Induction of superconductivity by a tunneling of quasiparticles involving photons in multilayer superconducting tunnel structures

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An S_hISIS_h tunnel structure for which the superconducting transition temperature of superconductor S, T_{c0}, is lower than that of the other superconductor, is analyzed. When an oscillating voltage is applied to this structure at $T \geqslant T_{c0}$, a gap Δ may be induced in S as the result of a tunneling of quasiparticles involving photons. The size of the gap and the critical current of the structure may reach values on the order of their equilibrium values corresponding to $T=0$ if the transmission of the barriers is not too small.

An interesting aspect of multilayer, in particular, three-layer, tunnel structures of the type S_hISIS_h (I is a layer of an insulator, and S_h is a superconductor whose transition temperature is higher than that of superconductor S) is that a superconductivity may be induced in S [which has the lower (equilibrium) transition temperature T_{c0}]. This effect occurs (at $T \geqslant T_{c0}$) because quasiparticles are extracted from S in a certain interval of values of a steady-state voltage (V) across the structure.[1–3] We wish to stress that we are talking about an induced superconductivity due to a deviation of quasiparticles from equilibrium—not a proximity effect, which is negligible at the transmission values of the tunneling barriers which ordinarily prevail experimentally. We show below that there is another possibility for the induction of a superconductivity in an S_hISIS_h structure, which results from a tunneling of quasiparticles involving photons when an oscillating voltage $V_\sim(t)$ with a frequency in a certain range is applied. We analyze the behavior of the gap as a function of the frequency for various temperatures (and for amplitudes $V_\sim$ which are appreciable). We also analyze the steady-state supercurrent which can flow through the structure under conditions of an induced superconductivity.

We consider an S_hISIS_h structure with a thin S layer (of thickness $d \ll \xi$) with a short mean free path $l \ll d$ and a low barrier transmission $D_j = \langle \alpha D_j(\alpha) \rangle 1$ ($\alpha = p_{Fx}/p_F$, $j=1,2$). We assume that the superconductors S_h are fairly massive and that, since D_j is small, their parameter values and the matrix Green's functions[4] $\check{G}_j$ are at their equilibrium values. For the Green's function $\check{G}_j \equiv \check{G}(t,t')$ in S we have the equation[2,5] (we assume that the potential in region S is zero, and we set $\hbar = 1$)

$$i\check{\tau}_z\frac{\partial\check{G}}{\partial t}+i\frac{\partial\check{G}}{\partial t'}\check{\tau}_z-\check{\Delta}(t)\check{G}+\check{G}\check{\Delta}(t')+\check{\Sigma}\check{G}-\check{G}\check{\Sigma}=0,$$

$$\check{G}^2(t,t')\equiv\int dt_1\check{G}(t,t_1)\check{G}(t_1,t')=\check{1}\delta(t-t'), \tag{1}$$

where $\check{\Sigma}=\check{\Sigma}_1+\check{\Sigma}_2+\check{\Sigma}_{ph}$, $\check{\Sigma}_j=i\epsilon_j\check{G}_j$, $\epsilon_j=D_jv_F/4d$,

$$\check{G}_j(t,t')=\check{S}_j(t)\check{G}_h(t-t')\check{S}_j^*(t')$$

$$\check{S}_j(t)=\exp(i\chi_j(t)\check{\tau}_z),\quad \chi_j(t)=\chi_j+(-1)^je\int_0^t V_j(t_1)dt_1, \tag{2}$$

V_j is the voltage across the jth barrier, and χ_j is the constant part of the phase.

We assume that an alternating voltage $V_\sim(t)=V_\sim\cos\omega t$ is applied to the S_hISIS_h structure. Assuming for simplicity that the transmission values of the barriers are the same ($\mathbf{D}_j=\mathbf{D}$), we find from (2)

$$\check{\Sigma}_1(t,t')+\check{\Sigma}_2(t,t')=\check{\Sigma}_{st}(t-t')+\check{\Sigma}_\sim(t,t'), \tag{3}$$

where the Fourier components of the steady-state part of the Keldysh, retarded (R), and advanced (A) matrices $\check{\Sigma}_{st}^{(R,A)}(\epsilon)$ are

$$\hat{\Sigma}_{st}^{(R,A)}(\epsilon)=i\epsilon_b\sum_{n=-\infty}^{\infty}J_n^2(a)\{a_h(\epsilon+n\omega)\hat{\tau}_z+(-1)^nf_h(\epsilon+n\omega)\cos(\varphi/2)i\hat{\tau}_y\}^{R,A}. \tag{4}$$

Here $\epsilon_b\equiv Dv_F/2d$, $J_n(a)$ is the Bessel function of the first kind of index n,

$$a=eV/2\omega,\quad g_h(\epsilon)=2f_0(\epsilon)\mathrm{Re}\,g_h^R(\epsilon),\quad f_h(\epsilon)=2f_0(\epsilon)\mathrm{Re}\,f_h^R(\epsilon),$$

$$f_0(\epsilon)=\tanh(\epsilon/2T),\quad g_h^R(\epsilon)=\epsilon/[(\epsilon+i0)^2-\Delta_h^2]^{1/2}=f_h^R(\epsilon)\epsilon/\Delta_h.$$

We have also taken into account the constant difference between the phases (φ) of the order parameter between the superconductors S_h, which is determined by the steady-state supercurrent flowing through the structure.

We consider the case of low barrier transmission values, in which we have

$$\epsilon_b\ll T_{c0}, \tag{5}$$

so we can ignore the proximity effect. For the modulus of the order parameter Δ in S we then find the following result from the self-consistency equation for frequencies which are not too low, $\omega\gg\epsilon_b$:

$$\Phi(\Delta)\equiv\ln(\Delta/\Delta_0)-\int_\Delta^\infty d\epsilon[f(\epsilon)-1]/(\epsilon^2-\Delta^2)^{1/2}=0. \tag{6}$$

Here Δ_0 is the equilibrium gap at $T=0$, $f(\epsilon)$ is the quasiparticle distribution function, which is related to the numbers of electron excitations, $n(\epsilon)$, and hole excitations, $n(-\epsilon)$. In this case of symmetric barriers we have $n(\epsilon)=n(-\epsilon)$ and $f(\epsilon)=[1-2n(\epsilon)]\mathrm{sig}\,n\epsilon$. From (1) and (4) we find the following equation for $f(\epsilon)$:

$$\epsilon_b \sum_{n=-\infty}^{\infty} J_n^2(a) k_n(\epsilon)[f_0(\epsilon+n\omega)-f(\epsilon)]=\nu(\epsilon) I_{\text{in}}, \tag{7}$$

where $k_n(\epsilon)=\nu(\epsilon)\nu_h(\epsilon+n\omega)-(-1)^n\rho(\epsilon)\rho_h(\epsilon+n\omega)\cos(\varphi/2)$, $\nu_{(h)}(\epsilon)=\text{Re}\, g_{(h)}^R(\epsilon)$, and $\rho_{(h)}(\epsilon)=\text{Re}\, f_{(h)}^R(\epsilon)$.

It is difficult to solve Eq. (7) in the general case, because of the complexity of the inelastic-collision integral I_{in} (Refs. 6, 4, and 7). However, if the tunneling scale time $\tau_b=\epsilon_b^{-1}$ is much shorter than the inelastic relaxation time τ_{in} (for quasiparticles with energies on the order of Δ_0), then it is a simple matter to solve the problem for amplitudes a which are not too small. Specifically, in this case it is possible to satisfy the condition

$$J_1^2(a)\epsilon_b \gg 1/\tau_{\text{in}}, \tag{8}$$

which means that the rate of tunneling processes involving a single photon is higher than the rate of energy relaxation. Let us assume that the frequency is in the region $|\omega-\Delta_h|\sim\Delta_0$ and that the amplitude a is not very large ($J_1^2(a)\gg J_n^2(a)$, $n\geqslant 2$). Retaining the main term in sum (7), and assuming $\epsilon<\Delta_h$, we then find

$$f(\epsilon)=f_0(\epsilon)\theta(\Delta_h-\epsilon-\omega)+[k_1(\epsilon)f_0(\epsilon+\omega)+k_{-1}(\epsilon)f_0(\epsilon-\omega)]/[k_1(\epsilon)+k_{-1}(\epsilon)]. \tag{9}$$

If we instead assume $\epsilon>\Delta_h$ we find $f(\epsilon)\simeq f_0(\epsilon)$. As the amplitude a increases, and condition (8) becomes satisfied, the distribution function thus becomes independent of a in the leading approximation. Let us consider the case in which the transition temperatures of the two superconductors are very different: $T_{c0}\ll T_{ch}$. Introducing $\Omega=\omega-\Delta_h$, we find the following result from (9) for frequencies $|\Omega|\sim\Delta_0$ ($T\ll T_{ch}$):

$$f(\epsilon)=\left\{f_0(\epsilon)\theta(-\Omega)+\left[1-2[\epsilon-\Delta c(\varphi)]\left(\left(\frac{\Omega-\epsilon}{\Omega+\epsilon}\right)^{1/2}[\epsilon+\Delta c(\varphi)]+\epsilon\right.\right.\right.$$
$$\left.\left.\left.-\Delta c(\varphi)\right)^{-1}\right]\theta(\Omega)\right\}\theta(|\Omega|-\epsilon)+f_0(\epsilon+\omega)\theta(\epsilon-|\Omega|),\quad \epsilon<\Delta_h. \tag{10}$$

Here $c(\varphi)=\cos(\varphi/2)$. Using (10), we find from (6) the following equation for the behavior of $\delta=\Delta/\Delta_0$ as a function of $w_1=(\omega-\Delta_h)/\Delta_0$ at temperatures $T\ll T_{ch}$:

$$\ln\delta+F_+(w_1,\delta)\theta(|w_1|-\delta)=0, \tag{11}$$

where the function $F_\pm(\omega,\delta)$ is given by

$$F_\pm(w,\delta)=2\int_\delta^{|w|} dx\left\{\theta(-w)\frac{1}{e^{\beta x}+1}+\theta(w)\left[\frac{x\pm c(\varphi)\delta}{x\mp c(\varphi)\delta}\left(\frac{w-x}{w+x}\right)^{1/2}\right.\right.$$
$$\left.\left.+1\right]^{-1}\right\}\frac{1}{(x^2-\delta^2)^{1/2}}. \tag{11a}$$

Here $\beta=\Delta_0/T$. It thus follows from (10) that the alternating voltage causes a decrease in the number of quasiparticles, $n(\epsilon)<n_0(\epsilon)=[1-f_0(\epsilon)]/2$, with energies on the order of Δ_0. A further consequence is the induction of a gap Δ in S. This gap can reach a value on the order of equilibrium value at $T=0$. At the same time, the flow of

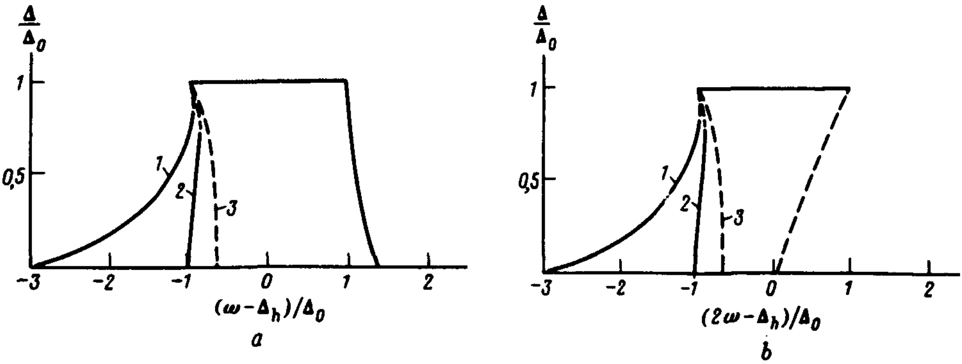

FIG. 1. Frequency dependence of the gap at amplitudes $a = \mathrm{eV}/2\omega$ which are not small (see the text proper) for various temperatures $T \gg T_{c0}$ (for the case with $T_{ch} = 10T_{c0}$ and $\varphi = 0$) for frequencies at which the effect is due to a tunneling of quasiparticles involving (a) one or (b) two photons. 1—$t = T/T_{c0} = 1$; 2—$t = 1.2$; 3—$t = 2$. The heavy lines correspond to stable solutions. [In the case $\varphi \neq 0$, the solutions differ from those shown here at (a) $(\omega - \Delta_h)/\Delta_0 \gtrsim 1$ and (b) $(2\omega - \Delta_h)/\Delta_0 > 0$.

a steady-state supercurrent becomes possible; we will come back to this subject below. At frequencies $\omega < \Delta_h$, Eq. (11) is the same, within the replacement of ω by eV/2, as the equation for the gap at steady-state voltages.[2] Figure 1(a) shows plots of $\Delta(\omega)$ found for the case $T_{ch}/T_{c0} = 10$ and $T \ll T_{ch}$. The heavy lines here show stable solutions corresponding to the maximum of the energy $U(\Delta)$ (Ref. 7, for example) or, equivalently, to the condition $\partial\Phi/\partial\Delta > 0$.

The frequency range discussed above is that in which a superconductivity may be induced as the result of a tunneling involving a single photon. It turns out that at frequencies defined by the condition $|2\omega - \Delta_h| \sim \Delta_0$ the induction of a gap in S may occur as the result of a tunneling of quasiparticles involving two photons. To see this, we assume $\omega \approx \Delta_h/2$, and we assume that the amplitude a is not large: $J_2^2(a)$, $1/(\epsilon_b\tau_{in}) \gg J_n^2(a)$, $n \geq 3$. For an energy interval large in comparison with Δ_0, i.e., $|\Delta_h - 2\omega| < \epsilon < \Delta_h - \omega$, the leading role is played by tunneling processes involving two photons, and Eq. (11) reduces to

$$\epsilon_b J_2^2(a)\{k_2(\epsilon)[f_0(\epsilon+2\omega) - f(\epsilon)] + k_{-2}(\epsilon)[f_0(\epsilon-2\omega) - f(\epsilon)] = \nu(\epsilon) I_{\text{in}}\,. \tag{12}$$

Using the model expression $I_{\text{in}} = [f(\epsilon) - f_0(\epsilon)]/\tau_{\text{in}}$ for the collision integral, we find $f(\epsilon)$ and an equation for Δ for an arbitrary value of the parameter $b = \tau_{\text{in}}\epsilon_b J_2^2(a)$. In particular, for $b \gg 1$ the solution which we find differs from (11) only in that w_1 is replaced by $w_2 = (2\omega - \Delta_h)/\Delta_0$, and the function $F_+(w_1,\delta)$ by $F_-(w_2,\delta)$. Figure 1(b) shows the solutions Δ as a function of $2\omega - \Delta_h$ (for $b \gg 1$).

When a superconductivity is induced in S, a steady-state supercurrent I_s can flow through the S_hISIS_h structure. Even at temperatures $T - T_{c0} \sim T_{c0}$, this current may become comparable in magnitude to the critical current at $T = 0$. In particular, at a low power level ($a \ll 1$) we find the dependence

$$I_s(\varphi)=I_1 \sin(\varphi/2)+I_2(\varphi), \qquad (13)$$

where $I_1=I_c(\Delta,\Delta_h)$ is the same as the expression for the critical current through an S_hIS tunnel junction with equilibrium distribution functions,[8] and

$$I_2(\varphi)=\sin(\varphi/2)(2\Delta_h\Delta/eR)\int_\Delta^{\Delta_h} d\epsilon[f(\epsilon)-f_0(\epsilon)]\frac{1}{[(\Delta_h^2-\epsilon^2)(\epsilon^2-\Delta^2)]^{1/2}},$$

where R is the resistance of the structure. It follows from (13) that in this case the quantity I_s is dominated by the first term, which leads to a critical current which, at $T_{c0}\leqslant T\ll T_{ch}$, may be essentially comparable to the equilibrium value at absolute zero, $I_c(\Delta_0,\Delta_h)$. The small supercurrent induced by the proximity effect[2,5,9] [by virtue of condition (5)] differs from (13) in that it is proportional to $\sin\varphi$.

When V has a steady-state component, the oscillating voltage can again lead to the induction of a superconductivity in S. The analysis in this case is like that outlined above. In particular, for frequencies which satisfy the conditions ω, $|n\omega-\omega_J|\gg\epsilon_b$, where $\omega_J=\text{eV}$ is the Josephson frequency of the current oscillations, we have the kinetic equation

$$\epsilon_b\sum_{n=-\infty}^{\infty} J_n^2(a)\{\nu_h(\epsilon_+ +n\omega)[f_0(\epsilon_+ +n\omega)-f(\epsilon)]+\nu_h(\epsilon_- +n\omega)$$

$$\times[f_0(\epsilon_- +n\omega)-f(\epsilon)]=1_{\text{in}}, \qquad (14)$$

where $\epsilon_\pm=\epsilon\pm\omega_J/2$. Solving (14) under condition (8), we find an equation for Δ which takes the following form in the frequency range defined by the condition $|u_1|\sim 1$, where $u_1=(\omega_J/2+\omega-\Delta_h)/\Delta_0$, with $\Delta_h-\omega_J/2\gg T$:

$$\ln\delta+F(u_1,\delta)\theta(|u_1|-\delta)=0.$$

The function $F(u_1,\delta)$ is the same as (11′) in the case $u_1<-\delta$, while at $u_1>\delta$ we have $F(u_1,\delta)=\tilde{F}(u_1/\delta)$, where

$$\tilde{F}(x)=\ln[(x^2-1)^{1/2}+x]+(\pi/2)-2x\arctan[1/(x+(x^2-1)^{1/2})].$$

At voltages $\text{eV}<2(\Delta_h-\Delta_0)$, for which we have $\Delta=0$ at $V_\sim=0$ ($T>T_{c0}$) (Ref. 2), a gap may thus be induced by single-photon processes involving quasiparticle tunneling at frequencies which satisfy the condition $\Delta_h-\Delta_0<\omega+\omega_J/2<\Delta_h+\Delta_0$.

It would be interesting to examine the effect experimentally. It might be detected by measuring the supercurrent. An induced superconductivity was recently observed[10] in an S_hISIS_h structure at nonzero steady-state voltages V.

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