Coherent resonances in the spectra of e^+–e^- pairs created by hard γ rays in aligned single crystals

Yu. V. Kononets and I. S. Tupitsyn
Research Science Center "Kurchatov Institute," 123182, Moscow, Russia

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New pronounced resonant-type coherent effects are predicted theoretically in e^+–e^- pair production spectra under conditions of transition from the axial channeling regime to a planar regime.

1. The space-energy density of e^+–e^- pairs created per unit time by a nonpolarized beam of γ rays in an averaged potential of atomic chains in a single crystal aligned near an axial direction is expressed in the quasi-classical Baier–Katkov method[1] as an integral in time of a definite function which depends on the transverse (relative to the singled-out axial family) classical coordinates $r_\perp^\pm$ and velocities $v_\perp^\pm$ of one of the constituents [a positron (+) or an electron (−)] of the formed pair:

$$\frac{d^2N_p}{d\xi_\pm dS_\pm}=\frac{e^2m^2c^3}{S_\perp^0\,\pi\hbar^3\omega}\left\{\int_0^\infty\left[\frac{\xi_+^2+\xi_-^2}{4\xi_+\xi_-}\frac{\gamma_\pm^2}{c^2}\,(v_\perp^\pm(\tau)-v_\perp^\pm(-\tau))^2-1\right]\frac{\sin f_\pm(\tau)}{\tau}\,d\tau+\frac{\pi}{2}\right\}, \tag{1}$$

$$f_\pm(\tau)=\frac{c}{l_c(\xi_\pm,\omega)}\left\{2\tau+\frac{\gamma_\pm^2}{c^2}\left[\int_{-\tau}^{\tau}[v_\perp^\pm(\tau')]^2d\tau'-\frac{1}{2\tau}\,[r_\perp^\pm(\tau)-r_\perp^\pm(-\tau)]^2\right]\right\}. \tag{2}$$

Here $\xi_\pm=\epsilon_\pm/\hbar\omega$ is the ratio of the pair-particle energy to the γ-ray energy, $\gamma_\pm=\epsilon_\pm/mc^2$, $S_\perp^0$ is the unit-cell area for the two-dimensional lattice corresponding to the crystal axis family under consideration, l_c is the coherence length for the pair-production process:

$$l_c(\xi_\pm,\omega)=2\xi_+\xi_-\lambda\!\!\!^{-}_c\hbar\omega/mc^2. \tag{3}$$

The electronic charge, rest mass, Compton wavelength, speed of light, and Planck's constant have standard notation.

Dependence of the pair-distribution density (1) on the coordinate $r_\perp$ of the "creation act" and on the γ-ray incidence angle is determined by the initial conditions for the coordinates and velocities in Eqs. (1) and (2):

$$r_\perp^\pm(0)=r_\perp\,,\quad v_\perp^\pm(0)=\cos k_\perp/k,\quad k_\perp=k\sin\theta, \tag{4}$$

where k is the γ-ray momentum. The integral in Eq. (1) over the two-dimensional unit-cell area ($dS_\perp=d^2r_\perp$) gives the spectrum $dN_p/d\xi_\pm$ of e^+–e^- pairs created by one incident γ ray per unit time.

2. Simple quantitative evaluations can be obtained for the pair-production rate in the field of a separate atomic chain in the region of fairly large angles θ, where the

integral in τ of the expression proportional to $[v_\perp^\pm(\tau)-v_\perp^\pm(-\tau)]^2$ gives the main contribution to (1), and additional corrections to 2τ in the functions $f_\pm(\tau)$ may be disregarded.

Taking into account that the difference $v_\perp(\tau)-v_\perp(-\tau)$ is appreciably different from zero only for those τ's which correspond to the particle found at distances $\lesssim a_{T-F}$ (the atomic Tomas–Fermi screening length) from the atomic-chain axis, and assuming that the condition

$$2a_{T-F}/l_c\theta \ll 1 \tag{5}$$

is valid, we easily find in the main region of coordinates $r_\perp$

$$\frac{d^2N_p}{d\xi_\pm dS_\perp} \cong \frac{e^2}{S_\perp^0\,\pi\hbar^3\omega}\,\frac{\xi_+^2+\xi_-^2}{4\xi_+\xi_-}\,\frac{\sin[2x/l_c(\xi_\pm,\omega)\theta]}{x\theta}\int_0^\infty [U(\tau)-U(-\tau)]^2 d\tau$$

$$\cong \frac{e^2}{S_\perp^0\,\pi\hbar^3 c\omega}\,\frac{\xi_+^2+\xi_-^2}{4\xi_+\xi_-}\,\frac{\sin[2x/l_c(\xi_\pm,\omega)\theta]}{x\theta^2}\,\Delta U^2(y)a(y). \tag{6}$$

Here $r_\perp \equiv (x,y)$, $x=r_\perp k_\perp/k_\perp$; $U(\tau)$ is the particle's potential energy at the time τ; $\Delta U^2(y)$ and $a(y)$ are, respectively, the characteristic potential energy squared and the spatial potential width on the trajectory with an impact parameter $|y|$ relative to the chain axis ($r_\perp=0$).

Equation (6) shows that under the conditions examined the quantity $d^2N_p/d\xi_\pm dS_\perp$, as a function of the coordinate $r_\perp$:

(i) is appreciably different from zero in a narrow strip, $\sim 2a_{T-F}$ wide, which contains the atomic chain and which is oriented along the vector $k_\perp$;

(ii) oscillates with increasing $|x|$, assuming both positive and negative values and having a decreasing amplitude. These oscillations are of a purely quantum-mechanical nature, and their existence means, among other things, that $d^2N_p/d\xi_\pm dS_\perp$ may be interpreted as a spatial density of created pairs only after averaging over the oscillation period in space.

Choosing the $k_\perp$ direction in a single crystal, we can achieve the conditions under which the mentioned coordinate strip does not touch the nearby atomic chains. For the pair spectrum in a monatomic crystal in this "single-string" case we find the following expression from Eqs. (5) and (6):

$$\frac{dN_p}{d\xi} \cong A\,\frac{e^2}{\hbar c}\,\frac{\pi a_{T-F}^2}{8S_\perp^1}\,\frac{U_0^2}{\hbar^2\omega\theta^2}\,\frac{\xi^2+(1-\xi)^2}{4\xi(1-\xi)} \quad \text{at} \quad \left|\xi-\frac{1}{2}\right| \lesssim \frac{1}{2}(1-\theta_m/\theta)^{1/2}, \tag{7}$$

where U_0 is the single-atomic-chain potential-well depth, $S_\perp^1$ is the transverse area per chain, A is a constant on the order of 1, and

$$\theta_m = \theta_m(\omega) = \frac{4a_{T-F}}{\lambda\!\!\!^{-}_c}\,\frac{mc^2}{\hbar\omega}. \tag{8}$$

Simple physical considerations involving the length of the effective interaction of a pair of particles with the atomic chain

$$l_{\text{int}}=l_\theta l_{cfa}/(l_\theta+l_{cfa}),\quad l_\theta=2a_{T-F}/\theta,\quad l_{cfa}=a_{T-F}mc^2/|U_0|, \tag{9}$$

lead us to the conclusion that the pair-density maximum in the spectrum center ($\xi=\frac{1}{2}$) and, together with it, the maximum of the total pair yield are achieved at $l_c(\frac{1}{2},\omega)\cong l_{\text{int}}$. At moderately high frequencies, which satisfy the condition $\theta_m(\omega)\gtrsim|U_0|/mc^2$, this means that the integral pair yield, as a function of the angle θ, has a maximum at $\theta\cong\theta_m(\omega)$, whose magnitude can be evaluated with the help of Eq. (7):

$$N_p(\theta_m)\cong\frac{e^2}{\hbar c}\frac{\pi a_{T-F}^2}{4S_\perp^1}\frac{U_0^2}{\hbar^2\omega\theta_m^2}=\frac{e^2}{\hbar c}\frac{\pi\lambda\!\!\!^{-2}_c}{64S_\perp^1}\frac{\omega U_0^2}{m^2c^4}. \tag{10}$$

The relations which we obtained solve the problem of the dependence of the angular position of the pair-creation rate maximum and its height on the crystal parameters and the γ-ray frequency.[1)] They are in good agreement with the published experimental data.[2]

3. When the transverse photon momentum $\mathbf{k}_\perp$ is directed along one of the crystallographic planes formed by the singled-out family of atomic chains, the coherent effects arise in the pair-creation processes. They are caused by the interference of the probability waves from the neighboring chains transfixed by the vector $\mathbf{k}$.

For the interference leading to an increase of the e^+–e^- pair yield, the pair-creation-probability amplitude phase difference for neighboring atomic chains

$$\Delta\phi\cong\mathbf{q}\Delta\mathbf{r} \tag{11}$$

is found to be divisible by 2π. Here $\hbar\mathbf{q}$ is the momentum transferred to the crystal in the pair-creation process

$$\hbar\mathbf{q}=\mathbf{k}-p_+-p_-\cong\hbar n_k/l_c(\xi_\pm,\omega),\quad n_k=\mathbf{k}/k, \tag{12}$$

and Δr is the coordinate difference for the points of intersection of the γ-ray "trajectory" with the neighboring chain axes, the distance between which is equal to d:

$$\Delta r\cong n_k d/\theta. \tag{13}$$

Thus, the condition for the interference intensification of the pair-creation rate has the form

$$l_c(\xi_\pm,\omega)\theta\cong\frac{d}{2\pi i}, \tag{14}$$

where i is an integer.

Since the "physical" transverse dimension of an atomic chain is $2a_{T-F}$, the changes in the values of $\xi_\pm$, ω and θ, which characterize the width of the "coherent resonance" (14), are determined by the relation

$$\Delta[l_c(\xi_\pm,\omega)\theta]/[l_c(\xi_\pm,\omega)\theta]_{\text{res}}\sim 4a_{T-F}/d. \tag{15}$$

4. It is worth noting that the theories of coherent pair production, both standard and modified,[1] cannot provide a true description of e^+–e^- pair spectrum properties in the region of action of the coherent resonance mechanism even at relatively moderate