Absorption of light by free carriers in a semiconductor in a static electric field

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The absorption of light by free carriers in a semiconductor in a static electric field is analyzed on the basis of a quantum kinetic equation. The dynamic effect of the field on the scattering of electrons leads to an oscillatory correction to the optical absorption coefficient. This correction is linear in the field.

The absorption of light by free carriers in a semiconductor can be altered by a static electric field. This effect has been studied in detail.[1] It has been shown that the change in absorption results primarily from a heating of current carriers by the electric field. At frequencies in the quantum region, at which the photon energy satisfies $\Omega \gg \bar{\epsilon}$ ($\bar{\epsilon}$ is the average electron energy), the heating leads to a small change ($\sim \bar{\epsilon}/\Omega$) in the absorption. Under these conditions, the contribution to the change in the absorption due to the dynamic effect of a static electric field on the scattering of electrons by phonons or impurities may become appreciable. In the present letter we show that the dynamic effect leads to an oscillatory correction to the absorption coefficient. This correction is linear in the static field.

The classical kinetic equation may not be applicable for studying transport phenomena in strong and rapidly varying electric fields. In this case, a quantum kinetic equation should be used,[2–6] with the field being dealt with exactly, while the electron-phonon interaction is dealt with by perturbation theory. This approach makes it possible to correctly study the effect of a field on the scattering of electrons, since this approach incorporates the circumstance that the electron scattering occurs between states which incorporate the dynamics of the electron motion in the electric field exactly.

We consider a system of nondegenerate electrons which are interacting with acoustic strain phonons in a uniform electric field $\mathbf{F}+\mathbf{E}_0 \sin \Omega t$. We describe the electron states by a generalized momentum $\mathbf{p}$, which incorporates only the high-frequency component of the field. In this representation, the quantum kinetic equation for the electron distribution function $n\mathbf{p}(t)$ is

$$\frac{\partial}{\partial t} n_{\mathbf{p}}(t) - e\mathbf{F}\frac{\partial}{\partial \mathbf{p}} n_{\mathbf{p}}(t) = -2\,\mathrm{Re}\left\{ \sum_{n,k=-\infty}^{+\infty} \sum_{\mathbf{q}} |C_{\mathbf{q}}|^2 (1+2N_{\mathbf{q}}) J_k(\mathbf{aq}) J_{n+k} \times (\mathbf{aq}) e^{in\Omega t} \int_{-\infty}^{0} d\tau [n_{\mathbf{p}-e\mathbf{F}\tau}(t+\tau) - n_{\mathbf{p}+\mathbf{q}-e\mathbf{F}\tau}(t+\tau)] \right.$$

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$$\times\exp\{-i[(\epsilon_{\mathbf{p}+\mathbf{q}}-\epsilon_{\mathbf{p}}+k\Omega+i\delta)\tau-\frac{e\mathbf{F}\mathbf{q}}{2m}\tau^2]\}\Big\}. \quad (1)$$

Here $C_{\mathbf{q}}$ is the matrix element of the electron–phonon coupling, $N_{\mathbf{q}}$ is the distribution of phonons with respect to states with wave vector $\mathbf{q}$, m is the effective mass of a conduction electron, $J_n(x)$ is the Bessel function of the first kind, $\mathbf{a}=e\mathbf{E}_0/m\Omega^2$ is the electron oscillation amplitude in the high-frequency field, $\delta\to+0$ is the parameter of the adiabatic application of the field at $t\to-\infty$, and Re $\{\ldots\}$ means the real part of the expression in braces. In the limit $F\to 0$, Eq. (1) becomes the quantum kinetic equation for electrons in a high-frequency electric field.[7-9] The energy of the phonon has been ignored in the argument of the exponential function.

In the high-frequency limit, $\Omega\bar{\tau}\gg 1$ ($\bar{\tau}$ is the momentum relaxation time of the electrons), the distribution function can be written as the sum of a steady-state component $\bar{n}_{\mathbf{p}}$ and a high-frequency component $\tilde{n}_{\mathbf{p}}$. From Eq. (1) we have $\tilde{n}_{\mathbf{p}}/\bar{n}_{\mathbf{p}}\sim(\Omega\bar{\tau})^{-1}\ll 1$. In the lowest approximation in $(\Omega\bar{\tau})^{-1}$, we find an equation for $\tilde{n}_{\mathbf{p}}$ from (1) by replacing $n_{\mathbf{p}}$ by $\bar{n}_{\mathbf{p}}$ in the collision integral. Solving the resulting equation, and substituting the distribution function found into the expression for the current, we find

$$j_\sim=-\frac{2e}{m\Omega}\,\mathrm{Re}\sum_{\mathbf{p},\mathbf{q}}|C_{\mathbf{q}}|^2(1+2N_{\mathbf{q}})\mathbf{q}\bar{n}_{\mathbf{p}}\sum_{n,k=-\infty}^{+\infty}J_k(\mathbf{aq})J_{k+n}(\mathbf{aq})\,\frac{\exp(in\Omega t)}{in}$$

$$\times\int_{-\infty}^{0}d\tau\exp\left\{-i\left[(\epsilon_{\mathbf{p}+\mathbf{q}}-\epsilon_{\mathbf{p}}+k\Omega+i\delta)\tau+\frac{e\mathbf{F}\mathbf{q}}{2m}\tau^2\right]\right\}. \quad (2)$$

Expression (2) contains, in addition to the current component at the fundamental frequency, terms with $n\neq\pm 1$, which describe a generation of harmonics.

Using (2), we find the following result for the absorption coefficient:

$$\alpha=\frac{16\pi\Omega}{c\bar{n}E_0^2}\sum_{\mathbf{p},\mathbf{q}}|C_{\mathbf{q}}|^2(1+2N_{\mathbf{q}})\sum_{k=-\infty}^{+\infty}J_k^2(\mathbf{aq})k\bar{n}_{\mathbf{p}}\,\mathrm{Re}\int_0^\infty d\tau$$

$$\times\exp\left\{i\left[(\epsilon_{\mathbf{p}+\mathbf{q}}-\epsilon_{\mathbf{p}}-k\Omega+i\delta)\tau+\frac{e\mathbf{F}\mathbf{q}}{2m}\tau^2\right]\right\}, \quad (3)$$

where $\bar{n}$ is the refractive index, and c is the velocity of light in vacuum.

In the limit $F\to 0$, the real part of the integral in (3) becomes $\pi\delta(\epsilon_{\mathbf{p}+\mathbf{q}}-\epsilon_{\mathbf{p}}-k\Omega)$ [$\delta(x)$ is the Dirac δ-function]. As a result, we find from (3) the standard expression for the multiphoton absorption coefficient.[10] The singular δ-function has been replaced by a smooth exponential function because of a broadening of the electronic states in a strong electric field.

Expression (3) for the absorption coefficient contains the unknown distribution function $\bar{n}_{\mathbf{p}}$, which is rather difficult to determine. However, we easily see that in the quantum limit, in which we are interested here, we can ignore the electron momentum in the argument of the exponential function. The absorption coefficient then becomes independent of $\bar{n}_{\mathbf{p}}$, and the summation over the electron momenta in (3) becomes an elementary calculation. As a result, we find

$$\alpha=\frac{16\pi N\Omega}{c\bar{n}E_0^2}\sum_{\mathbf{q}}|C_{\mathbf{q}}|^2(1+2N_{\mathbf{q}})\sum_{k=-\infty}^{+\infty}J_k^2(\mathbf{aq})k$$

$$\times\mathrm{Re}\left\{\left(\frac{\pi mi}{2e\mathbf{Fq}}\right)^{1/2}\exp\left[-\frac{im\left(\frac{q^2}{2m}-k\Omega\right)^2}{2e\mathbf{Fq}}\right]\right\}, \tag{4}$$

where N is the density of conduction electrons.

At this point we restrict the discussion to the limit of a weak optical field, and we expand the Bessel function in powers of its argument. The integration over $\mathbf{q}$ in (4) can be carried out by the stationary-phase method. For fields F which are not too strong we find

$$\alpha=\alpha_0\left\{1+\frac{\sqrt{6}}{8}\beta\cos^2\gamma\sin\left(\frac{4\sqrt{6}}{9\beta}\right)\right\}. \tag{5}$$

Here α_0 is the linear optical absorption coefficient for absorption by free carriers in the quantum limit in a situation with scattering by acoustic phonons,[1] and γ is the angle between the vectors $\mathbf{F}$ and $\mathbf{E}_0$. The parameter $\beta=eF/(m\Omega^3)^{1/2}$ is the ratio of the energy $[\approx eF(m\Omega)^{-1/2}]$ acquired by a photoexcited electron in the field F at the de Broglie wavelength $[\approx(m\Omega)^{1/2}]$ to its energy Ω. Numerical estimates show that at the wavelength of a CO_2 laser with $m\approx 0.1m_0$ (m_0 is the mass of a free electron) we would have a value $\beta\simeq 0.01$ at $F\simeq 5\times 10^3$ V/cm.

The second term in (5), which describes the change in α in the lowest approximation in the field F, corresponds to a transition accompanied by the emission of a photon by an electron [the term with $k=-1$ in (4)]. In the quantum limit, with $F=0$, the probability for a transition accompanied by the emission of a photon is exponentially small. In an electric field, on the other hand, a process which is indirect in real space—the process in which an electron acquires an energy Ω in the field F and then emits a photon—becomes possible. The oscillations in (5) stem from a quantum interference of the electron waves incident on and reflected from the potential wall (eFx) formed by the field F.

In summary, these calculations show that the changes caused in the optical absorption by the effect of a static electric field on the scattering of electrons by phonons is linear in F. For fields which are not too strong, this effect may become predominant, since the contribution to the change in absorption from the drift and heating of carriers in the field F is quadratic in the field.[1] In an experimental study of this effect, one should bear in mind its polarization dependence and also the circumstance that its rise time is much shorter than that of the heating effect.

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