Fluctuational conductivity of tunnel structures above the superconducting transition temperature

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The fluctuational resistance δR in SIS and SIN tunnel junctions at $T > T_c$ is calculated without the use of a perturbation theory in the transmission. In each case, δR has a maximum, whose position depends on the transmission of the barrier and on the damping in the spectrum of excitations.

In the high-T_c superconductors of the BSCCO type, the resistance R along the c axis has been observed[1–4] to increase with decreasing temperature T. When the superconductor has been in the superconducting phase, the increase in R has occurred at temperatures down to a certain T_m; below T_m, the resistance has decreased sharply, vanishing at $T = T_c$. Ioffe *et al.*[5] have suggested an explanation for the nonmonotonic $R(T)$ behavior in the layered high-T_c superconductors. According to this idea, the peak in $R(T)$ stems from superconducting fluctuations, which can be classified into two types. Fluctuations of one type lead to a decrease in the density of states at low energies because of the fluctuational formation of a virtual gap.[6] As a result, there is an increase ($\delta R_{DOS} > 0$) in the resistance of a Josephson tunnel junction or of a layered superconductor with a Josephson interaction between layers. The fluctuations of the other type lead to a contribution of virtual Cooper pairs to the conductivity (an Aslamasov–Larkin correction,[7] $\delta R_{AL} < 0$). These fluctuations increase more rapidly as the difference $(T - T_c)$ decreases, but they contain only a power of the transmission. Since these contributions differ in sign, in magnitude, and in temperature dependence, a maximum forms on the temperature dependence of the fluctuational resistance $\delta R(T) = \delta R_{DOS} + \delta R_{AL}$. The value of δR was calculated in Ref. 5 by the method of Ref. 7, except that the electron spectrum was assumed to be approximately two-dimensional (a corrugated cylinder). Although the fluctuational mechanism for a maximum in $R(T)$ should operate in layered superconductors, the applicability of this mechanism to a material of the BSCCO type is not obvious, since an increase in $R(T)$ with decreasing temperature is also observed in nonsuperconducting BSCCO samples. It thus becomes necessary to study this mechanism in more detail, in (for example) tunnel junctions. No previous calculation of δR for an SIS tunnel junction has taken fluctuations of both types into account. Varlamov and Dorin[6] calculated only the component δR_{DOS} for an SIS junction; that component leads to an increase in the resistance. Calculating the complete fluctuational resistance requires going beyond a perturbation theory in powers of the transmission.

In the present letter we use the exact joining conditions for the Green's functions $\check{G}$ at the barrier to calculate the fluctuational resistance of SIS and SIN junctions, $\delta R(T)$, at $T > T_c$. The calculation method can easily be generalized to the case of a

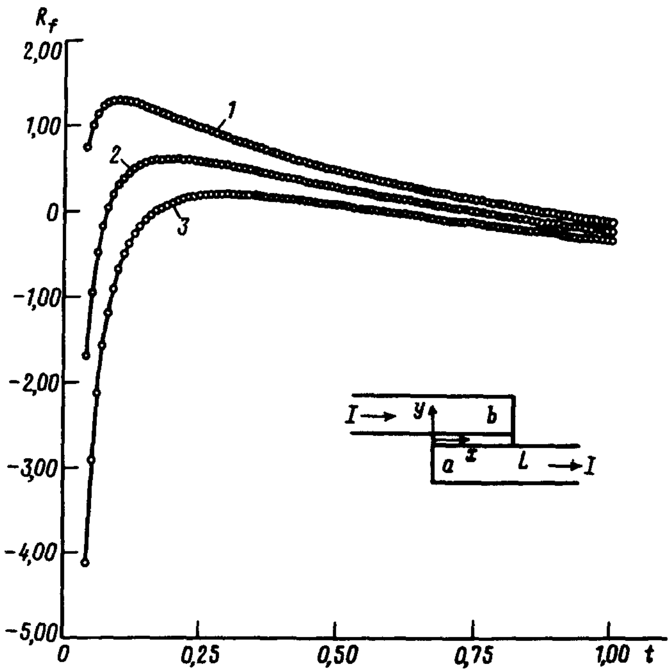

FIG. 1. Fluctuational resistance $\delta R_f = \delta\tilde{R}/\delta\tilde{R}_0$ versus the "temperature" $t=(T-T_c)/T_c$ for various values of the parameter $\tilde{\epsilon}_1 = 1.44\epsilon_0/\gamma$. *1*—0.1; *2*—0.2; *3*—0.3. The inset shows the system under discussion.

periodic SISI... system, which can be used to model a layered superconductor in which the frequency of jumps between layers is small.[8] (For example, in a BSCCO high-T_c superconductor at $T > T_c$, the motion of the electrons along the c axis is a hopping rather than a band motion.) Boundary conditions on $\check{G}$ in superconducting systems with a barrier have been derived by Zaĭtsev[9] in general form. These conditions were simplified in Ref. 10 for the dirty limit (these conditions were subsequently derived in Refs. 11 and 12 also). These conditions were used in Refs. 13–15 to calculate the conductivity of SININ systems[13-14] and SIN systems[15] at $T < T_c$. Going beyond the scope of a perturbation theory in the transmission had made it possible to explain the anomaly (the peak) in the conductivity at a zero bias voltage which has been observed in S–Sm and S–Sm–S junctions (S is a superconductor, N a normal metal, I an insulator, and Sm a semiconductor). An exact approach is also required for calculating the fluctuational resistance in tunnel junctions.

Let us examine an SIS (or SIN) junction (see the inset in Fig. 1). As in Ref. 15, we deal with the dirty case ($l < \xi$) and we assume that the electrode thicknesses a and b are small ($d_{a,b} < \xi_T \cong \sqrt{D/T}$). Taking an average of the equations for the Green's functions $\check{G}$ over the electrode thicknesses, and imposing the boundary conditions,[10] we then find an equation for the Green's function of electrode a:

$$D_a\partial_\perp(\check{G}\partial_\perp\check{G})_a+\epsilon_a[\check{G}_a,\check{G}_b]_-(\check{\sigma}_z\partial_t\check{G}_a+\partial_{t'}\check{G}_a\check{\sigma}_z)+i[\check{\Delta}(t)\check{G}-\check{G}\check{\Delta}(t')]_a$$
$$+i(V(t)\check{G}-\check{G}V(t'))_a=\check{L}_{ph}. \qquad (1)$$

Here D_a is the diffusion coefficient, $\partial_\perp=(\partial_x,0,\partial_z)$ and $\epsilon_a=D_a/(rld)_a$. The coefficient r characterizes the barrier transmission and is related to the resistance of the tunnel junction per unit area in its normal state, $R_\square=(rl/2\sigma)_a$, where σ_a is the conductivity of electrode a. The term on the right side of (1) describes the interaction with phonons. An equation for $\check{G}_b$ is found from (1) by interchanging the indices a and b. We can find the correction to the resistance by seeking the response of the system to fluctuations of the order parameter, $\hat{\Delta}(x,z,t)$. This approach was taken in Ref. 16 to find δR in a homogeneous system and in Ref. 17 to determine δR of a microbridge with a direct conductivity. In first order in $\hat{\Delta}$ we find from (1) the following expressions for the retarded (advanced) Green's functions:

$$\check{G}^{R(A)}_{1a}(\epsilon,\epsilon';q)=\hat{G}^{R(A)}_{1a}\delta(\epsilon-\epsilon'-\omega),\quad \check{G}^{R(A)}_{1a}=\hat{\Delta}_a(\omega,q)\alpha^{R(A)}_a+\hat{\Delta}_b(\omega,q)\beta^{R(A)}_a. \quad (2)$$

Here $\alpha^{R(A)}_{a,b}=(N_{b,a}/B)^{R(A)}$, $\beta^{R(A)}_{a,b}=i\epsilon_{a,b}/B^{R(A)}$, $N^{R(A)}_{a,b}=\pm[(\epsilon+\epsilon')/2\pm ia_{a,b}(q)/2]$, $a_{a,b}(q)=2\epsilon_{a,b}+(Dq^2+\gamma)_{a,b}$, $\gamma=\tau_\epsilon^{-1}+2\tau_s^{-1}$ is a coefficient describing the damping of superconducting fluctuations, and $B^{R(A)}=(N_aN_b)^{R(A)}+\epsilon_a\epsilon_b$. The second-order correction can be found easily from the orthogonality relation; it is expressed in terms of[17] $G^{R(A)}_{1a,b}$. We find the correlation functions $\langle\hat{\Delta}_a\hat{\Delta}_a\rangle$ and $\langle\hat{\Delta}_a\hat{\Delta}_b\rangle$ by substituting $G^{R(A)}_{1a,b,}$ into the condition for self-consistency with Langevin sources, which describe thermodynamic–equilibrium fluctuations. For brevity we reproduce here only the expressions for the correlation functions in the case of identical electrodes (an SIS system):

$$\langle\Delta_a(\omega,q)\Delta_a(\omega',q')\rangle=K_f[\tilde{\epsilon}_0^2(-L(\omega,q)L(\omega',q')]/[D(\omega,q)D(\omega',q')],$$
$$\langle\Delta_a(\omega,q)\Delta_b(\omega',q')\rangle=-K_f[i\tilde{\epsilon}_0(L(\omega,q)$$
$$+L(\omega',q'))]/[D(\omega,q)D(\omega',q')]. \qquad (3)$$

Here $\tilde{\epsilon}_0=2\epsilon_a/T_0=2\epsilon_b/T_0$, $T_0=8T/\pi$, $K_f=2\pi\delta(\omega+\omega')\delta_{qq'}\pi/(2\nu_F)$, $\nu_F=p_Fm/(2\pi^2)$ is the density of states, $L(\omega,q)=\tilde{\omega}+i(a_q/T_0=\kappa_0)$, $\kappa_0=\ln(T/T_{c0})$,

$$\tilde{a}_q\equiv a_q/T_0=\tilde{\epsilon}_0+\tilde{q}^2+\tilde{\gamma}^2,\quad \tilde{q}^2=Dq^2/T_0,\quad \tilde{\gamma}=\gamma/T_0,\quad D(\omega,q)=L^2(\omega,q)+\epsilon_0^2.$$

We now write equations for the Keldysh functions $\hat{G}=\hat{G}^R\hat{F}-\hat{F}\hat{G}^A$, i.e., we take the (1, 2) element of Eq. (1), multiply it by $\check{\sigma}_z$, and calculate the trace. Taking an average over the fluctuations, and integrating over the energy, we find the equation

$$\int d\epsilon\{D_a\partial_x[\partial_xF_a-\langle G^R\partial_xFG^A\rangle_a-\langle F^R\partial_xFF^A\rangle_a]-\epsilon_a\langle A(\epsilon)\rangle\}=0. \qquad (4)$$

Here $A(\epsilon)=G^R_aF_aG^A_b+G^R_bF_aG^A_a-F_aG^A_aG^A_b-G^R_bG^R_aF_a+F^R_bF^R_aF_a+F_aF^A_aF^A_b+F^R_aF_aF^A_b+F^R_bF_aF^A_a$, $\langle F^R_aF^A_b\rangle=-\langle\hat{G}^R_{1a}\hat{G}^A_{1b}\rangle$, $\langle G^{R(A)}_a\rangle=\pm1+\langle G^{R(A)}_{2a}\rangle$, etc. The function F_a is the distribution function in electrode a, which determines the electric potential V_a:

$$V_a=(1/4)\int d\epsilon(G^R-G^A)_aF_a. \qquad (5)$$

Equation (4) is the continuity equation for the current in electrode a. The first term is the divergence of the current averaged over the thickness; the second is the influx (outflux) due to the tunneling of quasiparticles through the barrier and the contribution of the proximity effect.[14,15] We have omitted from (4) the distribution function F_b in electrode b under the assumption that the potential is zero there and under the further assumption that the barrier resistance is large in comparison with the resistance of the electrodes. The length of the junction, L, is thus not very large: $L \ll l_0 \equiv \{[(\sigma d)_a^{-1} + (\sigma d)_b^{-1}]/R_\Box\}^{-1/2}$, where l_0 is a length scale of the spatial variation of the potential difference along the x axis in the normal state. Setting $G^{R(A)} = \pm 1$, $F^{R(A)} = 0$ in (4), and integrating (4) over x from 0 to L, we can express the junction resistance at $T \gg T_c$ (per unit length in the z direction) in terms of $\epsilon_{a,b}$: $R_N + R_\Box/L = (rl/2\sigma)_a/L = (rl/2\sigma)_b/L$. In a similar way we find the dimensionless resistance of the junction due to fluctuations:

$$\delta\tilde{R} \equiv [R(T) - R_N]/R_N = -(8V)^{-1} \int d\epsilon \langle \delta A(\epsilon) \rangle, \tag{6}$$

where $\delta A = A - A_N = A - 4F_a$, $F_a = [\tanh(\epsilon + V)\beta - \tanh(\epsilon - V)\beta]/2$.

Equations (2)–(6) determine the fluctuational resistance of the junction, $\delta\tilde{R}$, in a fairly general form. We find this resistance under the assumption that the typical tunneling energies $\epsilon_{a,b}$ are small in comparison with T_c. It follows from the expression for $\tilde{A}(\epsilon)$ that there are two types of fluctuations, which make different contributions to $\delta\tilde{R}$. The fluctuations of one type, which might be called "regular,"[17] arise from terms in A which have poles in one ϵ half-plane, i.e., from terms of the type $G_{2a}^R F_a$ and $F_{1a}^R F_a F_{1a}^R$. The contribution of these terms reduces to essentially a change in the density of states as the result of fluctuations: $\delta\nu \sim \langle \delta G_{2a}^R - \delta G_{2a}^A \rangle$. The fluctuations of the other type, called "anomalous,"[17] arise from those terms in A which have poles in different ϵ half-planes, i.e., from terms of the type $F_{1a}^R F_a F_{1a}^A$. These terms contain an extra power of $\epsilon_{a,b}$, but they increase more rapidly as $T \to T_c$. An analytic expression can be derived for $\delta\tilde{R}$ in the case $\epsilon_{a,b} \ll T_c$. That expression is quite unwieldy, so we will discuss here only the case of a low transmission, with $\epsilon_{a,b} \ll \gamma_{a,b}\,(T - T_c)$. In this case we have

$$\delta\tilde{R} = \delta\tilde{R}_0[\rho_r(t) - \rho_{an}(t)]. \tag{7}$$

Here $\delta\tilde{R}_0 = 84\zeta(3)/(\pi^2 p_F^2 ld) \simeq 10.2/(p_F^2 ld)$, $\rho_r(t) = -\ln\, t$ is the regular part, $\rho_{an}(t) = \tilde{\epsilon}_1/t$ is the anomalous part, $\tilde{\epsilon}_1 = 1.44\epsilon_0/\gamma$, $t = (T - T_c)/T$, and $T_c = T_{c0}(1 - \tilde{\gamma})$ is the critical temperature renormalized because of damping. The maximum of $\delta\tilde{R}(t)$ is reached at $t_m = \tilde{\epsilon}_1$; t_m depends strongly on γ. Figure 1 shows a plot of $\delta\tilde{R}(t)$. We see that the maximum of δR is defined most clearly in the case $\tilde{\epsilon}_1 \ll 1$.

Let us estimate some typical parameter values. For conventional superconductors, the energy ϵ_0 is conveniently written in the form $\epsilon_0 = 3\pi^2\hbar^3 j_c/(5k_F T_c\; emd)$. With $d = 1000$ A, $T_c = 4$ K, $j_c = 10^5$ A/cm^2, and $\tau_e = 10^{-10}$ s, we find $\epsilon_0 = 0.015$ K and $\gamma = 0.1$ K. The maximum of $\delta R(T)$ is reached at $T_m = 1.22T_c$. For the case of the BSCCO high-T_c superconductors, ϵ_0 can be written in the form $\epsilon_0 = (\hbar D/d^2 Z)$, where $Z = rl/2d$ is an anisotropy parameter. With $Z \simeq 10^5$, $d = 10^{-7}$ cm, $l = 10^{-6}$ cm and $\tau_\epsilon = 10^{-12}$ s we find $\epsilon_0 \simeq 0.2$ K and $\gamma = 10$ K. We then have $T_m = 1.03T_c$. Since we are probably dealing with the pure case in the case of high-T_c superconductors, the estimates for high-T_c superconductors are valid only in order of magnitude.

In the case of an SIN junction, it can be shown that the maximum of $\delta\widetilde{R}(t)$ is reached at $t_m=\widetilde{\epsilon}_1$ if the condition $\widetilde{\epsilon}_1 \gg \widetilde{\gamma}$ holds.

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