Dilaton at nonzero temperature and deconfinement in gluodynamics

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The temperature dependence of a gluon condensate is derived from an effective dilaton Lagrangian. A first-order phase transition occurs in the system. The temperature of this transition is estimated to be $T_c \sim 200$–250 MeV.

1. In gluodynamics, the lightest and therefore the stablest hadron is a glueball with the vacuum quantum numbers, $J^{PC}=0^{++}$. A low-energy Lagrangian for the interaction of a 0^{++} glueball (a dilaton) which was derived in Ref. 1 implements Ward gauge identities in the way that the chiral pion Lagrangian implements Ward chiral identities at the tree level. The effective Lagrangian of the dilaton is

$$L(\sigma)=\frac{1}{2}(\partial_\mu\sigma)^2-V(\sigma),\quad V(\sigma)=\frac{\lambda}{4}\sigma^4\left(\ln\frac{\sigma}{\sigma_0}-\frac{1}{4}\right). \tag{1}$$

The field σ is related to the trace of the energy-momentum tensor in gluodynamics by

$$\frac{m_0^4}{64|\varepsilon_v|}\sigma^4(x)=-\Theta_{\mu\mu}(x)=\frac{b\alpha_s}{4\pi}\mathrm{Tr}F_{\mu\nu}^2(x), \tag{2}$$

and the constants λ and σ_0 are expressed in terms of physical parameters,

$$\lambda=\frac{m_0^4}{16|\varepsilon_v|},\quad \sigma_0^2=\frac{16|\varepsilon_v|}{m_0^2}, \tag{3}$$

where $|\varepsilon_v|=-\frac{1}{4}\langle\Theta_{\mu\mu}\rangle$ is the nonperturbative vacuum energy density, and m_0 is the mass of the lightest excitation above the vacuum, i.e., the mass of a dilaton.

2. The regularized effective potential for the vacuum condensate, σ_T, at $T\neq 0$ is written in the form

$$V_{\mathrm{eff}}^R(\sigma_T)=V(\sigma_T)+\frac{1}{2}T\sum_{n=-\infty}^{+\infty}\int\frac{d^3k}{(2\pi)^3}\ln[k^2+(2\pi nT)^2+V^{(2)}(\sigma_T)]$$
$$-\frac{1}{2}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\int\frac{d^3k}{(2\pi)^3}\ln(k^2+\omega^2+m_0^2). \tag{4}$$

At $T=0$ the theory is unrenormalized because of the infinite number of vertices in the tree approximation. At $T\neq 0$, however, all the ultraviolet divergences cancel out in V_{eff}^R (the vacuum contribution at $T=0$ is subtracted out in the course of the regularization). As a result, we are left with a finite temperature contribution.

The equation for the vacuum condensate σ_T is found from the condition for a minimum of[2] V_{eff}^R:

$$\frac{\delta V_{\text{eff}}^{R}}{\delta\sigma}\Big|_{\sigma_T} = V^{(1)}(\sigma_T) + \frac{V^{(3)}(\sigma_T)}{4\pi^2}[F(m_T^2,T)+\Phi(m_T^2,m_0^2)] = 0, \quad (5)$$

$$F = \int_0^\infty \frac{k^2 dk}{\sqrt{k^2+m_T^2}\{\exp(\sqrt{k^2+m_T^2}/T)-1\}}, \quad \Phi = \frac{1}{8} m_T^2 \ln \frac{m_T^2}{m_0^2} + \frac{1}{8}(m_T^2 - m_0^2). \quad (6)$$

The term $\Phi(m_T^2,m_0^2)$ stems from radiative corrections, which in turn arise from the difference between the quasiparticle masses $m_{T\neq 0}^2$ and m_0^2.

3. At low temperatures, $T \ll m_0$, we can assume that the glueball mass is independent of T in Eq. (5). We then have $\Phi = 0$ and $m_T^2 = m_0^2 = V^{(2)}(\sigma_0) = \lambda\sigma_0^2$.

Equation (5) for σ_T becomes

$$\sigma_T^2 \ln \frac{\sigma_T^2}{\sigma_0^2} + \frac{3}{2\pi^2}\left(\ln \frac{\sigma_T^2}{\sigma_0^2} + \frac{5}{3}\right) F(T) = 0. \quad (7)$$

Under these conditions $m_0/T \gg 1$, Eq. (7) has a solution

$$\sigma_T^2 = \sigma_0^2[1-\alpha(T)], \quad (8)$$

$$\alpha(T) = \frac{\lambda}{2\pi^2}\int_1^\infty dx \frac{\sqrt{x^2-1}}{e^{(m_0/T)x}-1} bf$$

$$\simeq \frac{\lambda}{2\pi^2}\int_1^\infty dx \sqrt{x^2-1}\, e(^{-m_0/T})\, x = \frac{\lambda}{2\pi^2}\frac{T}{m_0} K_1\left(\frac{m_0}{T}\right).$$

We thus finally find the temperature dependence of the gluon condensate:

$$\langle \text{Tr} F_{\mu\nu}^2 \rangle_T = \langle \text{Tr} F_{\mu\nu}^2 \rangle_0 \left[1 - \frac{\lambda}{\sqrt{2\pi^3}}\left(\frac{T}{m_0}\right)^{3/2} e^{-m_0/T}\right]. \quad (9)$$

Even at $T \sim 200$ MeV with $m_0 \sim 1$ GeV, the change in the condensate is $\sim 0.1\%$ The gluon condensate is thus only a weak function of T all the way to the critical temperature T_c.

The contribution of the thermal excitation of glueballs to V_{eff}^R is well known and can be calculated easily.

Examining the second and third terms in (4) in the limit $m_0/T \gg 1$, we find

$$U_{ge}(m_0,T) = \frac{T^{5/2} m_0^{3/2}}{(2\pi)^{3/2}} e^{-m_0/T}. \quad (10)$$

4. As we know, the deconfinement phase in gluodynamics must contain massless gauge bosons. In the confinement phase, we should find the energy spectrum of massive glueballs and bound states of quarks. The mass gap (the difference between the energies of the ground state and the first excited state) is thus zero in one phase and nonzero in the other.

On the other hand, a vacuum condensate is not a suitable candidate for the role of a local order parameter, since even at $T > T_c$ we have $\sigma_T \neq 0$, because the vacuum contains nonperturbative color fluctuations and small instantons, which are well defined at $T\neq 0$. In the deconfinement phase we thus have $m_T=0$ but $\sigma_T\neq 0$. In Ref. 3, the vacuum gluon condensate $\langle \mathrm{Tr} F_{\mu\nu}^2\rangle$ was treated as a local parameter for gluon deconfinement, which vanishes at $T>T_c$ by analogy with a chiral phase transition, in which we have $\langle \bar{q}q\rangle=0$ in the phase with the reconstructed symmetry. However, the arguments above, the results of Refs. 4 and 5, and lattice calculations of the condensate by the Monte Carlo method[6] show that this analysis is not totally correct.

The effective potential must therefore be thought of as a function of the variables T, σ_T, and m_T. In order to analyze the phase transition in this model, we thus need to write a self-consistent system of equations for the vacuum condensate σ_T and the mass gap m_T. This system of equations is

$$\frac{\delta V_{\mathrm{eff}}^R}{\delta\sigma}\Big|_{\sigma T}=0, \tag{11}$$

$$\frac{\delta^2 V_{\mathrm{eff}}^R}{\delta\sigma^2}\Big|_{\sigma T}=m_T^2. \tag{12}$$

It is shown below that a first-order phase transition occurs in this system, with discontinuities in m_T and σ_T. To find the magnitude of the discontinuity in the condensate, we note that on the unstable branch in Eq. (12), it is sufficient to retain exclusively the "bubble" diagram, since the other term is suppressed by an amount $\sim m_T/T$ in comparison with it.

Introducing the dimensionless variables $f=\sigma_T^2/\sigma_0^2$, $g=m_T^2/m_0^2$, we can rewrite self-consistent equations (11) and (12) as

$$f\ln f+\frac{3\lambda}{2\pi^2}\left(\ln f+\frac{5}{3}\right)\left\{\frac{T^2}{m_0^2}F\left(\frac{m_0^2}{T^2}g\right)+\frac{1}{8}g\ln g-\frac{1}{8}g+\frac{1}{8}\right\}=0, \tag{13}$$

$$f+\frac{3}{2}f\ln f+\frac{3\lambda}{2\pi^2}\left(\frac{3}{2}\ln f+\frac{11}{6}\right)\left\{\frac{T^2}{m_0^2}F\left(\frac{m_0^2}{T^2}g\right)+\frac{1}{8}g\ln g-\frac{1}{8}g+\frac{1}{8}\right\}=g. \tag{14}$$

The function F is given by (6). From Eqs. (13) and (14) we find

$$g=f(1+\ln f)\Big/\left(1=\frac{3}{5}\ln f\right). \tag{15}$$

The mass gap thus vanishes ($g=0$) in the case $f=e^{-1}$. This case arises at a temperature T_1, which we can find by setting $f=e^{-1}$, $g=0$, $F(0)=\pi^2/6$ in Eq. (13):

$$\left(\frac{T_1}{m_0}\right)^2=\frac{6}{\lambda e}-\frac{3}{4\pi 2}. \tag{16}$$

It follows from numerical calculations that the temperature T_1 corresponds to the appearance of the second (unstable) branch, as is characteristic of a first-order phase transition. The temperature T_2 is the critical temperature for the hadron (glueball) phase. Specifically, at $T>T_2$ a hadron phase cannot exist even in a metastable state. We can also conclude from the numerical calculations that in QCD we have T_1 ($\sim$170 MeV) $<T_c<T_2$ ($\sim$270 MeV).

Equation (16) can be used to find an upper estimate of the mass of the 0^{++} glueball. Using (3), we find $m_0 < 4.642\,|\varepsilon_v|^{1/4}$. In QCD, with $|\varepsilon_v|_{QCD} \simeq 4\times 10^{-3}$ GeV (Ref. 7), we have $m_0 < 1170$ MeV. In gluodynamics, we have $|\varepsilon_v|_{GD} \simeq (2\text{–}4)\,|\varepsilon_v|_{QCD}$ and $m_0 < 1650$ MeV.

5. To find the critical temperature of the first-order phase transition, T_c, we need to compare the free energies of the two phases. In the confinement phase, the free energy is determined by the vacuum energy density plus the thermal excitations of glueballs. In the deconfinement phase, it is determined by the vacuum energy density (after the discontinuity) plus the "hot gas" of gluons.

An equation for T_c can thus be written in the form[5]

$$\eta|\varepsilon_v| + \frac{T_c^{5/2} m_0^{3/2}}{(2\pi)^{3/2}} e^{-m_0/T_c} \simeq (N_c^2-1)\frac{\pi^2}{45} T_c^4, \tag{17}$$

where

$$\eta = \frac{1}{|\varepsilon_v|}[V(\sigma_{T_c+0}) - V(\sigma_{T_c-0})] = 1 - 3e^{-2} \simeq 0.59.$$

For the critical temperature we thus find

$$T_c = \left(\frac{45}{\pi^2}\eta\frac{|\varepsilon_v|}{N_c^2-1}\right)^{1/4}\left(1+\frac{\text{const}}{N_c^2}\right), \quad \text{const} \ll 1.$$

In the chromodynamic case we find $T_c \simeq 192$ MeV. For gluodynamics we find $T_c \simeq 228\text{–}270$ MeV.

We have a comment regarding these results. If we redefine the field, $\sigma \to N_c\tilde{\sigma}$, in Lagrangian (1), we find $L(\tilde{\sigma}) = N_c^2\,[\frac{1}{2}(\partial_\mu\tilde{\sigma})^2 - V(\tilde{\sigma})]$, and in the potential $V(\tilde{\sigma})$ the constants satisfy $\tilde{\lambda}, \tilde{\sigma} \sim N_c^0$.

In the loop expansion, the propagator is thus $\sim 1/N_c^2K^2$, and the vertex $\sim N_c^2 V(4)(\tilde{\sigma}_T)$. The classical contribution to V_{eff} at the mean-field level is thus $\sim N_c^2$, that at the single-loop level is $\sim N_c^0$, that at the two-loop is $\sim N_c^{-2}$, and so forth.

In the limit $N_c \to \infty$, the phase-transition temperature T_c is thus determined completely by the discontinuity in the gluon condensate at this point and is independent of N_c.

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