JETP **32**, 493 (1970)]; J. Kosterlitz and D. Thouless, J. Phys. C **6**, 1181 (1973); J. Kosterlitz, J. Phys. C **7**, 1046 (1974).
[2] L. D. Landau and E. M. Lifshitz, *Quantum Mechanics: Non-Relativistic Theory*, Pergamon, New York, 1977.

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